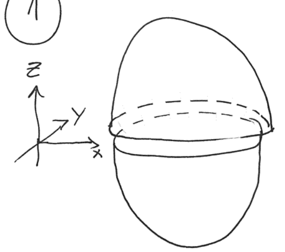
1



Tvær einsleitt hlaðnar hálf kúluskeljar

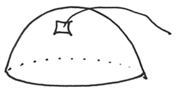
Heildarhleðsla beggja saman Q.

Flatarhleðslu þéttleiki ρ_s

$$4\pi R_0^2 \rho_s = Q \longrightarrow \rho_s = \frac{Q}{4\pi R_0^2}$$

Geisli R_0

Krafturinn milli skeljanna er samsíða samhverfuás þeirra $\hat{z}$



Reiknum kraftinn á þetta smáa flatarfrymi, ekki bara frá öllu neðra hvelinu, heldur frá allri kúluskelinni. 3. Lögmál Newtons styttir þá út innan hvelsins sem við finnum kraftinn á. Síðan þarf að heilda yfir öll flatarfrymi efra hvelsins til að finna heildarkraftinn á það.

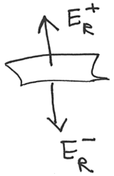
Rafsviðið fyrir utan kúluna (við yfirborðið) er

$$E_R(R_0^+) = \frac{Q}{4\pi\epsilon_0 R_0^2} = \frac{\rho_s}{\epsilon_0}$$

Rafsviðið rétt innan kúlunnar

$$E_R(R_0^-) = 0$$

þegar öll kúlan er til staðar. Ef við skoðum aðeins lítið frymi þá er rafsviðið sitt hvoru megin í andstæðar áttir ef aðeins er skoðað rafsviðið frá þessu frymi.



Frá öllum öðrum frymum snýst stefnan rafsviðsins hér ekki við.

Þegar við reiknum kraftinn á frymið frá allri kúlunni megum við ekki reikna „sjálfkraftinn" kraftinn á frymið frá því sjálfu. Til að losna við það notum við þessa hegðun sviðsins og notum ekki rafsviðið rétt utan skeljarinnar heldur meðaltal sviðsins rétt utan og innan hennar

$$\frac{1}{2}\left(E_R(R_0^+) + E_R(R_0^-)\right) = \frac{\rho_s}{2\epsilon_0} = E_R^{ave}$$

Fráhrindikraftur á efri skelina beint upp

$$dF_z = E_z^{ave} \cdot d\rho_s$$

Eða

$$F_z = \int_0^{\pi/2} \underbrace{E_R^{ave} \cdot \cos\theta}_{E_z^{ave}} \; \rho_s \; \underbrace{2\pi R_0^2 \sin\theta \, d\theta}_{ds}$$

$$= \frac{\rho_s^2}{2\epsilon_0} 2\pi R_0^2 \int_0^{\pi/2} \cos\theta \cdot \sin\theta \cdot d\theta$$

$$= \frac{\rho_s^2 \pi R_0^2}{\epsilon_0}\frac{1}{2} = \left(\frac{Q}{4\pi R_0^2}\right)^2 \frac{\pi R_0^2}{2\epsilon_0} = \frac{Q^2}{32\pi\epsilon_0 R_0^2}$$

Sjá Ananda DasGupta, Eur. J. Phys. 28, 705 (2007)

doi: 10.1088/0143-0807/28/4/010

(2)

$$V(\bar{R}) = A \frac{e^{-\lambda R}}{R}$$ fyrir einhverja hleðsludreifingu

Dæmið þarf að leysa í skrefum og fyrstu niður-stöður getur þurft að endurbæta!

Coulomb-lögmálið segir okkur að heildarhleðslan hlýtur að hverfa $Q=0$, ef ekki þá yrði að vera þáttur $\frac{1}{R}$ í mættinu, ánskýlingar.

Aðfelluformið þegar $R \to 0$, $V(R) \xrightarrow[R\to 0]{} \frac{A}{R}$, bendir til þess að inni í samfelldri hleðsludreifingu sé punkthleðsla með hleðslu $q = 4\pi\epsilon_0 A$

fyrir $R \neq 0$ er rafsviðið

$$\bar{E} = -\bar{\nabla} V(R)$$

eða hér

$$E_R = -\frac{\partial V}{\partial R} = -A\left\{-\frac{1}{R^2} - \frac{\lambda}{R}\right\} e^{-\lambda R}$$

$$\to \bar{E} = -A\,\hat{a}_R \left\{-\frac{1}{R^2} - \frac{\lambda}{R}\right\} e^{-\lambda R}$$

Til þess að finna hleðsludreifinguna alla (fyrir utan punkt-þáttinn) notum við

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

Í kúluhnitum hér

$$-\epsilon_0 \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 \frac{\partial}{\partial R} V\right) = \rho(R) \quad \text{ef } R \neq 0$$

$$\to \rho(R) = -A\epsilon_0 \frac{\lambda^2 e^{-\lambda R}}{R} \quad \text{þ. } R \neq 0$$

Skoðum heildarhleðsluna frá þessum þætti

$$\int dv\, \rho(R) = \left(4\pi \int_0^\infty R^2 dR\, \frac{e^{-\lambda R}}{R}\right)\cdot(-A\epsilon_0\lambda^2)$$

$$= -4\pi\epsilon_0 A \int_0^\infty x\,dx\, e^{-x} = -4\pi\epsilon_0 A$$

$$= -q$$

Því þurfum við í viðbót við $\rho(R) = -A\epsilon_0 \frac{\lambda^2 e^{-\lambda R}}{R}$ þ. $R \neq 0$ eina punkthleðslu í $R=0$ með stærð $q = 4\pi\epsilon_0 A$

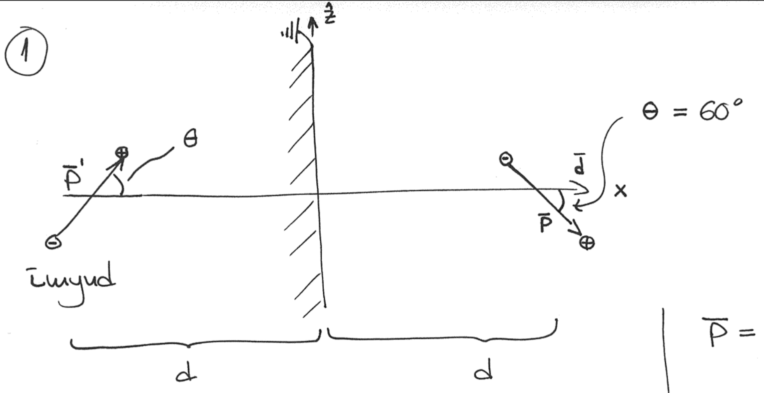
Heildarhleðslan er $Q = 0$. Það er líka gaman að sjá að $\oint E \cdot d\bar{s} \to 0$ þegar notað er kúluyfib. með vaxandi $R \to \infty$ og E sem við fundum.

Auka

Punkthleðsluna má tákna við þrívítt δ-fall

$$\rho_p(\bar{r}) = \epsilon_0 A\, 4\pi\, \delta^{(3)}(\bar{r}) = \epsilon_0 A\, 4\pi \frac{1}{R^2}\delta(R)\frac{1}{4\pi}$$

1



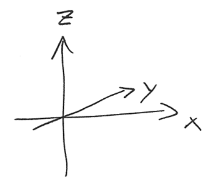
↑ Ímynd til p.a. losna við leiðandi slættuna úr dømunum

Tvístangið $\bar p$ virðist vera í sviði $\bar E'$ frá tvístangi $\bar p'$

$$\bar E' = \frac{p}{4\pi\epsilon_0 (2d)^3}\left(\hat x\, 2\cos\theta - \hat z \sin\theta\right)$$

$$\bar p = p(\cos\theta, 0, -\sin\theta)$$

$$\bar p' = p(\cos\theta, 0, \sin\theta)$$



2

c) finna $\bar\tau_E$ á $\bar p$

$$\bar\tau = \bar p \times \bar E' = p(\cos\theta, 0, -\sin\theta) \times (2\cos\theta, 0, -\sin\theta)\frac{p}{4\pi\epsilon_0 (2d)^3}$$

$$= \hat y \frac{p}{4\pi\epsilon_0 (2d)^3} \cos\theta \sin\theta$$

$$= \hat y \frac{p}{4\pi\epsilon_0 (2d)^3} \frac{\sqrt{3}}{4} = \hat y \frac{\sqrt{3}\, p}{128\pi\epsilon_0 d^3}$$

a) Finna orku kerfisins. Alment hótum við

$$\bar E' = \frac{p}{4\pi\epsilon_0 R^3}\left(\hat a_{R'}\, 2\cos\theta' + \hat a_{\theta'} \sin\theta'\right)$$

í hnitakerfi ~~miðað~~ við $\bar p'$.

$$2\cos\theta'\, \hat a_{R'} + \sin\theta'\, \hat a_{\theta'} = 3\cos\theta'\, \hat a_{R'} - \left(\cos\theta'\, \hat a_{R'} - \sin\theta'\, \hat a_{\theta'}\right)$$

$$= 3\cos\theta'\, \hat a_{R'} - \hat a_{z'}$$

3

og tí

$$\bar E' = \frac{1}{4\pi\epsilon_0} \frac{1}{R^3}\left(3(\bar p \cdot \hat R)\hat R - \bar p'\right)$$

áni þess at hnit partiað koma fyri.

