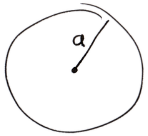
①  Gegnheil rafsvarandi kúla ~~með~~ skautun $\bar{P}(R) = P_0 \left(\frac{R}{a}\right)^2 \hat{a}_R$

① Finna jafngilda bol- og yfirborðsþéttleika

bolur:

$$\rho_P = -\bar{\nabla}\cdot\bar{P}$$ aðeins fall af R

$$\rightarrow \rho_P = -\frac{1}{R^2}\partial_R\left\{R^2 P_0 \left(\frac{R}{a}\right)^2\right\}$$ kúluhnit ef $R<a$

$$= -\frac{1}{R^2}\partial_R\left\{R^4\right\}\frac{P_0}{a^2}$$

$$= -4\left(\frac{R}{a^2}\right)P_0$$ ef $R<a$

og 0 fyrir $R>a$

② yfirborð:

$$\rho_{ps} = \bar{P}\cdot\hat{a}_n\Big|_{R=a} = \bar{P}\cdot\hat{a}_R\Big|_{R=a} = P_0$$

③ Heildar jafngilda hleðslan

$$Q_{ps} = \oint ds\,\rho_{ps} = 4\pi a^2 P_0$$ fyrir yfirborðið

$$Q_P = \int_V dv\,\rho_P = -\frac{4P_0}{a^2}4\pi\int R^2 dR\,R = -\frac{4P_0}{a^2}4\pi\frac{a^4}{4}$$

$$= -4\pi a^2 P_0$$

$\rightarrow Q_{ps} + Q_P = 0$, er það tilviljun?

lítum á

$$Q_P = \int_V dv\,\rho_P = -\int_V dv\,\bar{\nabla}\cdot\bar{P}$$

notum setningu Gauß $\longrightarrow -\oint_S d\bar{s}\cdot\bar{P} = -Q_{ps}$

$\rightarrow Q_P + Q_{ps} = 0$ alltaf

④ Rafsviðsmættið?

Jafngildu hleðsludreifingarnar eru kúlusamhverfar, óháðar hornum → notum lögmál Gauß og finnum fyrst sviðið

því fáumst við ekki við D hér hjálparsviðið.

Innan kúlu, $R<a$

$$\oint_{S(R)} \bar{E}\cdot d\bar{s} = \frac{Q(R)}{\epsilon_0}$$

Hleðslan innan yfirborðs S(R)

$$Q(R) = \int_{R'<R} dv'\,\rho(R') = 4\pi\int_0^R R'^2 dR'\left\{-4\frac{R'P_0}{a^2}\right\}$$

$$= -4\pi\frac{4P_0}{a^2}\int_0^R dR'\,(R')^3 = -4\pi\frac{4P_0}{a^2}\frac{R^4}{4} = -4\pi P_0\frac{R^4}{a^2}$$

Notum í lögmál Gauß

$$4\pi R^2 E(R) = -\frac{4\pi R^4}{\epsilon_0 a^2}P_0 \rightarrow \boxed{\bar{E}(R) = -\left(\frac{R}{a}\right)^2\frac{P_0}{\epsilon_0}\hat{a}_R \quad R<a}$$

utan kúlu $Q = 0$, kúlusamhverfa

$$\rightarrow \boxed{\bar{E} = 0, \quad R>a}$$

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$$\bar{E} = -\bar{\nabla} V = -\partial_R V(R)$$ vegna kúlusamhverfu

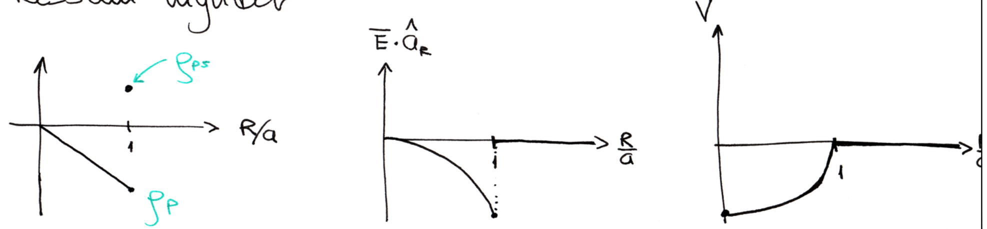
setjum $V=0$ þ. $R \to \infty$ $\to$ $V(R) = 0$ fyrir $R > a$

og $V(R) = \frac{a}{3}\left(\frac{R}{a}\right)^3 \frac{P_0}{\epsilon_0} + C$

$V(R)\big|_{R=a} = 0$ því $V(R)$ er samfellt í yfirborði kúlunnar

$$\to V(R) = \frac{P_0 a}{3\epsilon_0}\left\{\left(\frac{R}{a}\right)^3 - 1\right\}, \quad R < a$$

