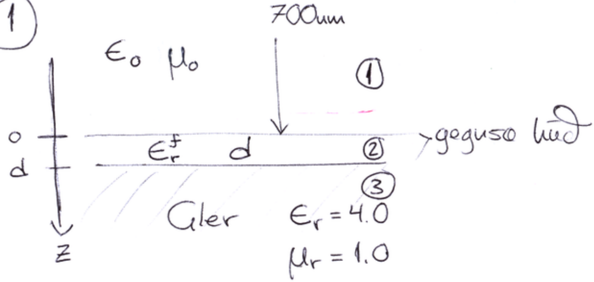
①



Þetta er því dæmi um ferð geisla um þunnt lag milli tveggja efniskerfa, þ.e. loft - lag - gler.

Sýnidæmi 8-12 í bók og Mynd 8-15 eiga við það.

Hér þarf að skeyta saman lausnum

$$\bar{E}_1 = \hat{a}_x \left\{ E_{i0} e^{-i\beta_1 z} + E_{r0} e^{+i\beta_1 z} \right\}$$

$$\bar{H}_1 = \hat{a}_y \frac{1}{\eta_1} \left\{ E_{i0} e^{-i\beta_1 z} - E_{r0} e^{+i\beta_1 z} \right\}$$

①

$$\bar{E}_2 = \hat{a}_x \left\{ E_2^+ e^{-i\beta_2 z} + E_2^- e^{+i\beta_2 z} \right\}$$

$$\bar{H}_2 = \hat{a}_y \frac{1}{\eta_2} \left\{ E_2^+ e^{-i\beta_2 z} - E_2^- e^{+i\beta_2 z} \right\}$$

②

$$\bar{E}_3 = \hat{a}_x E_3^+ e^{-i\beta_3 z}$$

$$\bar{H}_3 = \hat{a}_y \frac{E_3^+}{\eta_3} e^{-i\beta_3 z}$$

③

Með jaðarskilyrðum

$$\bar{E}_1(0) = \bar{E}_2(0) \qquad \bar{E}_2(d) = \bar{E}_3(d)$$

og

$$\bar{H}_1(0) = \bar{H}_2(0) \qquad \bar{H}_2(d) = \bar{H}_3(d)$$

Eg fer með höfunda bókar og skilgreini heildarsamviðnám bylgju

$$Z(z) = \frac{E_x(z)}{H_y(z)}$$

heildarsvið

Fyrir einn skilflöt er þá (í efni 1)

$$Z_1(z) = \eta_1 \frac{e^{-i\beta_1 z} + \Gamma e^{+i\beta_1 z}}{e^{-i\beta_1 z} - \Gamma e^{+i\beta_1 z}}, \quad \text{með } \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

fyrir lóðrétta innkomu bylgju

Reikna skrifar síðan $Z_1(-\ell)$ og í beinu framhaldi $Z_2(0) = Z_2(d-d)$

$$Z_2(0) = \eta_2 \frac{\eta_3 \cos(\beta_2 d) + i\eta_2 \sin(\beta_2 d)}{\eta_2 \cos(\beta_2 d) + i\eta_3 \sin(\beta_2 d)}$$

Speglunin í $z=0$ ~~roest~~ af

i ~~stae~~ η_2

$$\Gamma_0 = \frac{E_{r0}}{E_{i0}} = -\frac{H_{r0}}{H_{i0}} = \frac{Z_2(0) - \eta_1}{Z_2(0) + \eta_1}$$

[z_2 inniheldur upplýsingar um bylgjuna sum speglast í $z=d$]

~~Bødo~~ var um enga speglun þ.e. $\Gamma_0 = 0$ fyri vissa bylgjulengd.

Tá $\Gamma_0 = 0$. Til þess þarf $Z_2(0) = \eta_1$

$$\rightarrow \quad \eta_2\left\{\eta_3 \cos(\beta_2 d) + i\eta_2 \sin(\beta_2 d)\right\} = \eta_1\left\{\eta_2 \cos(\beta_2 d) + i\eta_3 \sin(\beta_2 d)\right\}$$

