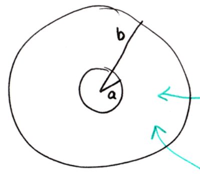
①



Kjörleiðandi kúluskeljar

$$\epsilon(R) = \epsilon_0\left\{1+\left(\frac{b}{R}\right)^2\right\}$$

$$\sigma(R) = \sigma_0\left\{1+\left(\frac{b}{R}\right)^2\right\}$$

① Finnum leiðni þéttisins

Hér er hvorki hægt að nota Laplace til að finna mættið fyrir rafsviðið né strauminn

↳ $\bar{\nabla}\cdot\bar{J} = 0$

sem er varðveisla straumsins í sístæðu ástandi.

①

Milli skeljanna gildir

$$\oint \bar{J}\cdot d\bar{s} = 0$$

inn eða út um skeljarnar kemur I

$$\rightarrow \bar{J} = \frac{I}{4\pi R^2}\hat{a}_R$$

veljum stefnu frá innri skel á þá ytri.

Ohms lögmál gefur okkur þá

$$\bar{E} = \frac{\bar{J}(R)}{\sigma(R)}$$

②

$$\bar{E} = \frac{\bar{J}(R)}{\sigma(R)} = \bar{a}_R\frac{I}{4\pi R^2\sigma_0\left\{1+\left(\frac{b}{R}\right)^2\right\}} = \frac{\hat{a}_R I}{4\pi\sigma_0\left\{R^2+b^2\right\}}$$

Næst getum við fundið spennuna skeljanna

$$V_a - V_b = -\int_b^a \bar{E}\cdot d\bar{l} = -\int_b^a \frac{I\,dR}{4\pi\sigma_0\left\{R^2+b^2\right\}}$$

$$= \frac{I}{4\pi\sigma_0}\frac{1}{b}\arctan\left(\frac{x}{b}\right)\Bigg|_a^b = \frac{I}{4\pi\sigma_0 b}\left\{\arctan(1)-\arctan\left(\frac{a}{b}\right)\right\}$$

$$= \frac{I}{4\pi\sigma_0 b}\left\{\frac{\pi}{4}-\arctan\left(\frac{a}{b}\right)\right\} > 0 \quad \text{ef } a<b$$

③

$$G = \frac{I}{V_a-V_b} = \frac{4\pi\sigma_0 b}{\left\{\frac{\pi}{4}-\arctan\left(\frac{a}{b}\right)\right\}}$$ leiðni þéttisins

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② Dreifing frjálsra hleðslna í þéttinum?

Rúmhleðslur má finna frá $\bar{\nabla}\cdot\bar{D} = \rho$

$$\bar{D} = \epsilon(R)\bar{E} = \epsilon_0\left\{1+\left(\frac{b}{R}\right)^2\right\}\frac{\hat{a}_R I}{4\pi R^2\sigma_0\left\{1+\left(\frac{b}{R}\right)^2\right\}}$$

$$= \frac{\hat{a}_R I\epsilon_0}{4\pi R^2\sigma_0}$$

$$\rho(R) = \bar{\nabla}\cdot\bar{D} = \frac{1}{R^2}\frac{d}{dR}\left\{\frac{I\epsilon_0}{4\pi\sigma_0}\right\} = \underline{0}$$ ← hverfandi rúmhleðsla

④

Á skeljunum í $R = b^-$ og a^+ safnast fyrir hleðslur sem viðhalda spennunni á þéttinum. Þessu er lýst með jaðarskilyrðunum

$$\hat{a}_{n2}\cdot(\bar{D}_1-\bar{D}_2) = \rho_s$$

$\underline{\bar{R} = a^+}$

$$\rho_s(a^+) = \epsilon(a)\bar{E}(a)\cdot\hat{a}_R = \frac{\epsilon_0\left\{1+\left(\frac{b}{a}\right)^2\right\}I}{4\pi\sigma_0 a^2\left\{1+\left(\frac{b}{a}\right)^2\right\}}$$

$$= \frac{\epsilon_0 I}{4\pi\sigma_0 a^2}$$

$\underline{\bar{R} = b^-}$

$$\rho_s(b^-) = -\epsilon(b)\bar{E}(b)\cdot\hat{a}_R = -\frac{\epsilon_0 2 I}{4\pi\sigma_0 b^2 2}$$

$$= -\frac{\epsilon_0 I}{4\pi\sigma_0 b^2}$$

Því sést strax að heildar frjálsu hleðslurnar styttast út.

③ Dreifing skautunarhleðsla í rafsvaranum

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P} \quad \rightarrow \quad \bar{P} = \bar{D} - \epsilon_0 \bar{E} = \{\epsilon(R) - \epsilon_0\} \bar{E}$$

$$\epsilon(R) - \epsilon_0 = \epsilon_0 \left(\frac{b}{R}\right)^2 , \quad \bar{E} = \frac{\hat{a}_R I}{4\pi\sigma_0 \{R^2 + b^2\}}$$

$$\rightarrow \quad \bar{P} = \frac{\hat{a}_R \epsilon_0 I b^2}{4\pi\sigma_0 R^2 \{R^2 + b^2\}}$$

þá skautunarhleðslur eru

$$\rho_P = -\bar{\nabla} \cdot \bar{P} = -\frac{1}{R^2}\left[\frac{d}{dR} \frac{\epsilon_0 I b^2}{4\pi\sigma_0 \{R^2 + b^2\}}\right]$$

$$\rightarrow \rho_P = \frac{1}{R^2} \frac{\epsilon_0 I b^2}{4\pi\sigma_0} \frac{2R}{\{R^2 + b^2\}^2} = \frac{\epsilon_0 I b^2}{2\pi\sigma_0 R \{R^2 + b^2\}^2}$$

skautunarhleðsla á yfirborði rafsvarans

$R = a^+$

$$\rho_{ps}(a^+) = \bar{P}(a^+) \cdot \hat{a}_n = -\bar{P}(a) = -\frac{\epsilon_0 I b^2}{4\pi\sigma_0 a^2 (a^2 + b^2)}$$

$R = b^-$

$$\rho_{ps}(b^-) = P(b) = \frac{\epsilon_0 I}{4\pi\sigma_0 2 (b^2)} = \frac{\epsilon_0 I}{8\pi\sigma_0 b^2}$$

Við höfum séð að heildar frjálsa hleðslan í þéttinum styttist út, en hvernig er heildar skautuðu hleðslunni háttað?

$$Q(a^+) = S_a \rho_{ps}(a^+) = \frac{\epsilon_0 I b^2}{\sigma_0 (a^2 + b^2)}$$

$$Q(b^-) = S_b \rho_{ps}(b^-) = \frac{\epsilon_0 I}{2\sigma_0}$$

$$Q_b = \int_a^b dv \, \rho_P = \frac{2\epsilon_0 I b^2}{\sigma_0} \int_a^b \frac{R\,dR}{\{R^2 + b^2\}^2}$$

$$\rightarrow Q_b = \frac{\epsilon_0 I}{\sigma_0} \left\{ \frac{b^2}{b^2 + a^2} - \frac{1}{2} \right\}$$

Því sést greinilega að $Q_b + Q(a^+) + Q(b^-) = 0$

skautunarhleðslan styttist út í þéttinum.