⑥ Rissum myndir



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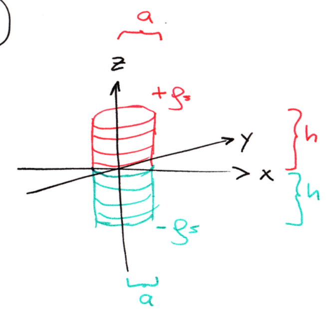
②



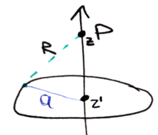
① Finna V á z-ás, allsstaðar

$$dV = \frac{\rho_l}{4\pi\epsilon_0 R}$$

$$\to V = \oint \frac{\rho_l \, dl'}{4\pi\epsilon_0 R} = \frac{\rho_l}{4\pi\epsilon_0 R}\oint dl'$$

$$= \frac{\rho_l a}{2\epsilon_0 R} = \frac{\rho_l a}{2\epsilon_0\sqrt{a^2 + (z-z')^2}}$$

Athugum hring:



Fjarlægðin frá hringnum að P er allsstaðar sú sama

því má finna V fyrir sívalning m.þ.a. „summa" upp fyrir hringi

$$dV(z) = \frac{\rho_s a \, dz'}{2\epsilon_0\sqrt{a^2 + (z-z')^2}}$$

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Efri sívalningur $(z > h)$

$$V_+(z) = \int_0^h \frac{\rho_s a \, dz'}{2\epsilon_0\sqrt{a^2 + (z-z')^2}} = \frac{\rho_s a}{2\epsilon_0}\int_0^h \frac{dz'}{\sqrt{a^2 + (z-z')^2}}$$

(GR 2.271.4)

Gerum heildið með aðgát og athugum seinna hvernig það lítur út fyrir mismunandi svæði

$(z > h)$ ofan

$(z < 0)$ neðan

$(h > z > 0)$ inni

$$= \frac{\rho_s a}{2\epsilon_0}\left\{-\ln\left[(z-z') + \sqrt{a^2 + (z-z')^2}\right]\Big|_0^h\right\}$$

$$= \frac{\rho_s a}{2\epsilon_0}\left\{-\ln\left[(z-h) + \sqrt{a^2 + (z-h)^2}\right] + \ln\left[z + \sqrt{a^2 + z^2}\right]\right\}$$

$$= \frac{\rho_s a}{2\epsilon_0}\left[-\ln\left\{\frac{(z-h) + \sqrt{a^2 + (z-h)^2}}{z + \sqrt{a^2 + z^2}}\right\}\right]$$ ef $z > h$

hér væri hreinlegast að nota innsetn. $z - z' = x$, $dx = -dz'$

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$V_+(z)$ þegar $z < 0$

breytuskipti $z' - z = x$, $dx = dz'$

$$V_+(z) = \int_0^h \frac{\rho_s a \, dz'}{2\epsilon_0\sqrt{a^2 + (z'-z)^2}} = \int_{-z}^{h-z} \frac{\rho_s a \, dx}{2\epsilon_0\sqrt{a^2 + x^2}}$$

$$= \frac{\rho_s a}{2\epsilon_0}\left\{\ln\left[x + \sqrt{a^2 + x^2}\right]\Big|_{-z}^{h-z}\right\}$$

$$= \frac{\rho_s a}{2\epsilon_0}\left\{\ln\left[\frac{(h-z) + \sqrt{a^2 + (h-z)^2}}{\sqrt{a^2 + z^2} - z}\right]\right\}$$

þegar $h > z > 0$

$$V_+(z) = \int_0^z \frac{\rho_s a \, dz'}{2\epsilon_0\sqrt{a^2 + (z-z')^2}} + \int_z^h \frac{\rho_s a \, dz'}{2\epsilon_0\sqrt{a^2 + (z'-z)^2}}$$

