Sívalningshol $\quad d = a$

Skrifum

$$\bar{E} = \bar{E}_T + \hat{a}_z E_z$$

$$\bar{H} = \bar{H}_T + \hat{a}_z H_z$$

$$\bar{\nabla} = \bar{\nabla}_T + \hat{a}_z \partial_z$$

Við höfum Maxwells jöfnurnar

(1) $\bar{\nabla} \times \bar{E} = -i\omega\mu \bar{H}$, $\bar{\nabla} \times \bar{H} = i\omega\epsilon \bar{E}$ (2)

(3) $\bar{\nabla} \cdot \bar{E} = 0$, $\bar{\nabla} \cdot \bar{H} = 0$ (4)

Þetta eru grófar útlínur lausnar. Ég hef mestan áhuga á að sjá hvernig sviðin líta út í holinu

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(3) $\rightarrow (\bar{\nabla}_T + \hat{a}_z \partial_z) \cdot (\bar{E}_T + \hat{a}_z E_z) = 0 \rightarrow \underline{\bar{\nabla}_T \cdot \bar{E}_T = -\partial_z E_z}$

(4) $\rightarrow \underline{\bar{\nabla}_T \cdot \bar{H}_T = -\partial_z H_z}$

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(1) $(\bar{\nabla}_T + \hat{a}_z \partial_z) \times (\bar{E}_T + \hat{a}_z E_z) = -i\omega\mu (\bar{H}_T + \hat{a}_z H_z)$

sem leiðir til

{Byrjum eins og fyrir bylgjuleiðara og lokum síðar}

$$\underline{\overline{\nabla}_T \times \overline{E}_T = -i\omega\mu H_z \hat{a}_z} \quad *$$

og

$$\overline{\nabla}_T \times (\hat{a}_z E_z) + \hat{a}_z \partial_z \times \overline{E}_T + \overline{\nabla}_T \times \overline{E}_T = -i\omega\mu \overline{H}_T - i\omega\mu \hat{a}_z H_z$$

notum *

$$\overline{\nabla}_T \times (\hat{a}_z E_z) + \hat{a}_z \partial_z \times \overline{E}_T = -i\omega\mu \overline{H}_T$$

eða

$$(\overline{\nabla}_T E_z) \times \hat{a}_z + \gamma \overline{E}_T \times \hat{a}_z = -i\omega\mu \overline{H}_T$$

Eins fæst

$$(\overline{\nabla}_T H_z) \times \hat{a}_z + \gamma \overline{H}_T \times \hat{a}_z = +i\omega\epsilon \overline{E}_T$$

$k^2 = \omega^2 \mu\epsilon$

h^2 eigingildi jöfnu

$k^2 = \gamma^2 + h^2$

sem má skrifa sem

$$\boxed{\begin{aligned} \overline{E}_T &= -\frac{1}{h^2}\left\{\gamma \overline{\nabla}_T E_z - i\omega\mu \hat{a}_z \times \overline{\nabla}_T H_z\right\} \\ \overline{H}_T &= -\frac{1}{h^2}\left\{\gamma \overline{\nabla}_T H_z + i\omega\epsilon \hat{a}_z \times \overline{\nabla}_T E_z\right\} \end{aligned}}$$

Setjum enda lokins á 0 og d, þá fáum við til viðbótar við jaðarskilyrðin á boguaflötinum (bylgustokknum)

$$H_z = 0 \quad \text{fyrir} \quad z=0,\ z=d \qquad \underline{TE}$$

$$\partial_z E_z = 0 \qquad -\|- \qquad \underline{TM}$$

Lokum lokinu

$\gamma \to i\frac{p\pi}{d}$

$$\omega^2 = \frac{1}{\mu\epsilon}\left(\left(\frac{x_{mn}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right)$$