Vegna vágisins á $\bar p$ hótum við stöðuorkuna

$U = -\bar p \cdot \bar E'$, allment fæst tí

$$U = \frac{1}{4\pi\epsilon_0} \frac{1}{R^3}\left(\bar p \cdot \bar p' - 3(\bar p \cdot \hat R)(\bar p' \cdot \hat R)\right)$$

Í okkar til felli fæst

$$\bar p \cdot \bar p' = p^2(\cos^2\theta - \sin^2\theta)$$

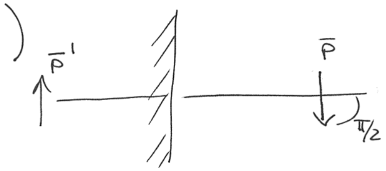
$$\bar p' \cdot \hat R = p\cos\theta$$

$$\bar p \cdot \hat R = p\cos\theta$$

4

Og tí

$$U = -\frac{1}{4\pi\epsilon_0}\frac{p^2}{(2d)^3}\left(1 + \cos^2\theta\right)$$



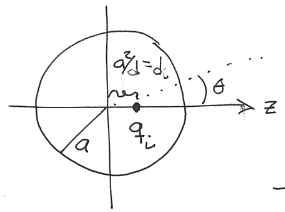
Lægst orka fæst fyri $\underline{\theta = \frac{\pi}{2}}$

d) tí mun tvístangið leita í javnvágis stöðuna $\pi/2 = \theta$ og svingast um hana

$$\Delta U = U(\theta) - U(\tfrac{\pi}{2}) = -\frac{p^2\cos^2\theta}{4\pi\epsilon_0 (2d)^3}$$

b) Eg í raun gert ráð fyri d í tvístanginum $\to 0$ og hef tí sleppt heildar kraftinum á tvístangið ....

(2) (1)



Sama dæmi og Ex - 4-5 á bls. 171 í bók

$$\rightarrow \rho_s = -\frac{q(d^2-a^2)}{4\pi a(a^2+d^2-2ad\cos\theta)^{3/2}}$$

↑ pólhornið

Heildar spanhleðslan

$$Q_i = a^2\int_0^{2\pi} d\phi \int_0^{\pi} \rho_s \sin\theta \cdot d\theta = -\frac{a}{d} q = q_i$$

c) Heildarorkan í eV ef $q = e$, $d = 2Å$, $a = 1Å$
Einfaldast að nota hleðslu myndina fyrir tvær punkthleðslur q og q_i

(2)

$$E_{tot} = V_q \cdot q_i = -\frac{1}{4\pi\epsilon_0}\frac{|q\cdot q_i|}{(d-d_i)}$$

$$= -\frac{1}{4\pi\epsilon_0}\frac{q^2 \frac{a}{d}}{\left(d-\frac{a^2}{d}\right)} = -\frac{q^2}{4\pi\epsilon_0}\frac{a}{d^2-a^2}$$

$$= -\frac{e^2}{4\pi\epsilon_0}\frac{a}{d^2-a^2}$$

Notum til stöðlunar Rydberg orkuna $E_R = \frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2$
og Bohr geislann $a_0 = \frac{\hbar^2}{m}\frac{4\pi\epsilon_0}{e^2}$

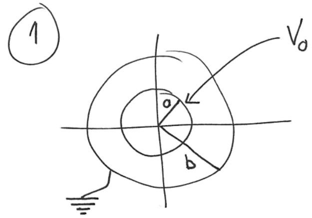
$$E_{tot} = -\frac{e^2}{4\pi\epsilon_0 a_0}\frac{\left(\frac{a}{a_0}\right)}{\left(\frac{d}{a_0}\right)^2-\left(\frac{a}{a_0}\right)^2} = -2E_R\cdot\frac{\left(\frac{a}{a_0}\right)}{\left(\frac{d}{a_0}\right)^2-\left(\frac{a}{a_0}\right)^2}$$

(3)

Nú er $E_R \sim 13.6$ eV og $a_0 \sim 0{,}53$ Å

$$E_{tot} = -2\cdot 13.6\ \text{eV}\cdot\frac{\left(\frac{1}{0{,}53}\right)}{\left(\frac{2}{0{,}53}\right)^2-\left(\frac{1}{0{,}53}\right)^2} \sim -4.8\ \text{eV}$$

(1) (1)



Finna rýmdina C með lausn á jöfnu Laplace

Kúlusamhverfa $\rightarrow$ $\nabla^2 V = 0$

verður $\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) = 0$

Bein heildum

$$\frac{d}{dR}\left(R^2\frac{dV}{dR}\right) = 0 \rightarrow R^2\frac{dV}{dR} = C_1$$

$$\rightarrow \frac{dV}{dR} = \frac{C_1}{R^2} \rightarrow V(R) = -\frac{C_1}{R} + C_2$$

Jaðarskilyrði eru $V(a) = V_0$ og $V(b) = 0$

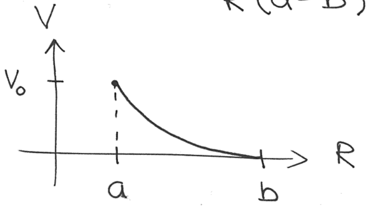
$$\rightarrow -\frac{C_1}{a} + C_2 = V_0 \quad \text{og} \quad -\frac{C_1}{b} + C_2 = 0$$

$$\begin{pmatrix} -\frac{1}{a} & 1 \\ -\frac{1}{b} & 1 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} V_0 \\ 0 \end{pmatrix} \to \quad C_1 = \frac{abV_0}{a-b}, \quad C_2 = \frac{aV_0}{a-b}$$

$$\to V(R) = -\frac{abV_0}{R(a-b)} + \frac{aV_0}{a-b} = \frac{aV_0(-b+R)}{R(a-b)}$$

$$= \frac{aV_0(R-b)}{R(a-b)}$$

sem greinilega uppfyllir jaðarskilyrðin



$$\bar{E} = -\bar{\nabla} V = -\hat{a}_R \frac{\partial V}{\partial R}$$

$$= -\frac{abV_0}{R^2(a-b)}\hat{a}_R$$

$$= \frac{abV_0}{R^2(b-a)}\hat{a}_R$$

Yfirborðshleðslan á innri skelinni er

$$\rho_s(a) = \epsilon_0 E_R(a) = \frac{\epsilon_0 abV_0}{a^2(b-a)} = \frac{\epsilon_0 bV_0}{a(b-a)}$$

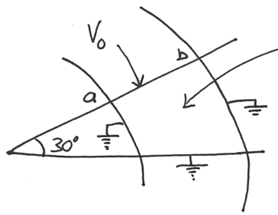
einsleit og óháð θ og φ

$$\to Q = \frac{4\pi\epsilon_0 bV_0 a^2}{a(b-a)} = \frac{4\pi\epsilon_0 ab V_0}{(b-a)}$$

er heildarhleðslan á skelinni

Rýmdin er
$$C = \frac{Q}{V_0} = \frac{4\pi\epsilon_0 ab}{(b-a)}$$

2



Finna hér V, $\bar{E}$ og $\bar{D}$

Sívalnings samhverfa

$$\frac{1}{r}\partial_r\left(r\partial_r V(r,\varphi)\right) + \frac{1}{r^2}\partial_\varphi^2 V(r,\varphi) = 0$$

Jafnan er aðgreinanleg

$$V(r,\varphi) = R(r)\Phi(\varphi)$$

$$\frac{d^2\Phi}{d\varphi^2} + k^2\Phi(\varphi) = 0$$

$$\frac{d^2}{dr^2}R + \frac{1}{r}\frac{d}{dr}R - \frac{k^2}{r^2}R = 0$$

$k \neq 0$, því lausnin

$R(r) \sim C_1 \ln(r) + C_2$

getur ekki uppfyllt jaðarst. $R(a)=0$ og $R(b)=0$

Lausnin fyrir $k^2>0$ er ekki góð því við getum ekki uppfyllt jaðarst. með $R(r) = A_n r^n + B_n r^{-n}$

<u>Skoðum $k^2<0$</u>

Setjum $k^2 = -\kappa^2$ með $\kappa \in \mathbb{R}$ þá fæst

$$\Phi(\varphi) = A_\kappa \cosh(\kappa\varphi) + B_\kappa \sinh(\kappa\varphi)$$

$$R(r) = C_\kappa r^{i\kappa} + D_\kappa r^{-i\kappa}$$

en $r^{i\kappa} = e^{i\kappa \ln r}$

$$\to R(r) = C_\kappa \cos(\kappa \ln(r)) + D_\kappa \sin(\kappa \ln(r))$$

fall með sveiflum, því ætti að vera hægt að uppfylla jaðarst. með því

Reynum $C_K = 0$, og lausn

$$V_K(r,\varphi) = B_K \sin\left(\kappa \ln\left(\frac{r}{a}\right)\right) \sinh(\kappa\varphi)$$

$$V_K(b,\varphi) = 0 \quad \to \quad \sin\left(\kappa \ln\left(\frac{b}{a}\right)\right) = 0$$

eða $\kappa \ln\left(\frac{b}{a}\right) = n\pi, \quad n = 1, 2, 3, \dots$

$$\kappa = \frac{n\pi}{\ln\left(\frac{b}{a}\right)}$$

Verðum að uppfylla líka

$V_K(r, 30^\circ) = V_0$ fyrir öll r

$= V_K\left(r, \frac{\pi}{6}\right)$

$$V(r,\varphi) = \sum_{n=1}^{\infty} B_{\kappa_n} \sin\left(\kappa_n \ln\left(\frac{r}{a}\right)\right) \sinh(\kappa_n\varphi)$$

$$V\left(r,\frac{\pi}{6}\right) = \sum_{n=1}^{\infty} B_{\kappa_n} \sin\left(\kappa_n \ln\left(\frac{r}{a}\right)\right) \sinh\left(\kappa_n \frac{\pi}{6}\right) = V_0$$

$$V_0 \int_a^b r\,dr \sin\left(\kappa_p \ln\left(\frac{r}{a}\right)\right) = \sum_{n=1}^{\infty} \sinh\left(\kappa_n \frac{\pi}{6}\right) B_{\kappa_n} \int_a^b r\,dr \sin\left(\kappa_n \ln\left(\frac{r}{a}\right)\right) \cdot \sin\left(\kappa_p \ln\left(\frac{r}{a}\right)\right)$$

$$= \sinh\left(\kappa_p \frac{\pi}{6}\right) B_{\kappa_p} \int_a^b r\,dr \sin^2\left(\kappa_p \ln\left(\frac{r}{a}\right)\right)$$

$$\to B_{\kappa_p} = \frac{V_0 \int_a^b r\,dr \sin\left(\kappa_p \ln\left(\frac{r}{a}\right)\right)}{\sinh\left(\kappa_p \frac{\pi}{6}\right) \int_a^b r\,dr \sin^2\left(\kappa_p \ln\left(\frac{r}{a}\right)\right)}$$