Í bókinni skrifar Cheng þessa jöfnu fyrir raun og þverhluta

$$\eta_3 \cos(\beta_2 d) = \eta_1 \cos(\beta_2 d) \qquad \text{Raun}$$

$$\eta_2^2 \sin(\beta_2 d) = \eta_1 \eta_3 \sin(\beta_2 d) \qquad \text{þver}$$

Við vitum að $\eta_3 \neq \eta_1$ (gler og loft)

$$\rightarrow \left\{ \begin{array}{l} \cos(\beta_2 d) = 0 \\ \quad \text{og} \\ \boxed{\eta_2^2 = \eta_1 \eta_3} \end{array} \right. \qquad \text{er lausn}$$

$$\boxed{\begin{aligned} \beta_2 d &= (2n+1)\frac{\pi}{2}, \quad n = 0, 1, 2, \ldots \\ \text{eða} \quad d &= (2n+1)\frac{\pi}{2\beta_2} \\ &= (2n+1)\frac{\lambda_2}{4} \end{aligned}}$$

því $\beta_2 = \dfrac{2\pi}{\lambda_2}$

pui fast innklitt

$$d = \frac{(2n+1)\lambda_2}{4} \quad \text{og} \quad \eta_2^2 = \eta_1 \eta_3 \quad \rightarrow \quad \frac{\mu_1}{\epsilon_2} = \sqrt{\frac{\mu_1 \mu_1}{\epsilon_1 \epsilon_3}}$$

$$\text{Sida} \quad \epsilon_{2r} = \sqrt{\epsilon_{1r}\,\epsilon_{3r}}$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{\omega\sqrt{\mu_1 \epsilon_2}} \rightarrow \lambda_2 = \frac{\lambda_0}{\sqrt{\epsilon_{2r}}}, \quad \lambda_0 = \frac{2\pi}{\omega\sqrt{\mu_0 \epsilon_0}}$$

I okkordomi er $\epsilon_{1r} = 1 \quad \rightarrow \quad \epsilon_{2r} = \sqrt{\epsilon_{3r}}$

$$\hookrightarrow d = \frac{(2n+1)}{4}\frac{\lambda_0}{\sqrt{\epsilon_{2r}}} = \frac{(2n+1)\lambda_0}{4\sqrt{\sqrt{\epsilon_{3r}}}} \qquad \epsilon_{3r} = 4$$

$\rightarrow$ minnsta pykkt $\quad d = \dfrac{\lambda_0}{4\cdot\sqrt{2}} = \dfrac{700\,\text{nm}}{4\sqrt{2}} \approx 124\ \text{nm}$

Reikna $\Gamma(\lambda_0)$. Við fundum ~~Z~~ eða ~~Z~~ í $z=0$

$$\Gamma_0 = \frac{Z_2(0) - \eta_1}{Z_2(0) + \eta_1} \quad , \quad 1 \text{ var loft} \rightarrow \eta_1 = \eta_0$$

$$\rightarrow \quad \Gamma_0 = \frac{\left(\frac{Z_2(0)}{\eta_0}\right) - 1}{\left(\frac{Z_2(0)}{\eta_0}\right) + 1} \qquad \eta_2 = \sqrt{\eta_1 \eta_3} = \sqrt{\eta_0 \eta_3}$$

$$\frac{Z_2(0)}{\eta_0} = \frac{\eta_2}{\eta_0} \frac{\eta_3 \cos(\beta_2 d) + i\eta_2 \sin(\beta_2 d)}{\eta_2 \cos(\beta_2 d) + i\eta_3 \sin(\beta_2 d)} = \sqrt{\frac{\eta_3}{\eta_0}} \frac{\eta_3 + i\eta_2 \tan(\beta_2 d)}{\eta_2 + i\eta_3 \tan(\beta_2 d)}$$

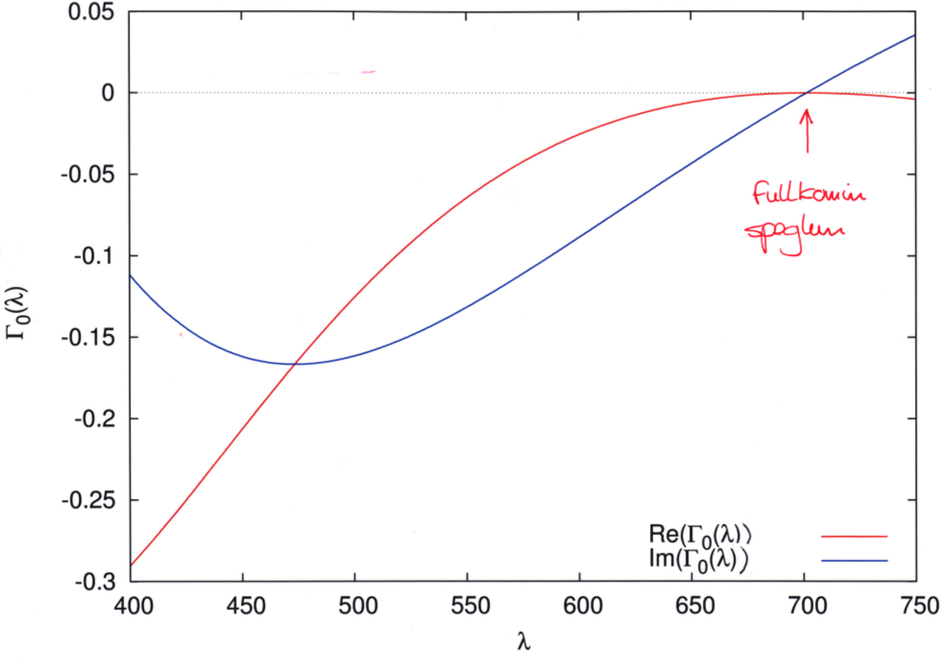
$$= \sqrt{\frac{\eta_3}{\eta_0}} \frac{\frac{\eta_3}{\eta_0} + i\sqrt{\frac{\eta_3}{\eta_0}} \tan(\beta_2 d)}{\sqrt{\frac{\eta_3}{\eta_0}} + i\frac{\eta_3}{\eta_0} \tan(\beta_2 d)}$$