Lausnirnar eru því með hjálp 10-5.2 og 10-5.3 í bók

$$E_z(r,\phi,z) = A_{mnp} J_m\left(x_{mn}\frac{r}{a}\right)\cos(m\phi)\cos\left(\frac{p\pi}{d}z\right) \qquad \boxed{TM_{mnp}}$$

x_{mn} er n-ta núllstöð J_m

$$m = 0,1,2,\dots \qquad n = 1,2,3,\dots \qquad p = 0,1,2$$

↑ uppfyllir jaðarskilyrði

$$\bar{E}_T = -\frac{1}{h^2}\gamma\,\bar{\nabla}_T E_z$$

$$\bar{H}_T = -\frac{i}{h^2}\omega\epsilon\,\hat{a}_z \times \bar{\nabla}_T E_z$$

$$\bar{\nabla}_T = \hat{a}_r \partial_r + \hat{a}_\phi \frac{1}{r}\partial_\phi$$

$$E_r = -A_{mnp}\frac{\gamma}{h} J_m'(hr)\cos(m\phi)\cos\left(\frac{p\pi}{d}z\right)$$

$$h = \frac{x_{mn}}{a}$$

$$E_\phi = +A_{mnp}\frac{\gamma m}{h^2 r} J_m(hr)\sin(m\phi)\cos\left(\frac{p\pi}{d}z\right)$$

$$H_\phi = -A_{mnp}\frac{i\omega\epsilon}{h} J_m'(hr)\cos(m\phi)\cos\left(\frac{p\pi}{d}z\right)$$

$$H_r = A_{mnp}\frac{m}{r} J_m\left(x_{mn}\frac{r}{a}\right)\sin(m\phi)\cos\left(\frac{p\pi}{d}z\right)$$

Fyrir TM 010 er því

$$E_z = A_{010} J_0 \left(x_{01} \frac{r}{a} \right)$$

$$H_\phi = -i A_{010} \frac{\omega \epsilon}{h} J_0' \left(x_{01} \frac{r}{a} \right) = i A_{010} \frac{\omega \epsilon}{h} J_1 \left(x_{01} \frac{r}{a} \right)$$

Til em TM_{110}, TM_{011} og TM_{111} en ég er forvitinn um TM_{110}

$$E_z = A_{110} J_1 \left(x_{11} \frac{r}{a} \right) \cos(\phi)$$

$$H_\phi = -A_{110} \frac{i \omega \epsilon}{h} \cdot J_1' \left(x_{11} \frac{r}{a} \right) \cdot \cos(\phi)$$

$$H_r = A_{110} \frac{1}{r} J_1 \left(x_{11} \frac{r}{a} \right) \sin(\phi)$$

p þarf að vera 1 til þess að fá E_r og E_ϕ

TE_{mnp}

$$\boxed{\overline{E}_T = \frac{i}{h^2}\omega\mu\hat{a}_z \times \overline{\nabla}_T H_z}$$

$$H_z(r,\phi,z) = B_{mnp}\, J_m\left(\frac{x'_{mn}}{a} r\right) \cos(m\phi) \sin\left(\frac{p\pi}{d} z\right)$$

$$m = 0, 1, 2, \ldots \qquad n = 1, 2, 3, \ldots \qquad \underline{p = 1, 2, 3, \ldots}$$

~~Skoda~~ TE_{111}

$$H_z = B_{111}\, J_1\left(\frac{x'_{11}}{a} r\right) \cos(\phi) \sin\left(\frac{\pi z}{d}\right)$$

$$E_\phi = B_{mnp}\, J'_m\left(\frac{x'_{mn}}{a} r\right) \cos(m\phi) \sin\left(\frac{p\pi}{d} z\right) \frac{i}{h}\omega\mu$$

$$E_r = +\frac{i\omega\mu}{h^2} B_{mnp}\, J_m\left(\frac{x'_{mn}}{a} r\right) \sin(m\phi) \sin\left(\frac{p\pi}{d} z\right)$$