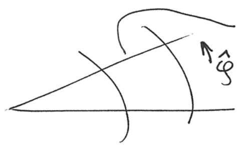
$$V(r,\varphi) = \sum_{n=1}^{\infty} B_{\kappa_n} \sin\left(\kappa_n \ln\left(\frac{r}{a}\right)\right) \sinh(\kappa_n\varphi)$$

$$\bar{E} = -\bar{\nabla} V(r,\varphi) = \hat{a}_r \frac{\partial V}{\partial r} + \hat{a}_\varphi \frac{\partial V}{r\,\partial\varphi}$$

$$= -\sum_{n=1}^{\infty} B_{\kappa_n} \frac{\kappa_n \cos\left(\kappa_n \ln\left(\frac{r}{a}\right)\right)}{r} \sinh(\kappa_n\varphi) \cdot \hat{a}_r$$

$$- \sum_{n=1}^{\infty} B_{\kappa_n} \sin\left(\kappa_n \ln\left(\frac{r}{a}\right)\right) \cosh(\kappa_n\varphi)\, \kappa_n \frac{\hat{a}_\varphi}{r}$$

$\bar{D} = \epsilon_0 \bar{E}$



Hleðsluþéttleiki

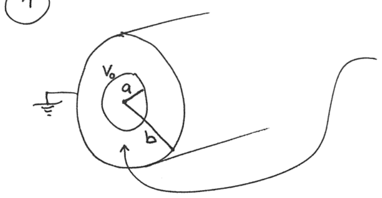
$$\rho_s = \epsilon_0 E_{\varphi}^{(30^\circ)} = -\epsilon_0 \sum_{n=1}^{\infty} B_{\kappa_n} \sin\left(\kappa_n \ln\left(\frac{r}{a}\right)\right) \cdot \cosh\left(\kappa_n \frac{\pi}{6}\right) \frac{\kappa_n}{r}$$

Þá þarf að finna heildar hleðsluna á lengd

$$\int_a^b \rho_s(r)\,dr \frac{L_z}{L_z} = Q'$$

Rýmdin er þá $C' = \frac{Q'}{V_0}$
(á lengd)

Þetta dæmi er nokkuð óvenjulegt, því hér þarf „lotubundin"-lausnarföll í r-áttina í stað φ-áttar. Þetta er ekki heppilegt prófdæmi, en það er gott til þess að minna okkur á að k-in í $k_r^2 + k_\varphi^2 = 0$ geta hvort fyrir sig verið þver- eða rauntölur, en ekki samtímis sömu tegundar. Það þarf að sannreyna að $\sin(k_n \ln(\frac{r}{a}))$ sé hornréttur fullkominn grunnur. Ég athugaði leiðnina, en sá að töflugildi fyrir leiðni getur ekki verið alls kostar rétt.



$\sigma = \frac{m}{r} + k$

Spennumunur V_0 milli sívalninga

Finna R, $\bar{J}$ og $\bar{E}$

Ef leiðarstraumurinn er I (enn óþekkt) þá er

$$\bar{J} = \frac{I}{2\pi r L}\hat{a}_r$$

Við höfum $\bar{J} = \sigma\bar{E}$ eða $\bar{E} = \frac{\bar{J}}{\sigma} = \frac{I}{2\pi L(m+kr)}\hat{a}_r$

$$V_0 = -\int_b^a \bar{E}\cdot d\bar{l} = -\int_b^a \frac{I\,dr}{2\pi L(m+kr)} = \frac{I}{2\pi L k}\ln\left\{\frac{m+kb}{m+ka}\right\}$$

og því

$$R = \frac{V_0}{I} = \frac{1}{2\pi L k}\ln\left\{\frac{m+kb}{m+ka}\right\}$$

Einnig til að fá leiðarstrauminn

$$I = \frac{V_0}{R} = 2\pi L k V_0\left\{\ln\left(\frac{m+kb}{m+ka}\right)\right\}^{-1}$$

og því

$$\bar{E} = \frac{I}{2\pi L(m+kr)}\hat{a}_r = \frac{k V_0 \hat{a}_r}{(m+kr)\ln\left(\frac{m+kb}{m+ka}\right)}$$

og

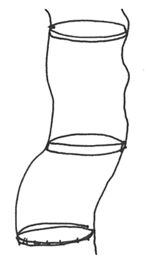
$$\bar{J} = \sigma\bar{E} = \frac{kV_0\hat{a}_r}{r\ln\left(\frac{m+kb}{m+ka}\right)}$$

Viðbót við skiladæmi

Um einsleitan leiðara með föstu þversniði gildir

$$R = \frac{l}{\sigma A}$$

Ef við getum „sneitt" niður leiðara með breytilegt þversnið og ~~eðlis~~leiðni þ.a. straumþéttleikinn sé jafn innan hverrar sneiðar, sem þýðir t.d. að J er hornrétt á sneiðarnar og σ getur aðeins breyst þvert á sneið þá má leggja saman viðnám sneiðanna



$$R = \sum_{i=1}^{n} R_i = \sum_{i=1}^{n}\frac{dl_i}{\sigma_i A_i}$$

2

Sem yfirfæra má í heildi (ferilheildi!)

$$R = \int_C \frac{dl}{\sigma A} \qquad (*)$$

því fengist

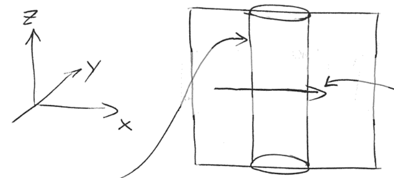
$$R = \int_a^b \frac{dr}{(m+kr)2\pi L} = \frac{1}{2\pi L k}\ln\left\{\frac{m+kb}{m+ka}\right\}$$

fyrir sívalningsleiðarann. Hér er mikilvægt að σ er aðeins fall af r, og að (almennt) R_i leggjast saman. Íhugið hvernig heildið væri sett upp fyrir G. Hafið í huga hve heildisjafnan $(*)$ er sterklega ~~tet~~ mörkuð. Myndin mín af leiðara er dæmi um leiðara sem ekki væri hægt að nota $(*)$ á!

3

P.5-23

hornréttur stór efniskubbur með leiðni σ



$$\bar J = \hat a_x J_0$$

borað gat með geisla b

finna nýjan straumþéttleika $\bar J'$

Gerum ráð fyrir að $\bar J = \sigma \bar E$

$\rightarrow \frac{\bar J}{\sigma}$ og $\bar E$ er lýst á sama hátt

$\rightarrow$ Finnum mættisfall fyrir $\bar E$ (þ.e.a.s. V) og notum síðan að $\bar J = -\sigma \bar\nabla V$

Leysum gátuna með sívalningshnitum þ.a. z-ás þeirra liggi eftir z-ásnum í upprunalegu kartísku hnitunum

4

Almenna lausnin fyrir V í kubbnum er (þegar gert er ráð fyrir lotubundinni lausn í φ-stefnu)

$$V(r,\varphi) = \sum_{n=0}^{\infty}\left(A_n r^n + B_n r^{-n}\right)\left(C_n\cos(n\varphi) + D_n \sin(n\varphi)\right)$$

Jaðarskilyrði

* Samhverfa um z-x-sléttu, $V(r,\varphi) = V(r,-\varphi)$

$\rightarrow D_n = 0$

* þegar $r \rightarrow \infty$ stefnir straumurinn á sitt fyrra gildi

$V(r,\varphi) \xrightarrow[r\to\infty]{} -\frac{J_0}{\sigma} r\cos\varphi$ því þá fæst

$$\bar J = J_0 \bar\nabla(r\cos\varphi) = J_0\left(\hat a_r \cos\varphi - \hat a_\varphi \sin\varphi\right) = J_0 \hat a_x$$