$$\frac{\eta_3}{\eta_0} = \sqrt{\frac{\mu_0\, \epsilon_0}{\epsilon_3\, \mu_0}} = \frac{1}{\sqrt{\epsilon_{3r}}} = \frac{1}{2}$$

$$\beta_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi\sqrt{\epsilon_{2r}}}{\lambda_0} = \frac{2\pi}{\lambda_0}\sqrt{\sqrt{\epsilon_{3r}}}$$

$$\rightarrow \beta_2 d = 2\pi\sqrt{\sqrt{\epsilon_{3r}}}\,\frac{d}{\lambda_0} = 2\pi\sqrt{2}\,\frac{d}{\lambda_0}$$

Sjā graf ā nästa sidan


0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
400
450
500
550
600
650
700
750
$\Gamma_0(\lambda)$
λ
$\mathrm{Re}(\Gamma_0(\lambda))$
$\mathrm{Im}(\Gamma_0(\lambda))$
fullkomin
speglun

2 Dæmi 8-45 í bók

umskrifa í

ϵ_{r1}, ϵ_{r2} og θ_i

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\frac{\epsilon_{r1}}{\epsilon_{r2}} = 2.25$$

$$\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Ljós úr þéttu efni í þunnt

gerum ráð fyrir að $\mu_1 = \mu_2 = \mu_0$

lögmál Snells

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta_i} = \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}$$

$$\Gamma_\perp = \frac{\frac{\eta_2}{\eta_1}\cos\theta_i - \cos\theta_t}{\frac{\eta_2}{\eta_1}\cos\theta_i + \cos\theta_t}$$

$$\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}} \rightarrow \frac{\eta_2}{\eta_1} = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$$

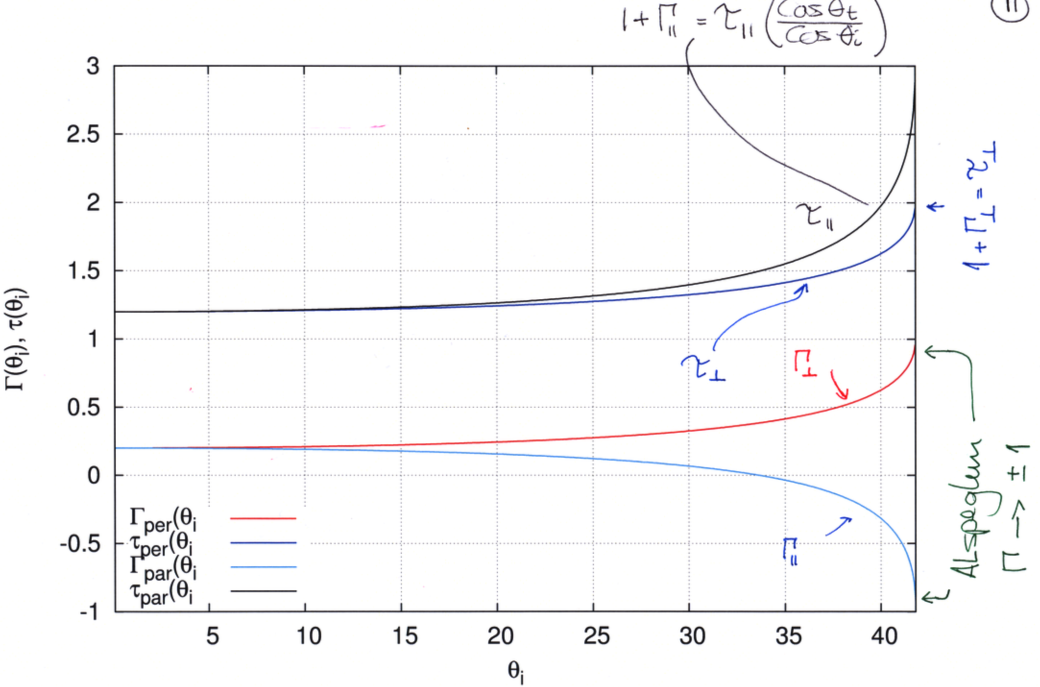
$$\Gamma_{\perp}(\theta_i) = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\, \cos\theta_i - \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}} \sin^2\theta_i}}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\, \cos\theta_i + \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}} \sin^2\theta_i}}$$

