TE-bylgjur milli samsíða platna

$E_z = 0$, x-samhverfa (forsenda)

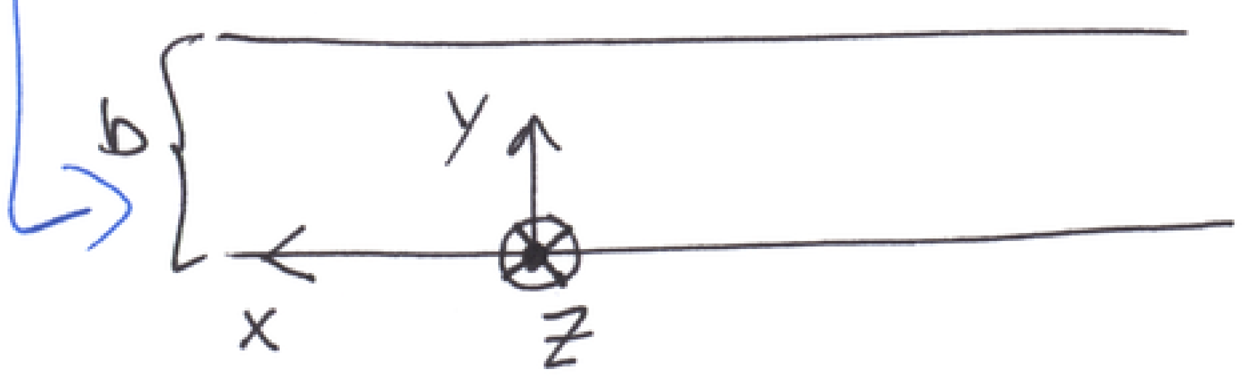
↳ $$\left(\frac{d^2}{dy^2} + h^2\right) H_z^0(y) = 0$$

$$H_z(y,z) = H_z^0(y)\, e^{-\gamma z}$$

$E_x = 0$ á plötunum

Maxwell gaf

$$E_x^0 = -\frac{i\omega\mu}{h^2}\frac{\partial H_z^0}{\partial y}$$



$$\frac{dH_z^0(y)}{dy} = 0 \quad \text{fyrir } y = 0, b$$

$$H_z^0(y) = B_n \cos\left(\frac{n\pi y}{b}\right)$$

Aðrir þættir eru (Maxwell)

$$H_y^0(y) = \frac{\gamma}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

$$E_x^0(y) = \frac{i\omega\mu}{h} B_n \sin\left(\frac{n\pi y}{b}\right)$$

$$\gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

↑ Eins og fyrir TM

2

Orkuflutningshraði

Vegna þröskulda í tíðni er ekki víst að grúpuhraði sé vel skilgreindur fyrir bylgjuleiðara

→ orkuflutningshraði

$$u_{en} = \frac{(P_z)_{ave}}{W'_{ave}}$$

með

$$(P_z)_{ave} = \int_S \bar{\mathcal{P}}_{ave} \cdot d\bar{s}$$

Meðaltalsaflið í þversniði S

og

$$W'_{ave} = \int_S \left[(w_e)_{ave} + (w_m)_{ave} \right] ds$$

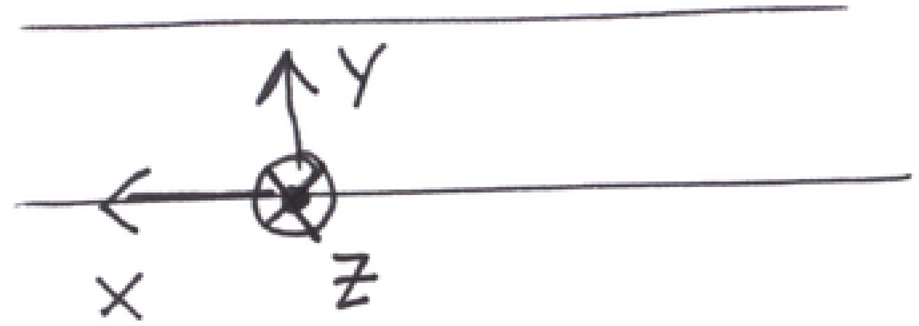
meðal orkan geymd í lengdareiningu leiðara