5

og því verður að gilda

Engir viðvörir fastar í lausn $\rightarrow A_0 = B_0 = 0$

$$\left.\begin{array}{l} A_n = 0 \\ C_n = 0 \end{array}\right\} \text{fyrir } n > 1$$

höfum því

$$V(r,\varphi) = A_1C_1 r\cos\varphi + \sum_{n=1} C_n B_n r^{-n}\cos(n\varphi)$$

$$\text{með } A_1C_1 = -\frac{J_0}{\sigma}$$

Straumurinn verður að hverfa í gatinu $\rightarrow J(b,\varphi) = 0$

$\rightarrow \left.\frac{\partial V}{\partial r}\right|_{r=b} = 0 \rightarrow$ í seinni summunni geta aðeins verið liðir með $n=1$

$$\rightarrow V(r,\varphi) = -\frac{J_0}{\sigma} r\cos\varphi + \frac{B_1C_1}{r}\cos\varphi$$

og $\left(-\frac{J_0}{\sigma} - \frac{B_1C_1}{b^2}\right)\cos\varphi = 0$ fyrir öll φ

$$\rightarrow B_1C_1 = -b^2\frac{J_0}{\sigma}$$

Svo lausnin er

$$V(r,\varphi) = -\frac{J_0}{\sigma}\left(r + \frac{b^2}{r}\right)\cos\varphi$$

Finnum $\bar{J} = -\sigma\bar{\nabla}V(r,\varphi)$

$$= -\sigma\left\{\hat{a}_r\frac{\partial}{\partial r} + \hat{a}_\varphi\frac{1}{r}\frac{\partial}{\partial\varphi}\right\}V(r,\varphi)$$

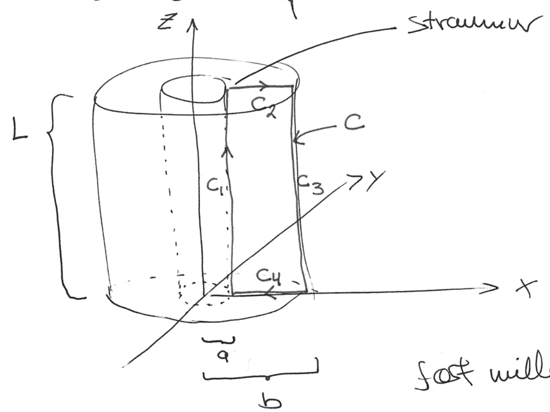
$$= \hat{a}_r\left\{J_0\left(1-\frac{b^2}{r^2}\right)\cos\varphi\right\} - \hat{a}_\varphi\left\{J_0\left(1+\frac{b^2}{r^2}\right)\sin\varphi\right\}$$

$$= J_0\left\{\hat{a}_r\cos\varphi - \hat{a}_\varphi\sin\varphi\right\} - \frac{J_0b^2}{r^2}\left\{\hat{a}_r\cos\varphi + \hat{a}_\varphi\sin\varphi\right\}$$

$$= \hat{a}_x J_0 - \frac{J_0b^2}{r^2}\left\{\hat{a}_r\cos\varphi + \hat{a}_\varphi\sin\varphi\right\}$$ utan sívalnings $r>b$

og $\bar{J} = 0$ innan sívalnings þ. $r<b$

1 Samása kapall



Reikna heildar segulflæðið milli leiðaranna

Lögmál Amperes sýnir okkur að sama segulsvið fæst milli sívalninganna og þá einum vír með I sem liggi á z-ás

→ Notum Ex-6-4 til að reikna $\bar{A}$

$$\bar{A} = \hat{a}_z\frac{\mu_0 I}{4\pi}\int_0^L\frac{dz'}{\sqrt{r^2+(z')^2}} = \hat{a}_z\frac{\mu_0 I}{4\pi}\left\{\ln\left(z'+\sqrt{(z')^2+r^2}\right)\right\}\Big|_0^L$$

$$\bar{A} = \hat{a}_z\frac{\mu_0 I}{4\pi}\left[\ln\left(\frac{L+\sqrt{L^2+r^2}}{r}\right)\right]$$

Heildar segulflæðið er

$$\Phi_B = \oint_{C=C_1+C_2+C_3+C_4}\bar{A}\cdot d\bar{l} = \oint_{C_1}\bar{A}\cdot d\bar{l} + \oint_{C_3}\bar{A}\cdot d\bar{l}$$

því á C_2 og C_4 er $\bar{A}$ hornrétt á $\bar{l}$

Í fjarlægð r er A fasti

$$\rightarrow \Phi_B = L\,A(a) - L\,A(b)$$

$$= \frac{\mu_0 I L}{4\pi}\left\{\ln\left(\frac{L+\sqrt{L^2+a^2}}{a}\right) - \ln\left(\frac{L+\sqrt{L^2+b^2}}{b}\right)\right\}$$

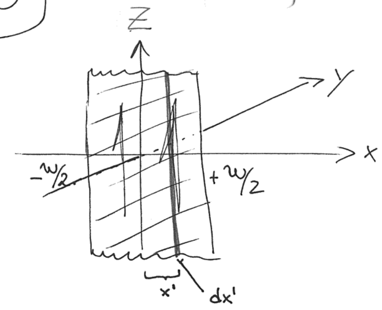
3

$$\Phi_B = \frac{\mu_0 I L}{4\pi}\left\{\ln\left(\frac{b\left[L+\sqrt{L^2+a^2}\right]}{a\left[L+\sqrt{L^2+b^2}\right]}\right)\right\}$$

Ef nú $L \gg a, b$ fengist

$$\Phi_B \to \frac{\mu_0 I L}{4\pi}\ln\left(\frac{b}{a}\right)$$

6-10 4



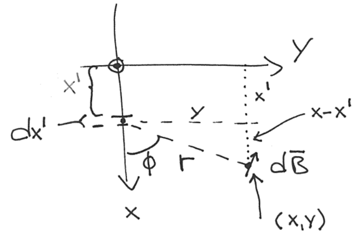
$$\bar{J}_s = \hat{a}_z J_{s0}$$

Finna $\bar{B}$ utan þynnunar

Þynnan er sett saman úr vírum með breidd dx' og straum

$dI = J_{s0}\cdot dx'$, hver þeirra gefur

$$\bar{B}_\phi = \hat{a}_\phi \frac{\mu_0 dI}{2\pi r}$$



$$\hat{a}_\phi = \left(-\hat{a}_x \sin\phi + \hat{a}_y \cos\phi\right)$$
$$= \left(-\hat{a}_x \frac{y}{r} + \hat{a}_y \frac{(x-x')}{r}\right)$$
$$r = \sqrt{(x-x')^2 + y^2}$$

5

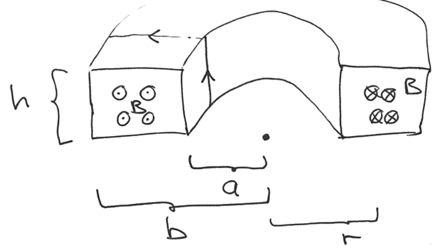
Nú þarf að summa upp alla „vírana"

$$B_x = -\frac{\mu_0 J_{s0}\, y}{2\pi}\int_{-w/2}^{w/2} dx' \left\{\frac{1}{(x-x')^2+y^2}\right\}$$
$$= -\frac{\mu_0 J_{s0}}{2\pi}\left\{\arctan\left(\frac{x+\frac{w}{2}}{y}\right) - \arctan\left(\frac{x-\frac{w}{2}}{y}\right)\right\}$$

$$B_y = \frac{\mu_0 J_{s0}}{2\pi}\int_{-w/2}^{w/2} dx' \left\{\frac{(x-x')}{(x-x')^2+y^2}\right\}$$
$$= \frac{\mu_0 J_{s0}}{4\pi}\ln\left\{\frac{\left(x+\frac{w}{2}\right)^2+y^2}{\left(x-\frac{w}{2}\right)^2+y^2}\right\}$$

$\bar{B} = (B_x, B_y, 0)$

1 1



Leyfi mér að athuga kleinuhring með kassalaga þverskurði

Sjá Ex. 6-14 til þess að reikna segulflæðisviðið

b) $\bar{B} = \hat{a}_\phi \frac{\mu_0 N I}{2\pi r}$, $\bar{H} = \frac{1}{\mu_0}\bar{B} = \frac{I N}{2\pi r}\hat{a}_\phi$ a)

c) $$\Phi_B = \int \bar{B}\cdot d\bar{s} = \frac{\mu_0 N I}{2\pi}\int_a^b \frac{dr'}{r'}\int_0^h dz$$
$$= \frac{\mu_0 N I h}{2\pi}\ln\left(\frac{b}{a}\right)$$

2

d) Orkuþéttleikinn

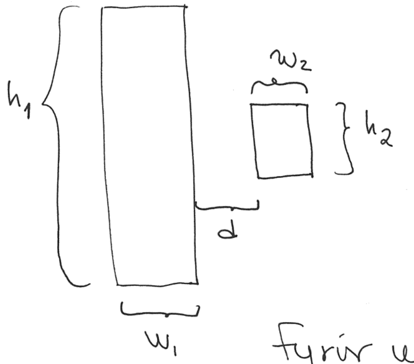
$$w_m = \frac{1}{2}\mu_0 H^2 = \frac{1}{8}\mu_0\left(\frac{NI}{\pi r}\right)^2$$

$$\to W_m = \frac{N^2 I^2}{8\pi^2}\mu_0 \int_a^b \frac{dr'}{r'} \int_0^{2\pi} d\phi' \int_0^h dz'$$

$$= \frac{\mu_0}{4\pi} N^2 I^2 h \ln\left(\frac{b}{a}\right)$$

3

6-40 finna víxlspan fyrir tvær lykkjur



Ef $h_1 >> h_2$

$h_2 > w_2 > d$

Gerum ráð fyrir að stóru lykkjuna megi nálga sem tvo samsíða víra með straum í sitt hvora áttina

