1. Langur sívalningur með geisla a ber $\bar{M} = M_0 \left(\frac{r}{a}\right)^2 \hat{a}_\phi$

Finna $\bar{B}$ innan og utan

Notum jafngilda strauma (þó svo komi í ljós að þeir séu óþarfir), en til að skilja betur uppsetninguna

$$\bar{J}_m = \bar{\nabla} \times \bar{M} \quad , \quad \bar{J}_{ms} = \bar{M} \times \hat{a}_n$$

$$J_m = \bar{\nabla} \times \bar{M} = M_0 \, \bar{\nabla} \times \left\{ \left(\frac{r}{a}\right)^2 \hat{a}_\phi \right\} = M_0 \frac{1}{r} \partial_r \left[r \left(\frac{r}{a}\right)^2 \right] \hat{a}_z$$

$$= \frac{3M_0}{a} \left(\frac{r}{a}\right) \hat{a}_z$$

á bognu hliðinni er $\hat{a}_n = \hat{a}_r$

$$\bar{J}_{ms} = M_0 \left(\frac{a}{a}\right)^2 \hat{a}_\phi \times \hat{a}_r = - M_0 \hat{a}_z$$

skoðum beins heildarstrauma (jafngilda)

í bol

$$I_m = \frac{3M_0}{a}\, 2\pi \int_0^a r\,dr\left(\frac{r}{a}\right) = \frac{3M_0\, 2\pi}{a^2}\,\frac{a^3}{3} = 2\pi M_0 a$$

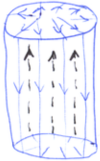
í stefnu $\hat{a}_z$

á yfirborði

$$I_{ms} = \bar{J}_{ms}\cdot \hat{a}_z \cdot 2\pi a = -2\pi M_0 a$$

$\rightarrow$ heildarstraumurinn er 0

Ef sívalningurinn er endanlegur sést að á endunum er straumurinn „radial"



Engir frjálsir straumar, $\bar{M} = 0$ utan sívalnings

$$\oint_c \bar{H}\cdot d\bar{l} = 0 \;\rightarrow\; \bar{H} = 0 \quad \text{og} \quad \bar{B} = 0 \;\; \underline{\text{utan}}$$

Innan

Enginn frjáls straumur $\rightarrow \bar{H} = 0$

$$\rightarrow \quad \frac{\bar{B}}{\mu_0} - \bar{M} = 0, \quad \rightarrow \quad \bar{B} = \mu_0 \bar{M}$$

$$= \mu_0 M \left(\frac{r}{a}\right)^2 \hat{a}_\phi$$

fyrir $r < a$

Athugum það sama, með jafngildu straumnum

$$I_m^{enc}(r) = \frac{3M_0}{a} 2\pi \int_0^r r' dr' \left(\frac{r'}{a}\right) = \frac{3M_0 2\pi}{a^2} \frac{r^3}{3} = 2\pi M_0 a \left(\frac{r^3}{a^3}\right)$$

Notum

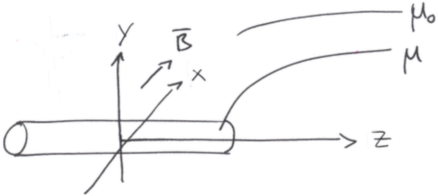
$$\oint_{C_r} \bar{B} \cdot d\bar{l} = \mu_0 I_m^{enc}(r) \rightarrow 2\pi r B = \mu_0 2\pi M_0 a \left(\frac{r}{a}\right)^3$$

$$\rightarrow B = \mu_0 M_0 \left(\frac{r}{a}\right)^2$$

og $\overline{B} = \mu_0 M_0 \left(\frac{r}{a}\right)^2 \hat{a}_\phi$ eins og áður sást.