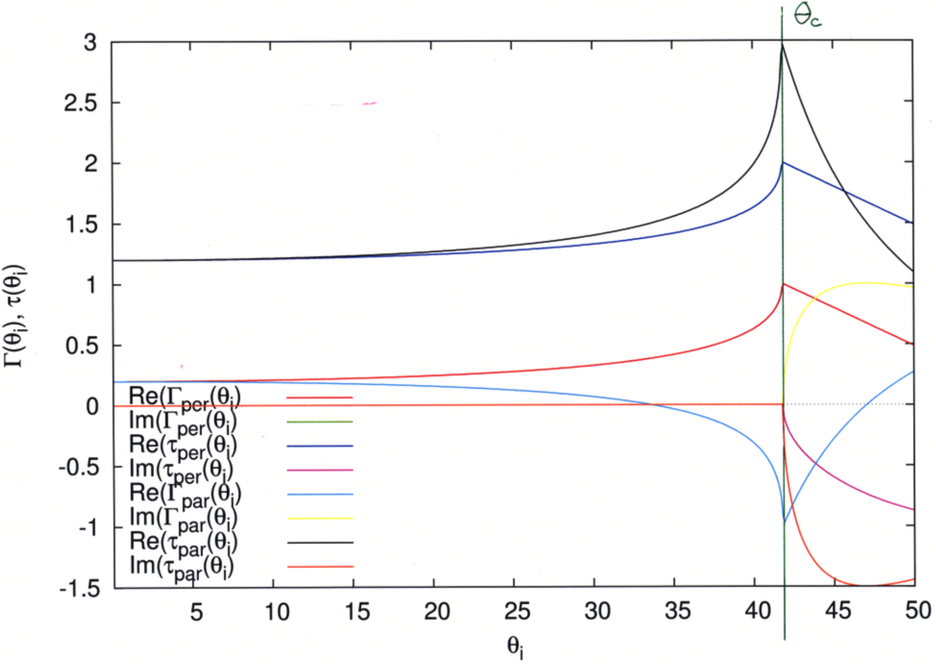
$$\tau_{\perp}(\theta_i) = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\, \cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\, \cos\theta_i + \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}} \sin^2\theta_i}}$$

Eins fast

$$\Gamma_{\parallel}(\theta_i) = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}} \sin^2\theta_i} - \cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}} \sin^2\theta_i} + \cos\theta_i}$$

$$\tau_{\parallel}(\theta_i) = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\, \cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}} \sin^2\theta_i} + \cos\theta_i}$$


$1 + \Gamma_{\parallel} = \tau_{\parallel} \left(\frac{\cos\theta_t}{\cos\theta_i} \right)$
$\tau_{\parallel}$
$1 + \Gamma_{\perp} = \tau_{\perp}$
$\tau_{\perp}$
$\Gamma_{\perp}$
$\Gamma_{\parallel}$
Alspeglen
$\Gamma \rightarrow \pm 1$
$\Gamma(\theta_i)$, $\tau(\theta_i)$
θ_i
$\Gamma_{per}(\theta_i$
$\tau_{per}(\theta_i$
$\Gamma_{par}(\theta_i$
$\tau_{par}(\theta_i$



θ_c
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
5
10
15
20
25
30
35
40
45
50
θ_i
$\Gamma(\theta_i)$, $\tau(\theta_i)$
$Re(\Gamma_{per}(\theta_i))$
$Im(\Gamma_{per}(\theta_i))$
$Re(\tau_{per}(\theta_i))$
$Im(\tau_{per}(\theta_i))$
$Re(\Gamma_{par}(\theta_i))$
$Im(\Gamma_{par}(\theta_i))$
$Re(\tau_{par}(\theta_i))$
$Im(\tau_{par}(\theta_i))$