Fyrir wír var segulflæðisviðið

$$\bar{B} = \hat{a}_\phi \frac{\mu_0 I}{2\pi r}$$

4

Því er flæðið í gegnum minni lykkjuna

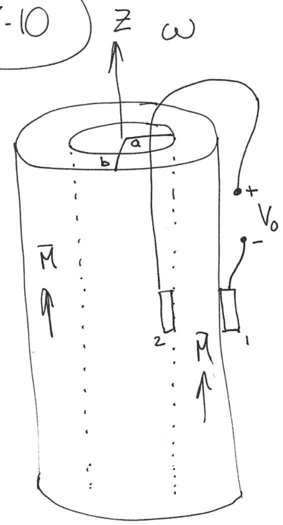
$$\Phi = \frac{\mu_0 I}{2\pi} \int_0^{h_2} dz \int_0^{w_2} dx \left\{ \frac{1}{d+x} - \frac{1}{w_1+d+x} \right\}$$

$$= \frac{\mu_0 I}{2\pi} h_2 \ln\left\{ \left(\frac{w_2+d}{d}\right) \cdot \left(\frac{w_1+d}{w_1+w_2+d}\right) \right\}$$

$$\to L = \frac{\Phi}{I} = \frac{\mu_0 h_2}{2\pi} \ln\left\{ \left(\frac{w_2+d}{d}\right)\left(\frac{w_1+d}{w_1+w_2+d}\right) \right\}$$

1

7-10



$\bar{M} = \hat{a}_z M_0$

$\mu_r = 5000$

$\sigma = 10^7$ S/m

a) finna $\bar{H}$ og $\bar{B}$ í seglinum

$$\bar{M} = \chi_m \bar{H} \to \bar{H} = \frac{\bar{M}}{\chi_M}$$

$$\mu_r = 1 + \chi_M \to \boxed{\bar{H} = \frac{\bar{M}}{\mu_r - 1}}$$

$$\bar{H} = \frac{\hat{a}_z M_0}{\mu_r - 1}$$

$$\bar{B} = \mu_0 \mu_r \bar{H}$$

$$\bar{B} = \mu_0 \mu_r \frac{\hat{a}_z M_0}{\mu_r - 1}$$

$$= \hat{a}_z \mu_0 M_0 \frac{\mu_r}{\mu_r - 1}$$

2

b) Finna V_0 í opinni rás — bursti 1 og 2

Milli burstanna er leiðari, þvert á hann ferðast segulflæðisviðið $\bar{B}$

$$V_{21} = \int_1^2 (\bar{u} \times \bar{B}) \cdot d\bar{l}$$

$\bar{B} = B\hat{a}_z$ og $\bar{u} = \omega r \hat{a}_\phi$

$$\to V_{21} = V_0 = \int_b^a \left(\hat{a}_\phi \omega r \times \hat{a}_z B\right) \cdot \hat{a}_r \, dr$$

$$= -\frac{\omega B}{2} r^2 \Big|_b^a = -\frac{\omega B}{2}(b^2 - a^2) = -\frac{\omega \mu_0 M_0 \mu_r}{2(\mu_r - 1)}(b^2 - a^2)$$

hægri h. regla

3

c) Straumurinn í lokaðri rás?

Lokaða rásin er bara með viðnám milli burstanna um segulsívalninginn. Snúningurinn veldur sömu íspennu og áður

$$\mathcal{E} = \int_1^2 (\bar{u} \times \bar{B}) \cdot d\bar{l} = -\frac{\omega B}{2}(b^2 - a^2) \quad \text{hér}$$

þessi íspenna rekur straum um viðnámið R (sem við eigum eftir að finna) i. Spennufallið um R vegna i verður að jafna út Σ.

$$\mathcal{E} - iR = 0$$

$$\to i = \frac{\mathcal{E}}{R} = -\frac{\omega B}{2R}(b^2 - a^2)$$

átt straumsins má snúa við með stefnu ω

4

Finnum R

Eðlisleiðnin σ er há $\to$ allur sívalningurinn leiðir

$$\bar{J}(r) = \frac{i\hat{a}_r}{A(r)h} = \frac{i\hat{a}_r}{2\pi r h}, \quad \bar{E}(r)\sigma = \bar{J}(r)$$

$$\to \bar{E}(r) = \frac{\hat{a}_r \, i}{2\pi r h \sigma}$$

Spennufallið yfir sívalninginn er þá

$$V = \int_b^a E(r)\,dr = -\frac{i}{2\pi h \sigma}\ln\left(\frac{b}{a}\right) = -iR$$

$$\to R = \frac{1}{2\pi h\sigma}\ln\left(\frac{b}{a}\right)$$

og því

$$\boxed{i = -\frac{\omega B (b^2 - a^2)}{2\ln\left(\frac{b}{a}\right)} 2\pi h \sigma}$$

5

7-11

Leiða út $\bar{\nabla} \cdot \bar{D} = \rho$ og $\bar{\nabla} \cdot \bar{B} = 0$

frá $\bar{\nabla} \times \bar{E} = -\partial_t \bar{B}$ og $\bar{\nabla} \times \bar{H} = \bar{J} + \partial_t \bar{D}$

ásamt $\partial_t \rho + \bar{\nabla} \cdot \bar{J} = 0$

Vitum að almennt gildir $\bar{\nabla} \cdot (\bar{\nabla} \times \bar{A}) = 0$

byrjum með Ampere-Maxwell lögmálið

$$\underbrace{\bar{\nabla} \cdot (\bar{\nabla} \times \bar{H})}_{=0} = \bar{\nabla} \cdot \bar{J} + \partial_t (\bar{\nabla} \cdot \bar{D}) = 0$$

berum saman við $\bar{\nabla} \cdot \bar{J} = -\partial_t \rho$

$$\to \partial_t (\bar{\nabla} \cdot \bar{D} - \rho) = 0 \to \boxed{\bar{\nabla} \cdot \bar{D} = \rho + C_1}$$

6

byrjum með Faraday lögmálið

$$\bar{\nabla} \times \bar{E} = -\partial_t \bar{B}$$

$$\underbrace{\bar{\nabla} \cdot (\bar{\nabla} \times \bar{E})}_{=0} = -\partial_t (\bar{\nabla} \cdot \bar{B}) = 0$$

$$\hookrightarrow \boxed{\bar{\nabla} \cdot \bar{B} = C_2}$$

Nú þarf að samfæra sig um að C_1 og C_2 hverfi án þess að nota niðurstöðuna. Við vitum að orku-innihald rafsviðs getur bilað ef við stöðum punktlind og markgildið $r \to 0$. Ef þessir fastar hverfa ekki leiða þeir til orkuvandræða þegar $r \to \infty$

P7-28

Á svæði án hleðslu gildir (og strauma)

$\bar{\nabla} \cdot \bar{E} = 0$, því er hægt að skilgreina

$\bar{A}_e$ þ.a. $\bar{E} = \bar{\nabla} \times \bar{A}_e$. Gera ráð fyrir lotubundinni $-i\omega t$

a) Finna tengslu $\bar{H}$ og $\bar{A}_e$

b) Sýna að $\bar{A}_e$ sé lausn á óhliðraðri jöfnu Helmholtz

Maxwell jöfnur

$$\bar{\nabla} \times \bar{E} = -i\omega\mu \bar{H}$$

$$\bar{\nabla} \times \bar{H} = i\omega\epsilon \bar{E}$$

$\bar{\nabla} \cdot \bar{E} = 0 \to$ höfum $\bar{A}_e$ þ.a. $\bar{E} = \bar{\nabla} \times \bar{A}_e$

Notum Faraday lögmálið

$$\hookrightarrow \bar{H} = \frac{i}{\omega\mu} \bar{\nabla} \times \bar{E} = \frac{i}{\omega\mu} \bar{\nabla} \times \bar{\nabla} \times \bar{A}_e$$

$$= \frac{i}{\omega\mu} \left\{ \bar{\nabla}(\bar{\nabla} \cdot \bar{A}_e) - \nabla^2 \bar{A}_e \right\} \quad (2)$$

Ampère-Maxwell

$$\bar{\nabla} \times \bar{H} = i\omega\epsilon \bar{E} = i\omega\epsilon \bar{\nabla} \times \bar{A}_e$$

$$\to \bar{\nabla} \times (\bar{H} - i\omega\epsilon \bar{A}_e) = 0$$

því er hægt að finna skalarmætti V_m þ.a.

$$\bar{H} - i\omega\epsilon \bar{A}_e = -\bar{\nabla} V_m$$

$$\to \bar{H} = i\omega\epsilon \bar{A}_e - \bar{\nabla} V_m \quad (1)$$

saman gefa (1) og (2)

$$i\omega\epsilon \bar{A}_e - \bar{\nabla} V_m = \frac{i}{\omega\mu} \left\{ \bar{\nabla}(\bar{\nabla} \cdot \bar{A}_e) - \nabla^2 \bar{A}_e \right\}$$

nota „Lorentz-kvörðun"

$$\bar{\nabla} \cdot \bar{A}_e = i\omega\mu V_m$$

þá fæst

$$\boxed{\nabla^2 \bar{A}_e + \omega^2 \mu\epsilon \bar{A}_e = 0}$$

sem svar við b-lið

Notum :

$$\bar{H} = i\omega\epsilon \bar{A}_e - \bar{\nabla} V_m$$

$$\to \boxed{\bar{H} = i\omega\epsilon \bar{A}_e + \frac{i}{\omega\mu} \bar{\nabla}(\bar{\nabla} \cdot \bar{A}_e)}$$

sem er svarið við a-lið.