---

(2)



(1) finna $\overline{B}, \overline{H}, \overline{M}$ innan og utan sívalnings

Engir frjálsir straumar $\rightarrow \overline{\nabla} \times \overline{H} = 0$

Einnig gildir $\overline{\nabla} \cdot \overline{B} = 0$

því er til ϕ_m þannig að $\overline{H} = -\overline{\nabla} \phi_m$

og $\boxed{\nabla^2 \phi_m = 0}$

þetta er því farið að líkjast rafsvarandi sívalningi í ytra rafsviði

Jaðarskilyrði

$$\bar{B} = B_0 \hat{a}_x \rightarrow \Phi_m(r,\phi) = -\frac{1}{\mu_0} B_0 r \cos\phi \quad , \quad r >> a$$

1

og í segulsviði gildir

$$\hat{a}_{n_2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s \leftarrow$$ hjá okkur er enginn frjáls yfirborðsstraumur

$$\rightarrow \hat{a}_r \times (\bar{H}^i - \bar{H}^o) = 0$$

$$\rightarrow -\frac{1}{a}\partial_\phi \Phi_m^i \Big|_{r=a} = -\frac{1}{a}\partial_\phi \Phi_m^o \Big|_{r=a}$$

2

Einnig er Φ_m „samfellt" í sívalnings-yfirborðinu" þannig að $B_n^i = B_n^o \leftarrow (\mu H_n^i = \mu_0 H_n^o)$

3

Almenna lausnin er

$$\phi_m^n(r,\phi) = r^n\left[A_n \sin(n\phi) + B_n \cos(n\phi)\right]$$

$$r^{-n}\left\{A_n' \sin(n\phi) + B_n' \cos(n\phi)\right\} \qquad \text{ef } n \neq 0$$

1 -> jafnstætt í horninu ϕ

$$\phi_m^i(r,\phi) = \sum_{n=1}^{\infty} B_n r^n \cos(n\phi), \qquad \text{ef } \underline{r<a}$$

$$\phi_m^o(r,\phi) = \sum_{n=1}^{\infty} B_n' r^{-n} \cos(n\phi) \underbrace{-\frac{1}{\mu_0} B_0 r \cos\phi}, \qquad \underline{r>a}$$

hér erum við búin að nota 1

3 $-\mu \partial_r \Phi_m^i(a,\phi) = -\mu_0 \partial_r \Phi_m^o(a,\phi)$

$$\mu \sum_{n=1}^{\infty} n B_n a^{n-1} \cos(n\phi) = \mu_0 \sum_{n=1}^{\infty} (-n) B_n' a^{-n-1} \cos(n\phi) - B_0 \cos\phi$$

2 $\rightarrow$ $$\sum_{n=1}^{\infty} n B_n a^{n-1} \sin(n\phi) = \sum_{n=1}^{\infty} n B_n' a^{-n-1} \sin(n\phi) - \frac{1}{\mu_0} B_0 \sin\phi$$

þetta þarf að gilda fyrir öll horn ϕ

3 $\rightarrow$ $$\begin{cases} \mu B_1 = -\mu_0 B_1' a^{-2} - B_0 & \text{ef } n = 1 \\ \mu B_n a^{n-1} n = \mu_0 B_n' a^{-n-1} (-n) & \text{ef } n \neq 1 \end{cases}$$

$$② \longrightarrow \begin{cases} B_1 = B_1' a^{-2} - \frac{1}{\mu_0} B_0 & \text{ef } n=1 \\ B_n a^{n-1} n = B_n' a^{-n-1} n & \text{ef } n \neq 1 \end{cases}$$

Seinni tvö skilyrðin í hvert skiptið ganga ekki upp saman $\longrightarrow$ $B_n = B_n' = 0$ ef $n \neq 1$