↑ Einingar eru þá réttar fyrir u_{en}

Meðaltal m.t.t. tíma

Dæmi

Reiknum Uen fyrir TM_n



$$E_z^0(y) = A_n \sin\left(\frac{n\pi y}{b}\right)$$

$$H_x^0(y) = \frac{i\omega\epsilon}{h} A_n \cos\left(\frac{n\pi y}{b}\right)$$

$$E_y^0(y) = -\frac{\gamma}{h} A_n \cos\left(\frac{n\pi y}{b}\right), \quad \gamma = i\beta$$

$$\bar{\mathcal{P}}_{ave} = \frac{1}{2}\mathrm{Re}\left(\bar{E}\times\bar{H}^*\right)$$

$$= \frac{1}{2}\mathrm{Re}\left(-\hat{a}_z E_y^0 H_x^{0*} + \hat{a}_y E_z^0 H_x^{0*}\right)$$

$$\rightarrow \bar{\mathcal{P}}_{ave}\cdot\hat{a}_z = -\frac{1}{2}\mathrm{Re}\left(E_y^0 H_x^{0*}\right)$$

$$= \frac{\omega\epsilon\beta}{2h^2} A_n^2 \cos^2\left(\frac{n\pi y}{b}\right)$$

$$\rightarrow (P_z)_{ave} = \int_0^b \bar{\mathcal{P}}\cdot\hat{a}_z\,dy$$

$$= \frac{\omega\epsilon\beta b}{4h^2} A_n^2$$

á einingarbreidd leiðara (x-stefna)

$$(w_e)_{ave} = \frac{\epsilon}{4} \Re\left(\bar{E} \cdot \bar{E}^*\right)$$

$$(w_m)_{ave} = \frac{\mu}{4} \Re\left(\bar{H} \cdot \bar{H}^*\right)$$

$$(w_e)_{ave} = \frac{\epsilon}{4} A_n^2 \left\{ \sin^2\left(\frac{n\pi y}{b}\right) + \frac{\beta^2}{h^2} \cos^2\left(\frac{n\pi y}{b}\right) \right\}$$

$$\rightarrow \int_0^b (w_e)_{ave}\, dy = \frac{\epsilon b}{8} A_n^2 \left\{ 1 + \frac{\beta^2}{h^2} \right\}$$

$$= \frac{\epsilon b}{8h^2} A_n^2 \left\{ h^2 + \beta^2 \right\} = \frac{\epsilon b}{8h^2} k^2 A_n^2$$

og eins fyrir segulsviðið

$$\int_0^b (w_m)_{ave}\, dy = \frac{\epsilon b}{8h^2} k^2 A_n^2$$

því fæst

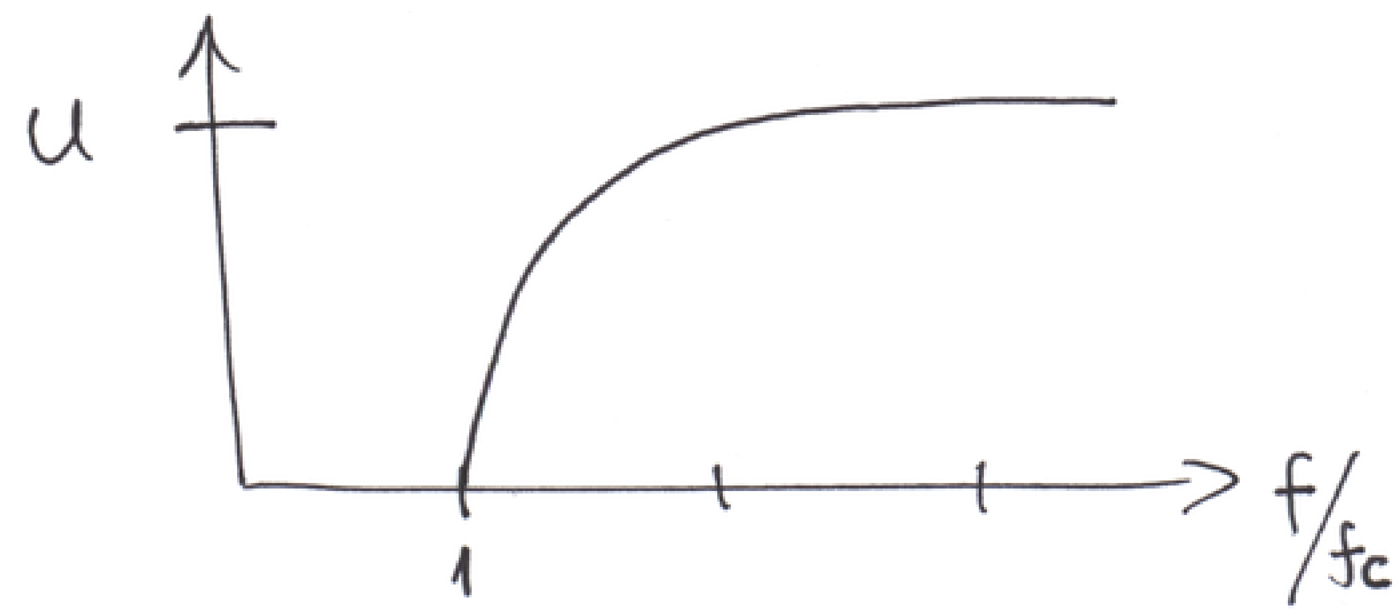
$$u_{en} = \frac{\frac{\omega \epsilon \beta b}{4h^2} A_n^2}{\frac{\epsilon b}{4h^2} k^2 A_n^2} = \frac{\omega \beta}{k^2}$$