(1)

8-11 y-skautuð flöt bylgja í x-átt $\epsilon = 2.5$

$f = 3\,GHz$ "Loss tangent $= 10^{-2}$"

$$\tan\delta_c = \frac{\epsilon''}{\epsilon'}$$

Ef $\epsilon = \epsilon' - i\epsilon''$ með $\epsilon' = 2.5$ og $\epsilon'' = +\frac{\sigma}{\omega}$

þá fæst $\tan\delta_c = \frac{\sigma}{\omega\epsilon'}$. Tengjum þetta við

$$\gamma = \alpha + i\beta \approx i\omega\sqrt{\mu\epsilon'}\left\{1 - i\frac{\epsilon''}{2\epsilon'} + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right\}$$

fyrir efni með litlu tapi

$$\alpha \simeq \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}$$

$$\beta \simeq \omega\sqrt{\mu\epsilon'}\left(1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)$$

(2)

a) Finna helmingunar ~~vegalengd bylgjunnar~~

Bylgjan er með dofnunarþátt $e^{-\alpha x}$

fyrir helmingunarvegalengd $x_{1/2}$ gildir $e^{-\alpha x_{1/2}} = \frac{1}{2}$

$$\to x_{1/2} = \frac{\ln 2}{\alpha}$$

$$\alpha = \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}$$

notum að $\sqrt{\mu\epsilon_0} = \frac{1}{c}$ $\mu \sim \mu_0$

$$\to \sqrt{\mu\epsilon_0} = \sqrt{\mu\frac{\epsilon'}{\epsilon_r}} \quad \text{því } \epsilon_0\epsilon_r = \epsilon'$$

$$= \frac{1}{c} \to \sqrt{\mu} = \sqrt{\frac{\epsilon_r}{\epsilon'}}\frac{1}{c}$$

$$\to \alpha = \frac{\omega\epsilon''}{2}\frac{\sqrt{\epsilon_r}}{\sqrt{\epsilon'}\,c}\frac{1}{\sqrt{\epsilon'}}$$

$$= \frac{\omega}{2}\left(\frac{\epsilon''}{\epsilon'}\right)\frac{\sqrt{\epsilon_r}}{c}$$

$$= \frac{\omega}{2}(\tan\delta_c)\frac{\sqrt{\epsilon_r}}{c} = \frac{\pi f}{c}(\tan\delta_c)\sqrt{\epsilon_r} \sim 0.497\ m^{-1}$$

(3)

$$\to x_{1/2} = 1.4\ m$$

b) Ákvarða η_c, λ, u_p og u_g

(8.50) gefur að
$$\eta_c \simeq \sqrt{\frac{\mu}{\epsilon'}}\left(1 + i\frac{\epsilon''}{2\epsilon'}\right)$$

$$\simeq \sqrt{\frac{\mu_0}{\epsilon_r\epsilon_0}}\left(1 + i\frac{\epsilon''}{2\epsilon'}\right)$$

$$= \frac{1}{\sqrt{\epsilon_r}}\eta_0\left(1 + \frac{i}{2}\tan\delta_c\right) = \frac{1}{\sqrt{2.5}}377\left(1 + \frac{i}{2}\cdot 10^{-2}\right)\Omega$$

$$\simeq 238\left(1 + i\cdot 0.5\cdot 10^{-2}\right)\Omega$$

(4)

$$\lambda = \frac{2\pi}{\beta}, \quad \beta = \omega\sqrt{\mu\epsilon'}\left(1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)$$

$$\omega\sqrt{\mu\epsilon'} = \omega\sqrt{\frac{\epsilon_r}{\epsilon'}\epsilon'}\frac{1}{c} = \frac{\omega}{c}\sqrt{\epsilon_r}$$

$$= \frac{2\pi f}{c}\sqrt{\epsilon_r}$$

$$\to \lambda = \frac{c}{f\sqrt{\epsilon_r}\left(1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)}$$

$$\simeq \frac{c}{f\sqrt{\epsilon_r}}\left(1 - \frac{1}{8}(\tan\delta_c)^2\right) \simeq 0{,}063\ m$$

5

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon'}\left(1+\frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)} \approx \frac{c}{\sqrt{\epsilon_r}}\left(1-\frac{1}{8}(\tan\delta_c)^2\right)$$

$$\approx 1{,}9\cdot 10^8 \ \mathrm{m/s}$$

$$u_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = \left(\sqrt{\mu\epsilon''}\left(1+\frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)\right)^{-1} \approx \frac{c}{\sqrt{\epsilon_r}}\left(1-\frac{1}{8}(\tan\delta_c)^2\right)$$

$$\approx u_p$$

c) Ef í $x=0$

$\overline{E} = \hat{a}_y\, 50 \sin(6\pi 10^9 t + \frac{\pi}{3})\ \frac{V}{m}$ hvað er þá $\overline{H}$

6

$$\overline{H} = \frac{1}{\eta_c}\,\hat{a}_x \times \overline{E}$$ útbreiðslu stefnan

fyrir fasorinn er þá $\overline{E} = \hat{a}_y e^{i\pi/3}$

$$\overline{H} \approx \hat{a}_z \cdot \frac{50}{|\eta_c|}\, e^{i\left(\pi/3 - \arctan\left(\frac{\eta_c''}{\eta_c'}\right)\right)}$$

$$\approx \hat{a}_z \cdot 0{,}21 \cdot e^{i\left(\frac{\pi}{3} - 0{,}0016\pi\right)}$$

$$\overline{H}(x,t) \approx \hat{a}_z\, 0.21 \cdot e^{-\alpha x} \sin(6\pi 10^9 t - \beta x + \tfrac{\pi}{3} - 0{,}0016\pi)$$

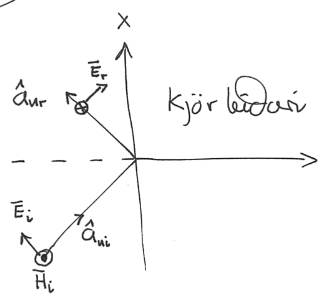
$$\approx \hat{a}_z\, 0.21 \cdot e^{-0.497x} \sin(6\pi 10^9 t - 31.6\pi x + 0{,}332\pi)\ \frac{A}{m}$$

$\frac{\pi}{3} - 0{,}0016\pi$

1

8-25

a) Finna $\overline{E}_1(xzt)$ og $\overline{H}_1(xzt)$



fyrir fasorana gildir

$$\overline{E}_1(xz) = -2E_{i0}\left\{\hat{a}_x\, i\cos\theta_i \sin(\beta_1 z\cos\theta_i) + \hat{a}_z \sin\theta_i \cos(\beta_1 z \cos\theta_i)\right\} \cdot e^{-i\beta_1 x \sin\theta_i}$$

$$\overline{E}_1(xzt) = \mathrm{Re}\left[\overline{E}_1(xz)e^{i\omega t}\right]$$

$$= 2E_{i0}\left\{\hat{a}_x \cos\theta_i \sin(\beta_1 z\cos\theta_i)\sin(\omega t - \beta_1 x\sin\theta_i) - \hat{a}_z \sin\theta_i \cos(\beta_1 z\cos\theta_i)\cos(\omega t - \beta_1 x \sin\theta_i)\right\}$$

2

$$\overline{H}_1(xz) = \hat{a}_y\, 2\frac{E_{i0}}{\eta_1}\cos(\beta_1 z\cos\theta_i)\, e^{-i\beta_1 x\sin\theta_i}$$

$$\overline{H}_1(xzt) = \mathrm{Re}\left[\overline{H}_1(xz)e^{i\omega t}\right] = \hat{a}_y \frac{2E_{i0}}{\eta_1}\cos(\beta_1 z\cos\theta_i)\cos(\omega t - \beta_1 x \sin\theta_i)$$

b)

$$\overline{\mathcal{P}}_{ave,1} = \frac{1}{2}\mathrm{Re}\left[\overline{E}_1(xz)\times\overline{H}_1^*(xz)\right]$$

$$= \hat{a}_x \frac{2E_{i0}^2}{\eta_1}\sin\theta_i \cos^2(\beta_1 z\cos\theta_i)$$

Ath

Ég hef sérstaklega notað (8-128) og (8-129) og horft framhjá öllu um „sine-reference“!

"Sine-reference" fæst við tí at steigreina

$$A(t) = \mathrm{Im}\left[A e^{+i\omega t}\right]$$

ef tað er gjørt fæst broyting á $\cos(\omega t - \beta_1 \cdots) \Leftrightarrow \sin(\omega t \cdots)$

ein peikar $\overline{S}_{ave}$, er rætnað fæst aftur

sama niðurstøða.

8-46

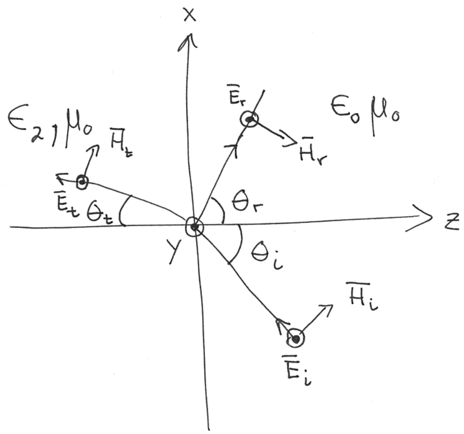
$$\overline{E}_i(x,z) = \hat{a}_y E_{i0}\, e^{-ik_0(x\sin\theta_i - z\cos\theta_i)}$$

Ath

f: tíðni

$$\epsilon_2 = \epsilon' - i\epsilon''$$

per pólur



Innstrálan er

$$\hat{a}_i = \hat{a}_x \sin\theta_i - \hat{a}_z\cos\theta_i$$

$$\overline{H}_i = \frac{1}{\eta_0}\hat{a}_i \times \overline{E}_i$$

$$= \frac{1}{\eta_0}\left\{\hat{a}_x\cos\theta_i + \hat{a}_z\sin\theta_i\right\} \cdot e^{-ik_0(x\sin\theta_i - z\cos\theta_i)}$$