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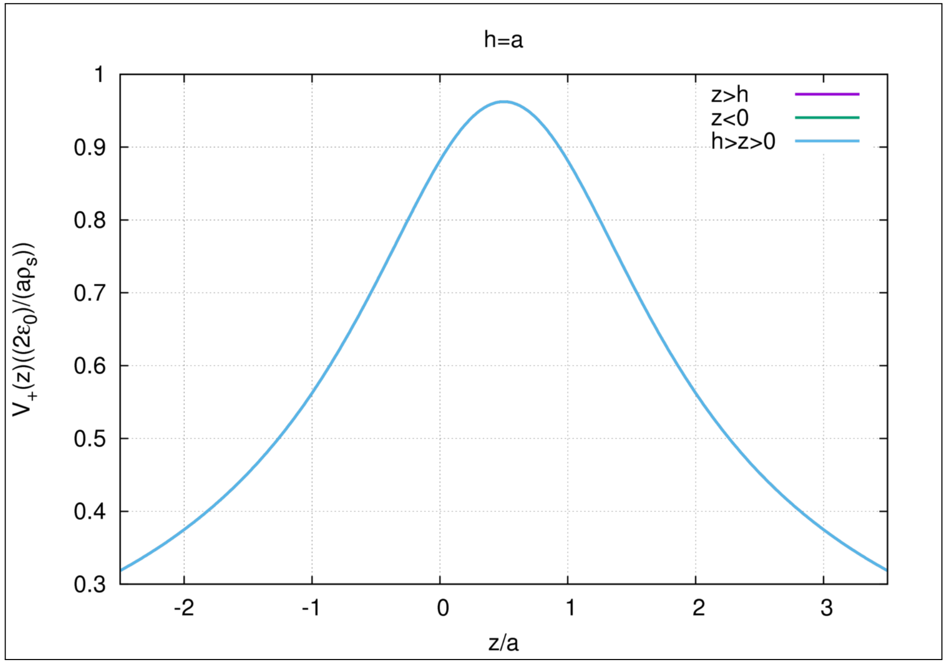
$$= \frac{\rho_s a}{2\epsilon_0}\left\{ -\ln\left[x+\sqrt{a^2+x^2}\right]\Big|_z^{0} + \ln\left[x+\sqrt{a^2+x^2}\right]\Big|_0^{h-z}\right\}$$

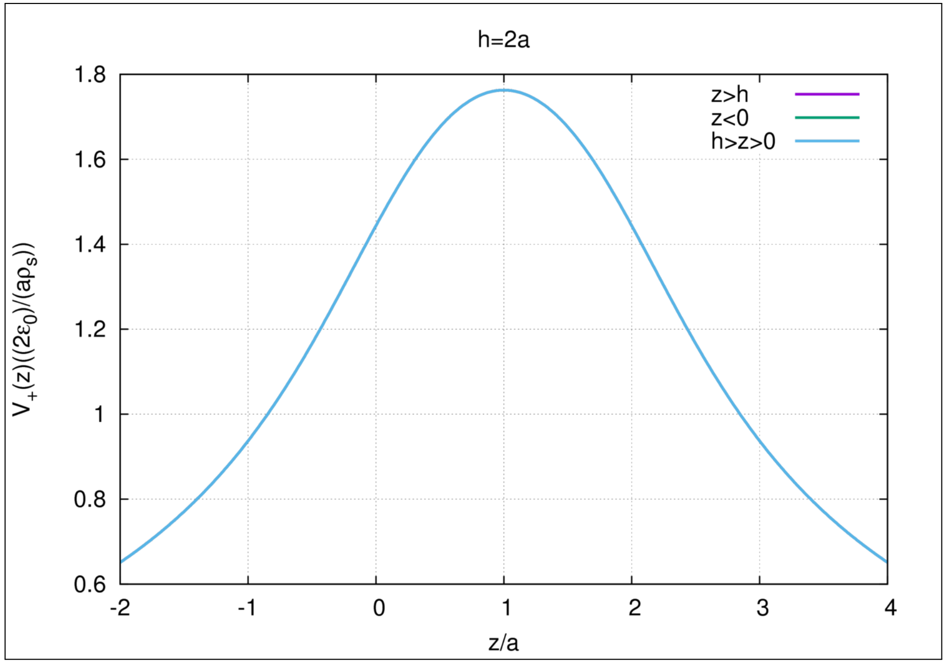
$$= \frac{\rho_s a}{2\epsilon_0}\left\{ -\ln a + \ln\left[z+\sqrt{a^2+z^2}\right] + \ln\left[(h-z)+\sqrt{a^2+(h-z)^2}\right] - \ln a\right\}$$

$$\to V_+(z) = \frac{\rho_s a}{2\epsilon_0}\left\{ \ln\left[\frac{\left(z+\sqrt{a^2+z^2}\right)\left((h-z)+\sqrt{a^2+(h-z)^2}\right)}{a^2}\right]\right\}$$

þegar $h>z>0$

Á næstu tveimur síðum sést að jöfnurnar fyrir $V_+(z)$ eru í raun jafngildar á öllum svæðunum