$\vdots$

$$H_z = B_{111} J_1\left(\frac{x'_{11}}{a} r\right) \cos\phi \sin\left(\frac{\pi z}{d}\right)$$

$$E_\phi = \frac{i}{hr}\omega\mu B_{111} J'_1\left(\frac{x'_{11}}{a} r\right) \cos\phi \sin\left(\frac{\pi z}{d}\right)$$

$$E_r = \frac{i\omega\mu}{h^2} B_{111} J_1\left(\frac{x'_{11}}{a} r\right) \cos\phi \sin\left(\frac{\pi z}{d}\right)$$

$$H_\phi = \cdots$$

Er hægt að sjá hér tvær hringskautaðar bylgjur leggjast saman? Önnur fer í $+Z$ og hin í $-Z$ stefnu?

Loftnet, longd L með streym

$$i(z,t) = \mathrm{Re}\left\{ I_0 \sin\left(\frac{2\pi |z|}{L}\right) e^{i\omega t} \right\} \quad \text{fyrir } |z| \leq \frac{L}{2}$$

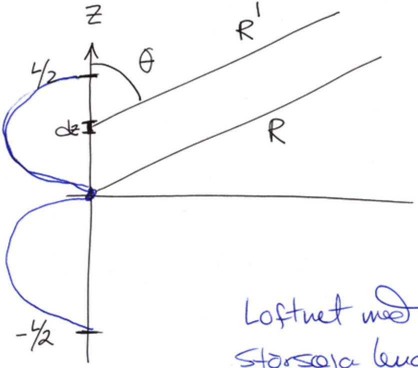
1 Teikna graf av streymi $i(z)$

$$i(z) = I_0 \sin\left(\frac{2\pi |z|}{L}\right)$$

Teiknum tað

$$\frac{i\left(\frac{z}{L}\right)}{I_0} = \sin\left(2\pi \left|\frac{z}{L}\right|\right)$$

Sjá næstu síðu



Loftnet með størsøga longd

Kønnum fjarsviðin

2 Finna geislunarmynstur

$$i(z) = I_0 \sin\left(\frac{2\pi |z|}{L}\right)$$

fjarsvið, notum að fræði Cheng, $R \gg L$

$$R' = \left\{R^2 + z^2 - 2Rz\cos\theta\right\}^{1/2} \simeq R - z\cos\theta$$ fyrir fasaþátt

og $$\frac{1}{R'} \sim \frac{1}{R}$$

$$dE_\theta = \eta_0 \, dH_\phi = i \, \frac{i(z)\,dz}{4\pi}\left\{\frac{e^{-i\beta R'}}{R'}\right\}\eta_0 \beta \sin\theta$$

og því með nálguninni verður fjarsviðið

$$E_\theta = \eta_0 H_\phi = i\,\frac{I_0 \eta_0 \beta \sin\theta}{4\pi R}\, e^{-i\beta R} \int_{-L/2}^{+L/2} dz\, e^{i\beta z\cos\theta} \sin\left(\frac{2\pi|z|}{L}\right)$$

skoDum heiZdict

$$\int_{-L/2}^{L/2} dz\, e^{i\beta z\cos\theta} \sin\left(\frac{2\pi|z|}{L}\right) = -\int_{-L/2}^{0} dz\, e^{i\beta z\cos\theta} \sin\left(\frac{2\pi z}{L}\right) + \int_{0}^{L/2} dz\, e^{i\beta z\cos\theta} \sin\left(\frac{2\pi z}{L}\right)$$

$$= \int_0^{L/2} dz\, \sin\left(\frac{2\pi z}{L}\right)\left\{ e^{-i\beta z\cos\theta} + e^{i\beta z\cos\theta} \right\}$$

$$\cancel{= \frac{1}{2i}\int_0^{L/2} dz\, \left\{ e^{i\frac{2\pi z}{L}} - e^{-i\frac{2\pi z}{L}} \right\}\left\{ e^{i\beta z\cos\theta} + e^{-i\beta z\cos\theta} \right\}}$$