í lutí 2 galdi at $\epsilon_2 = \epsilon' - i\epsilon''$

$$k_2 = \omega\sqrt{\mu_0\epsilon_2} = \omega\underbrace{\sqrt{\mu_0\epsilon_0}}_{k_0}\sqrt{\frac{\epsilon'}{\epsilon_0} - i\frac{\epsilon''}{\epsilon_0}} = k_0\sqrt{\epsilon_r' - i\epsilon_r''}$$

a) Finna $\overline{E}_t$ og $\overline{H}_t$

Fyrir þverstandandi galdi (8-207)

$$\tau_\perp = \frac{2\left(\frac{\eta_2}{\eta_0}\right)\cos\theta_i}{\left(\frac{\eta_2}{\eta_0}\right)\cos\theta_i + \cos\theta_t}$$

$\mu_2 = \mu_1 = \mu_0$

$$\hookrightarrow \frac{\eta_2}{\eta_0} = \sqrt{\frac{\epsilon_0}{\epsilon_2}} = \frac{1}{\sqrt{\epsilon_r' - i\epsilon_r''}}$$

$$\overline{E}_t = \hat{a}_y \tau_\perp E_{i0}\, e^{-ik_2(x\sin\theta_i - z\cos\theta_i)}$$

$$\overline{H}_t = \frac{1}{\eta_2}\hat{a}_t \times \overline{E}_t = \frac{1}{\eta_2}\left(\hat{a}_x\cos\theta_t + \hat{a}_z\sin\theta_t\right)\tau_\perp \overline{E}_t$$, men θ_t?

finna θ_t

Almennt galdi þ. $\mu_1 = \mu_2 = \mu_0$

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}} = \frac{1}{\sqrt{\epsilon_{r_2}}}$$

$$\rightarrow \sin\theta_t = \frac{\sin\theta_i}{\sqrt{\epsilon_{r_2}}} = \frac{\sin\theta_i}{\sqrt{\epsilon_r'' - i\epsilon_r''}} \in \mathbb{C}$$

tí er líka ~~Sin~~ $\cos\theta_i \in \mathbb{C}$

$\rightarrow$ x og z-pættir $\overline{H}_t$ eru við minkandi útslag og úr fasa (sporbangsstandur)

P10-2 Hornréttur bylgjustokkur

a) Teikna $\frac{u_g}{u}$ og $\frac{\beta}{k}$ vs $\frac{f}{f_c}$

Eq. (10-38)

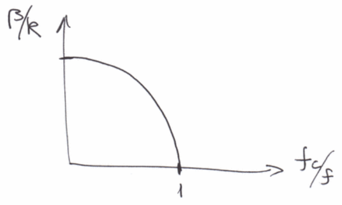
$$\beta = k\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

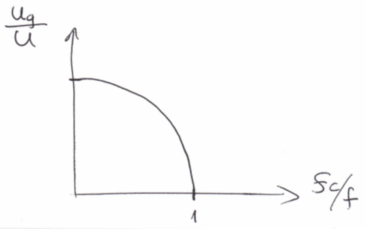
Eq. (10-43)

$$u_g = u\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\left(\frac{\beta}{k}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1$$

$$\left(\frac{u_g}{u}\right)^2 + \left(\frac{f_c}{f}\right)^2 = 1$$





b) Teikna $\frac{u}{u_p}$, $\frac{\beta}{k}$, og $\frac{\lambda_g}{\lambda}$ vs f/f_c

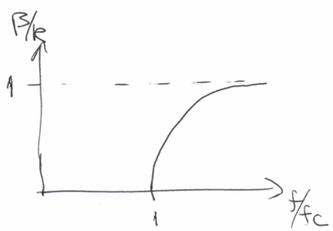
Eq. (10-42)

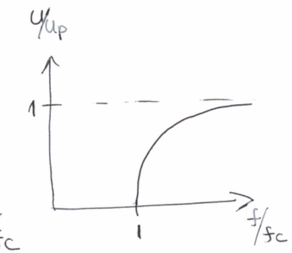
$$\frac{u_p}{u} = \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \rightarrow \left(\frac{u}{u_p}\right)^2 = 1-\left(\frac{f_c}{f}\right)^2 = 1-\frac{1}{\left(\frac{f}{f_c}\right)^2}$$

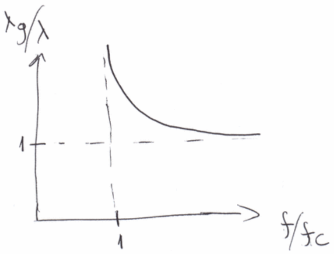
$$\left(\frac{\beta}{k}\right)^2 = 1-\left(\frac{f_c}{f}\right)^2 = 1-\frac{1}{\left(\frac{f}{f_c}\right)^2}$$

Eq. (10-43)

$$\left(\frac{\lambda}{\lambda_g}\right)^2 = 1-\left(\frac{f_c}{f}\right)^2 \rightarrow \left(\frac{\lambda_g}{\lambda}\right)^2 = \frac{1}{1-\left(\frac{f_c}{f}\right)^2} = \frac{\left(f/f_c\right)^2}{\left(\frac{f}{f_c}\right)^2-1}$$







c) $\frac{u_p}{u}$, $\frac{u_g}{u}$, $\frac{\beta}{k}$, og $\frac{\lambda_g}{\lambda}$ við $f = 1.25 f_c$

$\rightarrow$ $u_p/u = 1.67$

$u_g/u \simeq 0{,}60$

$\beta/k = 0{,}60$

$\lambda_g/\lambda = 1{,}67$

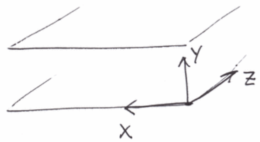
P10-7

u_{En} fyrir TE_n í taplausum stokki milli samsíða platna

$\gamma = i\beta$

Bera saman við Ex. 10-6 fyrir TM_n

Svörin eru gefin í (10-83,85)



$$H_z^0(y) = B_n \cos\left(\frac{n\pi y}{b}\right)$$

$$H_y^0(y) = \frac{\gamma}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

$$E_x^0(y) = \frac{i\omega\mu}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

$$\bar{S}_{ave} = \frac{1}{2}\Re\left(\bar{E}\times\bar{H}^*\right) = \frac{1}{2}\Re\left(\hat{a}_z E_x^0 H_y^{0*} - \hat{a}_y E_x H_z^{0*}\right)$$

$$\bar{S}_{ave}\cdot\hat{a}_z = \frac{1}{2}\Re\left(E_x^0 H_y^{0*}\right) = \frac{\omega\mu\beta}{2h^2} B_n^2 \sin^2\left(\frac{n\pi y}{b}\right)$$

$$(P_z)_{ave} = \int_0^b \bar{S}_{ave}\cdot\hat{a}_z\, dy = \frac{\omega\mu\beta b}{4h^2} B_n^2$$

á lengdareiningu í x-átt

Orkuþéttleiki

$$(w_e)_{ave} = \frac{\epsilon}{4}\Re\left(\bar{E}^0\cdot\bar{E}^{0*}\right) = \frac{\epsilon\omega^2\mu^2}{4h^2} B_n^2 \sin^2\left(\frac{n\pi y}{b}\right)$$

$$(W_e)_{ave} = \int_0^b (w_e)_{ave}\, dy = \frac{\epsilon\omega^2\mu^2 b}{8h^2} B_n^2 \leftarrow$$

og sama fæst fyrir $(W_m)_{ave}$

$$u_{en} = \frac{(P_z)_{ave}}{(W_e)_{ave} + (W_m)_{ave}} = \frac{\omega\mu\beta b}{\epsilon\omega^2\mu^2 b} = \frac{\beta}{\epsilon\mu\omega} = \frac{\omega\beta}{\epsilon\mu\omega^2}$$

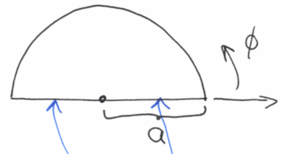
$$= \frac{\omega\beta}{k^2} = u\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

sama og fyrir TM

$$k^2 = \omega^2\mu\epsilon$$

P10-29

Hálfsívalningur sem bylgjuleiðari



a) finna E_z^0 fyrir TM

lausnin er $E_z^0(r\phi) = A_n J_n(hr)\sin(n\phi)$

því þá er jaðarskilyrðið $E_z^0 = 0$ líka uppfyllt fyrir $\phi = 0, \pi$

$J_n(ha) = 0$

ákvarðar h

J_0 er ekki möguleg lausn því þá fæst aðeins 0-lausn

$\rightarrow$ Enginn TM$_{0p}$-hættir

b) finna H_z^0 fyrir TE

$$H_z^0 = A_n J_n(hr)\cos(n\phi)$$

$$\hookrightarrow\ E_r^0 = \frac{i\omega\mu n}{h^2 r} A_n J_n(hr)\sin(n\phi)$$

$$E_\phi^0 = \frac{i\omega\mu}{h} A_n J_n'(hr)\cos(n\phi)$$

$$J_n'(ha) = 0$$

$E_r^0 = 0$ fyrir $\phi = 0, \pi$

Enginn þáttur $\bar{E}$ samsíða leiðara

Hér er $n = 0$ möguleg lausn

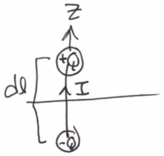
c) Eigingildi hátta

TM: $J_n(ha) = 0 \rightarrow (h)_{TM_{np}} = X_{np}/a$, $n = 1, 2, 3$

TE: $J_n'(ha) = 0 \rightarrow (h)_{TE_{np}} = X'_{np}/a$, $n = 0, 1, 2, \ldots$

P11-2

Hertz-tvískaut



$$\bar A = \frac{\mu_0}{4\pi}\int_{V'} \frac{\bar J e^{-ikR}}{R}\, dv', \qquad V = \frac{1}{4\pi\epsilon_0}\int_{V'} \frac{\rho e^{-ikR}}{R}\, dv'$$