Eftir stendur

$$\begin{cases} \mu B_1 + \mu_0 B_1' a^{-2} = -B_0 \\ B_1 - B_1' a^{-2} = -B_0 \frac{1}{\mu_0} \end{cases} , \qquad \mu_r = \frac{\mu}{\mu_0}$$

$$\to B_1 = -\frac{2B_0}{\mu_0 + \mu} , \qquad B_1' = -\frac{a^2 B_0 (\mu_0 - \mu)}{(\mu_0 + \mu)\mu_0}$$

$$= \frac{a^2 B_0 (\mu - \mu_0)}{(\mu_0 + \mu)\mu_0}$$

Sidan

$$B_1 = -\frac{2B_0}{\mu_0(1+\mu_r)} \,, \qquad B_1' = \frac{a^2 B_0(\mu_r - 1)}{\mu_0(1+\mu_r)}$$

Svo að lausnin er

$$\boxed{\begin{aligned} \phi_m^i(r,\phi) &= -\frac{2B_0 r}{\mu_0(1+\mu_r)}\cos\phi \\ \phi_m^o(r,\phi) &= \frac{a^2 B_0(\mu_r-1)}{\mu_0(1+\mu_r)r}\cos\phi - \frac{B_0}{\mu_0} r\cos\phi \end{aligned}}$$

Hér sést að $\phi_m^i(a,\phi) = \phi_m^o(a,\phi)$

$\bar{B}_n$ er samfellt í yfirborðinu, $\bar{\nabla}\cdot\bar{B} = 0$

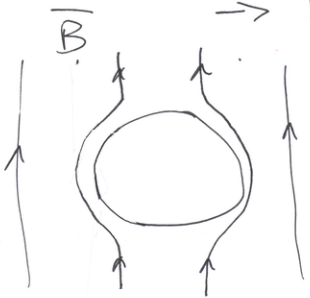
og $\lim\limits_{\mu_r \to 1} \phi_m^o(r,\phi) = -\frac{B_0}{\mu_0} r\cos\phi$, $\lim\limits_{\mu_r \to 1} \phi_m^i(r,\phi) = -\frac{B_0}{\mu_0} r\cos\phi$

$$\bar{B} = -\mu \bar{\nabla} \phi_m$$

$$\rightarrow \bar{B}^i = \hat{a}_r \frac{2B_0 \mu_r}{(1+\mu_r)} \cos\phi - \frac{2B_0 \mu_r}{(1+\mu_r)} \sin\phi \cdot \hat{a}_\phi$$

Sterkleg andseglum: $\mu_r \rightarrow 0$

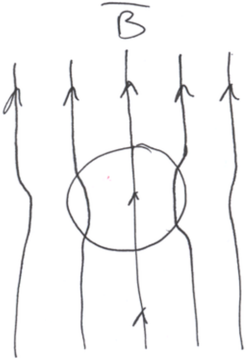
$$\rightarrow \bar{B}^i \rightarrow 0$$



Segulflóði sviðið forðast sterkliga andseglandi efni, eins og ofurleiðara

Sterkt meðseglun (járnseglun)

$\mu_r >> 1$



Segulsviðið dregst að sívalningnum, er segulviðnámið (reluctance) þar lægra

Nú gildir

$$\overline{H} = \frac{\overline{B}}{\mu_0} - \overline{M} \quad \rightarrow \quad \overline{M} = \frac{\overline{B}}{\mu_0} - \overline{H}$$

fyrir utan sivalningin er $\overline{M}^o = 0$, þui þar er $\overline{H}^o = \frac{\overline{B}^o}{\mu_0}$, en fyrir innan er $\overline{M}^i = \frac{\overline{B}^i}{\mu_0} - \overline{H}^i$

$$\rightarrow \overline{M}^i = \frac{\overline{B}^i}{\mu_0} - \frac{\overline{B}^i}{\mu} = \overline{B}^i\left(\frac{1}{\mu_0} - \frac{1}{\mu}\right) = \frac{\overline{B}^i}{\mu}(\mu_r - 1)$$

$\rightarrow \overline{M}^i \rightarrow 0$ <u>þegar $\mu_r \rightarrow 1$</u>

og <u>fyrir sterka andseglum</u> $\mu_r \approx 0$ $\quad \overline{M}^i = -\frac{\overline{B}^i}{\mu}$

sum nógvir til at eyða ytrasíðini innan efnins