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(2) Því kemur ekki á óvart að

$$\bar{E} = -\hat{a}_z \frac{\partial V_+(z)}{\partial z} = \hat{a}_z \frac{\rho_s a}{2\epsilon_0}\left\{\frac{1}{\sqrt{a^2+(z-h)^2}} - \frac{1}{\sqrt{a^2+z^2}}\right\}$$

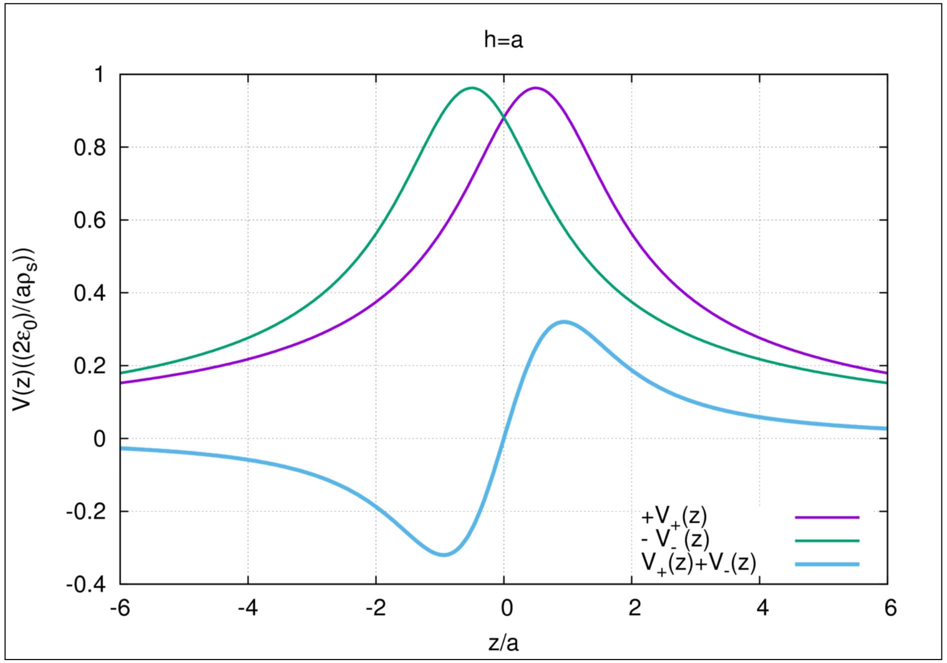
á öllum svæðunum

(3)

$$V_-(z) = \frac{\rho_s a}{2\epsilon_0}\left[-\ln\left\{\frac{(z+h)+\sqrt{a^2+(z+h)^2}}{z+\sqrt{a^2+z^2}}\right\}\right]$$

hér er mínusinn $-\rho_s$

Á næstu síðu er graf af $V(z) = V_+(z) + V_-(z)$



(4) (14)

$$V(z) = -\frac{\rho_s a}{2\epsilon_0}\left[\ln\left\{\frac{\left((z-h)+\sqrt{a^2+(z-h)^2}\right)\left((z+h)+\sqrt{a^2+(z+h)^2}\right)}{\left(z+\sqrt{a^2+z^2}\right)^2}\right\}\right]$$

Viljum finna ð fellu formið þegar $|z| >> h, a$

$$V(z) \longrightarrow -\frac{\rho_s a}{2\epsilon_0}\left[\ln\left\{\frac{\left(\left(1-\frac{h}{z}\right)+\sqrt{\left(\frac{a}{z}\right)^2+\left(1-\frac{h}{z}\right)^2}\right)\left(\left(1+\frac{h}{z}\right)+\sqrt{\left(\frac{a}{z}\right)^2+\left(1+\frac{h}{z}\right)^2}\right)}{\left(1+\sqrt{\left(\frac{a}{z}\right)^2+1}\right)^2}\right.\right.$$

$$\left(1-\frac{h}{z}\right)+\sqrt{\left(\frac{a}{z}\right)^2+\left(1-\frac{h}{z}\right)^2} \longrightarrow 1-\frac{h}{z}+1+\frac{1}{2}\left(\frac{a}{z}\right)^2+\frac{1}{2}\left(\frac{h}{z}\right)^2-\frac{h}{z}$$

notum WxMaxima til ð finna röð fyrir $\frac{a}{z}, \frac{h}{z}$, þá fæst

$$V(z) \longrightarrow \frac{\rho_s a}{2\epsilon_0}\left\{\frac{h^2}{z^2} - \frac{(3h^2a^2-h^4)}{2z^4} + \cdots\right\}$$

Tvískauts ð fella