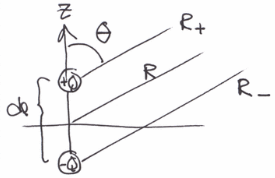
$$\bar E = -\bar\nabla V - i\omega \bar A$$

Úr bók $\bar A = \hat a_z \frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)$, $\hat a_z = \hat a_R \cos\theta - \hat a_\theta \sin\theta$

$$\rightarrow \quad A_R = \frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\cos\theta$$

$$A_\theta = -\frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-i\beta R}}{R}\right)\sin\theta$$

$$A_\phi = 0$$



$$\rightarrow V = \frac{Q}{4\pi\epsilon_0}\left\{\frac{e^{-i\beta R_+}}{R_+} - \frac{e^{-i\beta R_-}}{R_-}\right\}$$

$$R_\pm = \left(R^2 + \frac{dl^2}{4} \mp R\, dl \cos\theta\right)^{1/2} \simeq R \mp \tfrac{1}{2} dl \cdot \cos\theta$$

(11-10) $\rightarrow Q = \frac{I}{i\omega}$, gefið $(dl)^2 << R^2$

$$V \simeq \frac{I e^{-i\beta R}}{4\pi\epsilon_0 i\omega}\frac{1}{R^2}\left\{\left(R + \frac{dl}{2}\cos\theta\right)e^{i\beta\frac{dl}{2}\cos\theta} - \left(R - \frac{dl}{2}\cos\theta\right)e^{-i\beta\frac{dl}{2}\cos\theta}\right\}$$

$$= \frac{I e^{-i\beta R}}{4\pi\epsilon_0 i\omega R^2}\left\{2iR\sin\underbrace{\left(\frac{\beta dl\cos\theta}{2}\right)}_{1 \leqq \frac{\beta dl}{2}} + 2\left(\frac{dl}{2}\cos\theta\right)\cos\underbrace{\left(\frac{\beta dl}{2}\cos\theta\right)}_{1 \leqq \frac{\beta dl}{2}}\right\}$$

$$V \simeq \frac{I e^{-i\beta R}}{4\pi\epsilon_0 i\omega R^2}\left\{2iR\cdot\frac{\beta dl}{2}\cos\theta + dl\cdot\cos\theta\right\}$$

$$= \frac{I dl\cos\theta}{4\pi R^2}\eta_0\left(R + \frac{1}{i\beta}\right)e^{-i\beta R}$$

$$\left.\begin{aligned}\beta &= k_0 = \frac{\omega}{c} = \omega\sqrt{\epsilon_0\mu_0}\\ \eta_0 &= \frac{\omega\mu_0}{k_0} = \sqrt{\frac{\mu_0}{\epsilon_0}}\end{aligned}\right\} \frac{\beta}{\epsilon_0\omega} = \sqrt{\epsilon_0\mu_0}\,\frac{1}{\epsilon_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0$$

Rafsviðið kemur frá

$$\bar E = -\bar\nabla V - i\omega\bar A$$

$$\hookrightarrow \quad E_R = -\frac{\partial V}{\partial R} - i\omega A_R, \qquad E_\theta = -\frac{\partial V}{R\partial\theta} - i\omega A_\theta, \qquad E_\phi = -\frac{\partial V}{R\sin\theta\,\partial\phi} - i\omega A_\phi$$

$$\rightarrow E_R = -\frac{I dl}{4\pi}\eta_0\beta^2 2\cos\theta\left\{\frac{1}{(i\beta R)^2} + \frac{1}{(i\beta R)^3}\right\}e^{-i\beta R}$$

$$E_\theta = -\frac{I dl}{4\pi}\eta_0\beta^2\sin\theta\left\{\frac{1}{i\beta R} + \frac{1}{(i\beta R)^2} + \frac{1}{(i\beta R)^3}\right\}e^{-i\beta R}$$

$$E_\phi = 0$$

Eins og í bók (11-16a-c)

og því ljóst að hin aðferðin er styttri

P11-7

miðfatt loftnet með lengd $2h$, $h \ll \lambda$

$$I(z) = I_0\left(1 - \frac{|z|}{h}\right)$$

a) finna fjar E og H-svið

Í samanburði við (11-55) fæst

$R_1 \simeq R - z\cos\theta$

$$E_\theta = i\,\frac{I_0 \eta_0 \beta \sin\theta}{4\pi R}\, e^{-i\beta R} \int_{-h}^{h} \left(1 - \frac{|z|}{h}\right) e^{i\beta z \cos\theta}\, dz$$

$$= i\,\frac{I_0 \eta_0 \beta \sin\theta}{2\pi R}\, e^{-i\beta R} \int_{0}^{h} \left(1 - \frac{z}{h}\right) \cos(\beta z \cos\theta)\, dz$$

$$= \frac{i\,60 I_0}{(\beta h) R}\, e^{-i\beta R} F(\theta), \quad F(\theta) = \frac{\sin\theta\left[1 - \cos(\beta h \cos\theta)\right]}{\cos^2\theta}$$

$$H_\phi = \frac{E_\theta}{\eta_0} = \frac{i I_0}{(\beta h) 2\pi R}\, e^{-i\beta R} F(\theta)$$

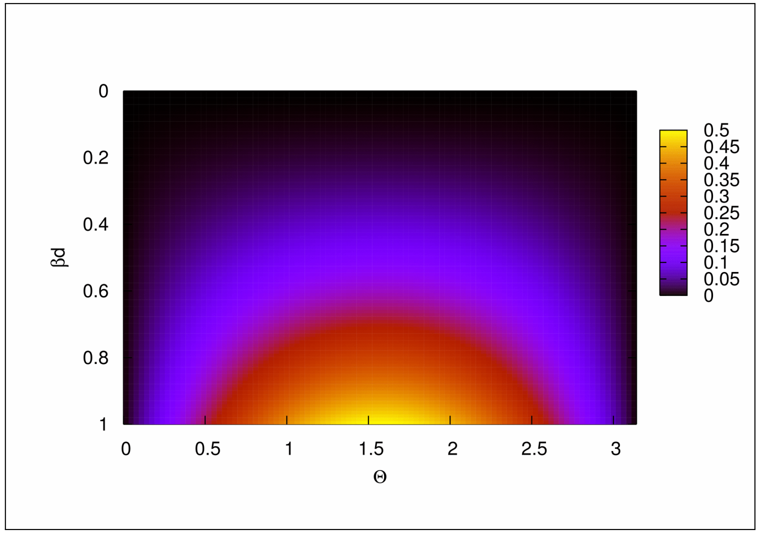
Þegar $h \ll \lambda \;\rightarrow\; \beta h \ll 1$

$$\cos(\beta h \cos\theta) \approx 1 - \frac{1}{2!}(\beta h \cos\theta)^2 + \cdots$$

$$\rightarrow F(\theta) \simeq \frac{1}{2}(\beta h)^2 \sin\theta$$

$$E_\theta = \frac{i\,30\,\beta h}{R}\, I_0 e^{-i\beta R} \sin\theta$$

$$H_\phi = \frac{i\beta h}{4\pi R}\, I_0 e^{-i\beta R} \sin\theta$$



b) finna R_r

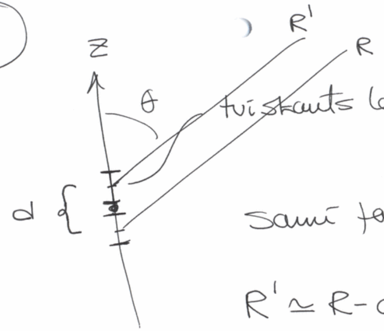
$$P_r = \frac{1}{2}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta\, R^2 E_\theta H_\phi^* = \frac{I_0^2}{2}\left[80\pi^2 \left(\frac{h}{\lambda}\right)^2\right]$$

$$R_r = \frac{P_r}{\frac{1}{2} I_0^2} = 20\pi^2 \left(\frac{2h}{\lambda}\right)^2$$

c)

$$D = \frac{4\pi |E_{max}|^2}{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta\, |E_\theta(\theta)|^2} = \frac{2}{\int_0^{\pi} \sin^3\theta} = 1{,}5$$

P11-15



tvískauts loftnet $2h \ll \lambda$

$d > 2h$

sami fasi og styrkur

$R' \simeq R - d\cos\theta$

R'

a)
$$E_\theta = \frac{iI(2h)}{4\pi}\eta_0 \beta \sin\theta\, e^{i\beta R}\left\{\frac{1}{R} + \frac{e^{i\beta d\cos\theta}}{R'}\right\}$$

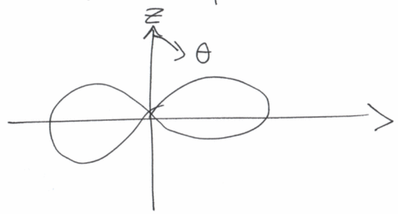
$$= \frac{i60Ih}{R} 2\beta\, e^{-i\beta(R-\frac{d}{2}\cos\theta)} F(\theta)$$

$\frac{1}{R} \simeq \frac{1}{R'}$ í fjarsíði

með

$$F(\theta) = \sin\theta \cos\left(\frac{\beta d}{2}\cos\theta\right)$$

b)
$$|F(\theta)| = |\sin\theta \cos\left(\tfrac{\pi}{2}\cos\theta\right)|$$



c)
$$|F(\theta)| = |\sin\theta \cos(\pi\cos\theta)|$$