$$= 2 \int_0^{L/2} dz \, \mathrm{Sin}\left(\frac{2\pi z}{L}\right) \mathrm{Cos}\left(\beta z \, \mathrm{Cos}\theta\right)$$

$$= 2L \int_0^{1/2} du \, \mathrm{Sin}(2\pi u) \, \mathrm{Cos}(\beta L u \, \mathrm{Cos}\theta)$$

$$= 2L \, \frac{2\pi \left\{ 1 + \mathrm{Cos}\left(\frac{\beta L}{2} \mathrm{Cos}\theta\right) \right\}}{(2\pi)^2 - (\beta L \, \mathrm{Cos}\theta)^2}$$

bør sem òg notat i wxMaxima og bør saman ved (GR- 2.533-1) med korrettingu

$$\int dx \, \mathrm{Sin}(ax) \, \mathrm{Cos}(bx) = -\frac{\mathrm{Cos}\{(a+b)x\}}{2(a+b)} - \frac{\mathrm{Cos}\{(a-b)x\}}{2(a-b)}$$

ef $a \neq b$

því fæst

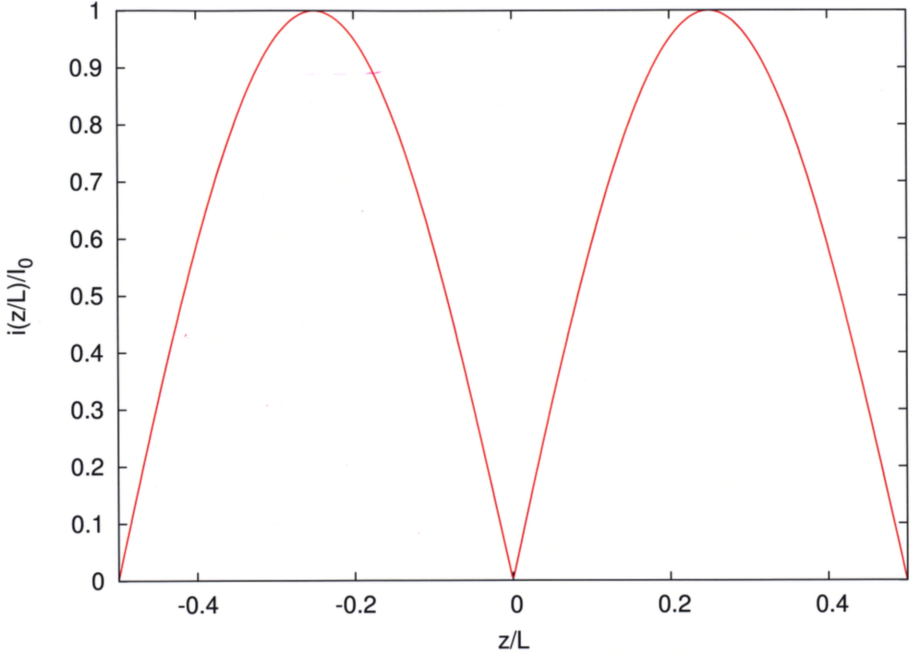
$$E_\theta = i\,\frac{L I_0 \eta_0 \beta}{R}\, e^{-i\beta R}\, \frac{\left[1+\cos\left(\frac{\beta L}{2}\cos\theta\right)\right]}{(2\pi)^2-(\beta L\cos\theta)^2}\,\sin\theta$$