$$= \frac{\omega}{k} \cdot \frac{\beta}{k}$$

$$= \frac{\omega}{\omega\sqrt{\mu\epsilon}} \cdot \frac{k\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{k}$$

$$= u\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= u_g$$



Rétthyrndur bylgjuleiðari

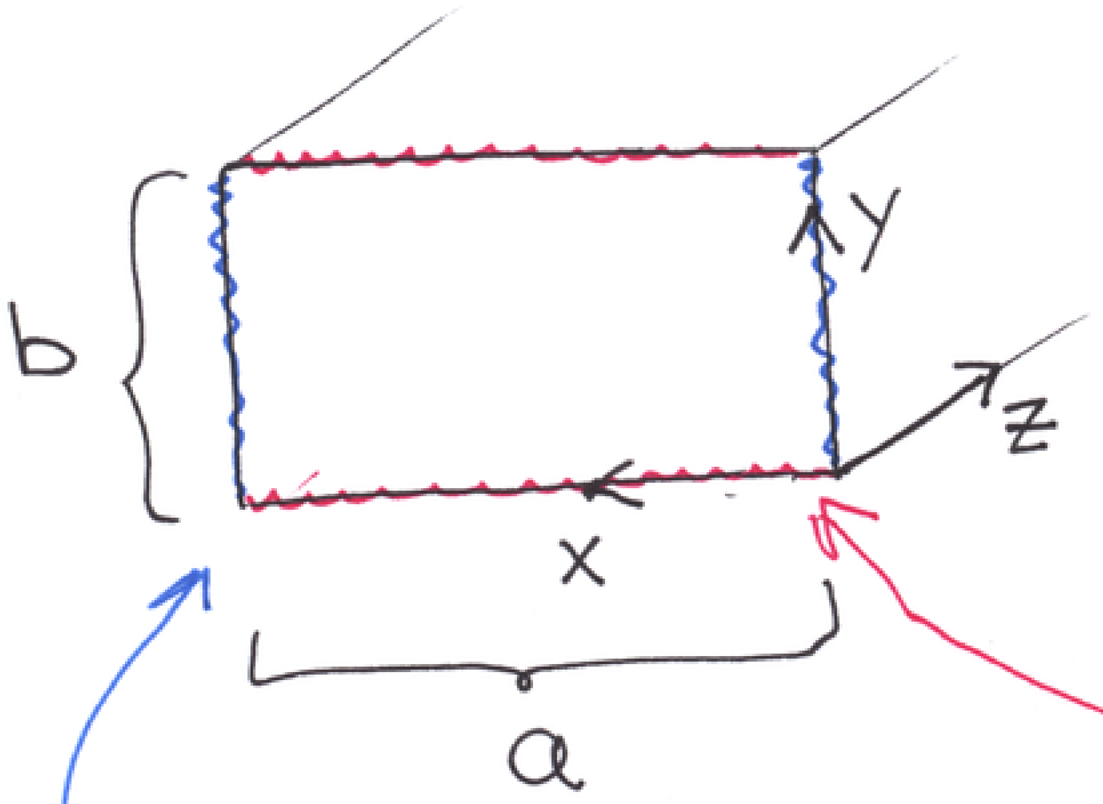
TM-bylgjur

$H_z = 0$

$$E_z(x,y,z) = E_z^0(x,y)\, e^{-\gamma z}$$

með

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) E_z^0(x,y) = 0$$



Lögunum leitir aðgreiningu breytistærða

$$E_z^0(x,y) = X(x)\, Y(y)$$

Þá fæst

$$\left.\begin{aligned} X''(x) + k_x^2 X(x) &= 0 \\ Y''(y) + k_y^2 Y(y) &= 0 \end{aligned}\right\}$$

