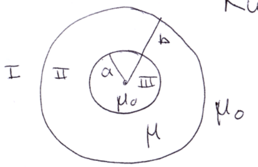
Kúlustel í ytra segulflæði $\bar{B} = B_0 \hat{a}_z$ (1)



Engir frjálsir straumar $\rightarrow$ $\nabla \times \bar{H} = 0$

Engin einskaut $\rightarrow$ $\nabla \cdot \bar{B} = 0$

$\rightarrow$ til er stalermætti ϕ_m þ.a. $\bar{H} = -\nabla \phi_m$

og $\nabla^2 \phi_m = 0$ (Gerum ráð fyrir að $\bar{H} = \frac{1}{\mu}\bar{B}$)

Jaðarskilyrði: fyrir $R >> b$ verður að gilda að

$$\bar{B} = B_0 \hat{a}_z \longrightarrow \phi_m(R,\theta) = -\frac{B_0}{\mu_0} R\cos\theta$$

eins verður að gilda fyrir bæði yfirborðin (þ. $R = a$ og b)

$$\hat{a}_{n_2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s$$

$\bar{B}_n$ er samfellt í yfirborði

(2)

Yfirborð $R = b$

$\hat{a}_R \times (\bar{H}_I - \bar{H}_{II}) = 0 \leftarrow$ engir frjálsir yfib. straumar

$\phi_m = \phi_m(R,\theta)$ $\quad \bar{H} = -\nabla \phi_m(R,\theta) = -\hat{a}_R \frac{\partial \phi_m}{\partial R} - \hat{a}_\theta \frac{1}{R}\frac{\partial \phi_m}{\partial \theta}$

$$\rightarrow -\frac{\hat{a}_\phi}{R}\left\{\partial_\theta \phi_m^I(b^+,\theta) - \partial_\theta \phi_m^{II}(b^-,\theta)\right\} = 0$$

$$\rightarrow \partial_\theta \phi_m^I(b,\theta) = \partial_\theta \phi_m^{II}(b,\theta) \quad (1)$$

auk $-\mu_0 \partial_R \phi_m^I(b,\theta) = -\mu \partial_R \phi_m^{II}(b,\theta)$ (2) $\leftarrow$ B_n samfellt

Yfirborð $R = a$ $\qquad -\hat{a}_R \times (\bar{H}_{III} - \bar{H}_{II}) = 0$

$$\rightarrow \partial_\theta \phi_m^{III}(a,\theta) = \partial_\theta \phi_m^{II}(a,\theta) \ (3), \text{ og } -\mu_0 \partial_R \phi_m^{III}(a,\theta) = -\mu \partial_R \phi_m^{II}(a,\theta) \ (4)$$

(3)

Í kúluhnitum er almenna lausn Laplace

$$\phi_m(R,\theta) = \sum_{n=0}^{\infty} \left\{A_n R^n + B_n R^{-(n+1)}\right\} P_n(\cos\theta)$$

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fyrir $R > b$ verður að fella formið $\phi_m^I(R,\theta) = -H_0 R\cos\theta$ að ráða þ. R verður stórt m.v. b

$$\rightarrow \phi_m^I(R,\theta) = -H_0 R\cos\theta + \sum_{n=0}^{\infty} B_n^I R^{-(n+1)} P_n(\cos\theta)$$

Fyrir $a < R < b$

$$\phi_m^{II}(R,\theta) = \sum_{n=0}^{\infty} \left\{A_n^{II} R^n + \frac{B_n^{II}}{R^{n+1}}\right\} P_n(\cos\theta)$$

(4)

Fyrir $R < a$, Enginn sérstöðupunktur

$$\rightarrow \phi_m^{III}(R,\theta) = \sum_{n=0}^{\infty} A_n^{III} R^n P_n(\cos\theta)$$

Eins og fyrir rafsvarandi kúlu í ytra rafsviði kemur í ljós að aðeins stuðlar með $n=1$ eru mögulegir

$$\rightarrow \phi_m^I(R,\theta) = -H_0 R\cos\theta + \frac{B_1^I \cos\theta}{R^2}$$

$$\phi_m^{II}(R,\theta) = \left\{A_1^{II} R + \frac{B_1^{II}}{R^2}\right\}\cos\theta$$

$$\phi_m^{III}(R,\theta) = A_1^{III} R\cos\theta$$

⑤

① → $-H_0 b + \frac{B_1^{I}}{b^2} = A_1^{II} b + \frac{B_1^{II}}{b^2}$

② → $-\mu_0\left\{-H_0 - \frac{2B_1^{I}}{b^3}\right\} = -\mu\left\{A_1^{II} - \frac{2B_1^{II}}{b^3}\right\}$

③ → $A_1^{III} a = A_1^{II} a + \frac{B_1^{II}}{a^2}$

④ → $-\mu_0\left\{A_1^{III}\right\} = -\mu\left\{A_1^{II} - \frac{2B_1^{II}}{a^3}\right\}$