Ef loftnetið er lítið $L \ll \lambda$, $\beta L \ll 1$ þá fæst

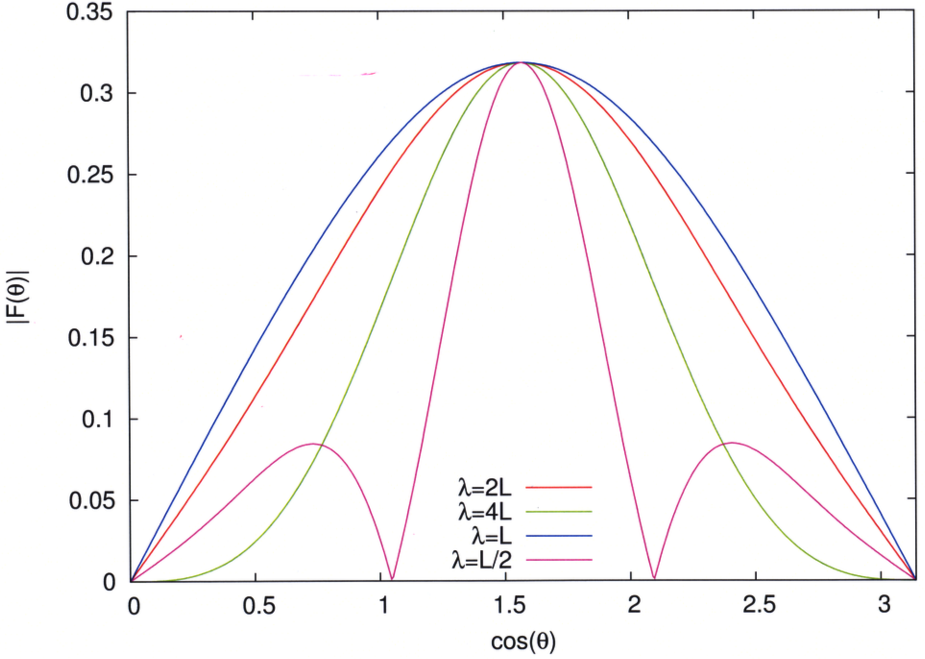
$$E_\theta \sim \frac{i\,\eta_0 I_0 \beta L}{2\pi^2 R}\, e^{-i\beta R}\,\sin\theta$$

þá geislar loftnetið eins og tvískaut

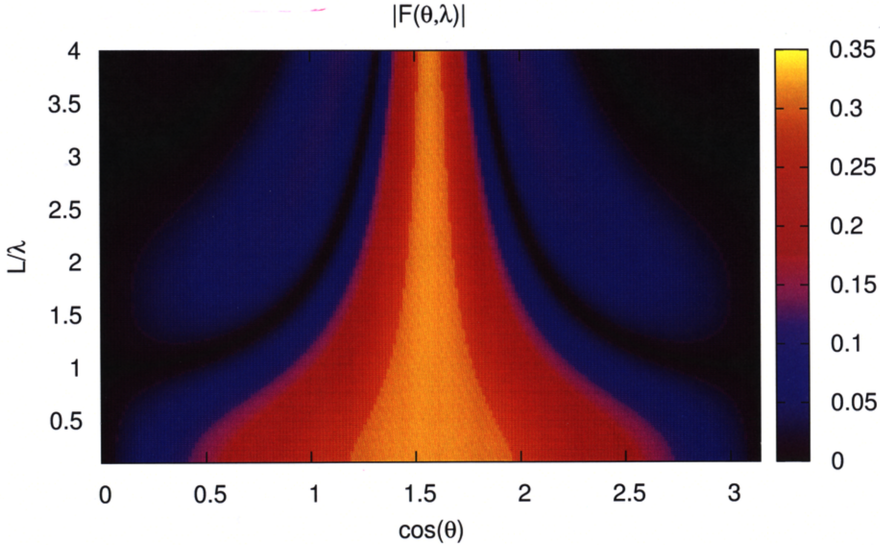
Skoðum önnur tilvik á næstu gröfum



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
i(z/L)/I0
-0.4
-0.2
0
0.2
0.4
z/L



0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
|F(θ)|
0
0.5
1
1.5
2
2.5
3
cos(θ)
λ=2L
λ=4L
λ=L
λ=L/2


|F(θ,λ)|
L/λ
4
3.5
3
2.5
2
1.5
1
0.5
0
0.5
1
1.5
2
2.5
3
cos(θ)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0