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① → $B_1^{I} - A_1^{II} b^2 - B_1^{II} = b^3 H_0$

② → $2B_1^{I}\mu_0 + \mu A_1^{II} b^3 - 2\mu B_1^{II} = -b^3 H_0 \mu_0$

⑥

③ → $A_1^{II} a^3 + B_1^{II} - A_1^{III} a^3 = 0$

④ → $\mu A_1^{II} a^3 - 2\mu B_1^{II} - \mu_0 A_1^{III} a^3 = 0$

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4 jöfnur B_1^{I}, B_1^{II}, A_1^{II} og A_1^{III} óþekktir þættir

$$\begin{pmatrix} 1 & -1 & -b^3 & 0 \\ 2\mu_0 & -2\mu & \mu b^3 & 0 \\ 0 & 1 & a^3 & -a^3 \\ 0 & -2\mu & \mu a^3 & -\mu_0 a^3 \end{pmatrix} \begin{pmatrix} B_1^{I} \\ B_1^{II} \\ A_1^{II} \\ A_1^{III} \end{pmatrix} = \begin{pmatrix} b^3 H_0 \\ -\mu_0 b^3 H_0 \\ 0 \\ 0 \end{pmatrix}$$

⑦

Deilum með μ_0 í gegnum jöfnu ② og ③ og skilgreinum $\mu_r = \frac{\mu}{\mu_0}$

$$\begin{pmatrix} 1 & -1 & -b^3 & 0 \\ 2 & -2\mu_r & \mu_r b^3 & 0 \\ 0 & 1 & a^3 & -a^3 \\ 0 & -2\mu_r & \mu_r a^3 & -a^3 \end{pmatrix} \begin{pmatrix} B_1^{I} \\ B_1^{II} \\ A_1^{II} \\ A_1^{III} \end{pmatrix} = \begin{pmatrix} b^3 H_0 \\ -b^3 H_0 \\ 0 \\ 0 \end{pmatrix}$$

með lausn

⑧

$$B_1^{I} = \frac{\left\{b^3 a^3\left(-2\mu_r^2 + \mu_r + 1\right) + b^6\left(2\mu_r^2 - \mu_r - 1\right)\right\} H_0}{b^3\left(2\mu_r^2 + 5\mu_r + 2\right) - 2a^3\mu_r^2 + 4a^3\mu_r - 2a^3}$$

$$= \frac{-a^3\left(2\mu_r^2 - \mu_r - 1\right) + b^3\left(2\mu_r^2 - \mu_r - 1\right)}{\left(2\mu_r^2 + 5\mu_r + 2\right) + \frac{a^3}{b^3}\left(-2\mu_r^2 + 4\mu_r - 2\right)} H_0$$

$$= \left\{\frac{(2\mu_r + 1)(\mu_r - 1)}{(2\mu_r + 1)(\mu_r + 2) - 2\left(\frac{a}{b}\right)^3(\mu_r - 1)^2}\right\}(b^3 - a^3) H_0$$

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og

$$A_1^{III} = -\left\{\frac{9\mu_r}{(2\mu_r + 1)(\mu_r + 2) - 2\left(\frac{a}{b}\right)^3(\mu_r - 1)^2}\right\} H_0$$

Utan kúluskeljar var $\overline{B} = B_0 \hat{a}_z$ fast svið

Kúluskelin bætir við tvístautssviði fyrir utan kúlust.

Í réttu hlutfalli við B_1^{I}

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Innan kúluskeljar myndast

$$\phi_m^{III} = A_1^{III} r \cos\theta = A_1^{III} z$$

$$\overline{H}^{III} = -\overline{\nabla} \phi_m^{III} = -A_1^{III} \hat{a}_z \quad \text{fast segulsvið!}$$

$$\overline{B}^{III} = -\mu_0 A_1^{III} \hat{a}_z = \frac{9 \mu H_0}{(2\mu_r+1)(\mu_r+2) - 2\left(\frac{a}{b}\right)^3 (\mu_r-1)^2}$$

$$= \frac{9\mu_r B_0}{\mu_r^2 \left\{ \left(2+\frac{1}{\mu_r}\right)\left(1+\frac{2}{\mu_r}\right) - 2\left(\frac{a}{b}\right)^3 \left(1-\frac{1}{\mu_r}\right)^2 \right\}}$$

$\longrightarrow \quad B^{III} \longrightarrow 0$ þegar $\mu_r = \frac{\mu}{\mu_0} \longrightarrow \infty$

fyrir járnseglandi efni er $\mu_r \sim 10^3 - 10^6$

Við sjáum því segulskýlingu

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fyrir sterka andseglun fæst $\mu = 0, \; \mu_r = 0$

$$B^{III} = \frac{9\mu_r B_0}{(2\mu_r+1)(\mu_r+2) - 2\left(\frac{a}{b}\right)^3 (\mu_r-1)^2} \longrightarrow 0$$