með $k_y^2 + k_x^2 = h^2$

jaðargildi

$$E_z^0(0,y) = 0 \qquad E_z^0(x,0) = 0$$

$$E_z^0(a,y) = 0 \qquad E_z^0(x,b) = 0$$

Einu lausnirnar fyrir X og Y verða

$$\sin k_x x\,,\quad \sin k_y y$$

með

$$k_x = \frac{m\pi}{a}\,,\quad m = 1, 2, 3, \dots$$

$$k_y = \frac{n\pi}{b}\,,\quad n = 1, 2, 3, \dots$$

$$\rightarrow \boxed{E_z^0(x,y) = E_0 \sin(k_x x) \sin(k_y y)}$$

og

$$\boxed{h^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\gamma = i\beta = i\sqrt{k^2 - h^2}$$

$$= i\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Til viðbótar fæst

$$E_x^0(x,y) = -\frac{\gamma}{h^2} k_x E_0 \cos(k_x x)\sin(k_y y)$$

$$E_y^0(x,y) = -\frac{\gamma}{h^2} k_y E_0 \sin(k_x x)\cos(k_y y)$$

$$H_x^0(x,y) = \frac{i\omega\epsilon}{h^2} k_y E_0 \sin(k_x x)\cos(k_y y)$$

$$H_y^0(x,y) = -\frac{i\omega\epsilon}{h^2} k_x E_0 \cos(k_x x)\sin(k_y y)$$

$$(f_c)_{mn} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$(\lambda_c)_{mn} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

lesa sjálf um TE í rétthyrndum leiðar

$$\boxed{\text{Þar kemur í ljós að } TE_{10} \text{ er ríkjandi háttur í rétthyrndum leiðurum ef } a > b}$$

TEM-bylgjur eru ekki til í þeim

$\rightarrow$ TE og TM-bylgjur

$$\nabla^2_{r\phi} E_z^0 + (\gamma^2 + k^2) E_z^0 = 0$$

$$E_z = E_z^0 e^{-\gamma z}$$

$$E_z(r, \phi, z) = E_z^0(r, \phi) e^{-\gamma z}$$

Lausnin er

$$\boxed{E_z^0(r, \phi) = C_n J_n(hr) \cos(n\phi)}$$

Bessel fall

Hringlaga bylgjustokkar

Hnitakerfið er sívalningshnit

$$\overline{E} = \overline{E}_T + \hat{a}_z E_z$$

$$\overline{H} = \overline{H}_T + \hat{a}_z H_z$$

þverþættir

$$(E_T^o)_{TM} = \hat{a}_r E_r^o + \hat{a}_\phi E_\phi^o = -\frac{\gamma}{h^2}\overline{\nabla}_T E_z^o$$

$$= -\frac{\gamma}{h^2}\left(\hat{a}_r \frac{\partial}{\partial r} + \hat{a}_\phi \frac{\partial}{r\partial\phi}\right) E_z^o$$

$$\longrightarrow \quad E_r^o = -\frac{i\beta}{h} C_n J_n'(hr)\cos(n\phi)$$

$$E_\phi^o = \frac{i\beta n}{h^2 r} C_n J_n(hr)\sin(n\phi)$$

$$H_r^o = -\frac{i\omega\epsilon n}{h^2 r} C_n J_n(hr)\sin(n\phi)$$

$$H_\phi^o = -\frac{i\omega\epsilon}{h} C_n J_n'(hr)\cos(n\phi)$$

$$H_z^o = 0$$

Eigingildi TM háttanna finnast frá

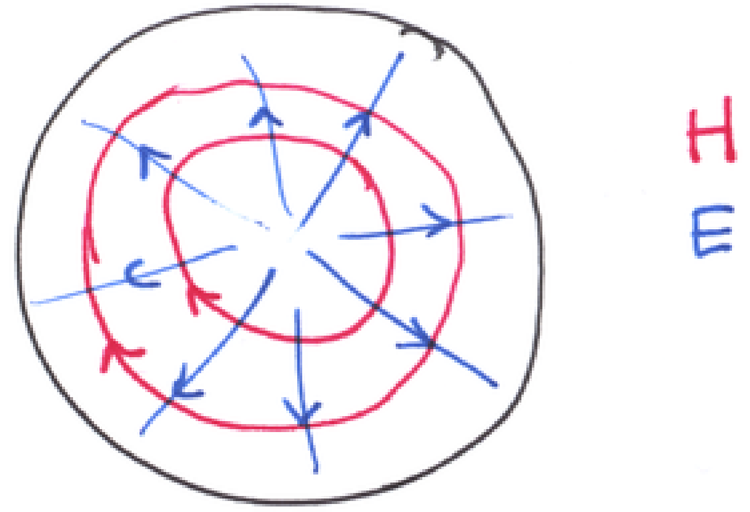
$$\boxed{J_n(ha) = 0}$$

því $E_z^o(a,\phi) = 0$

Gildin á mögulegum „h" ákvarðast frá núllstöðvum J_n

Lægsta núllstöðin er fyrir $J_0 \longrightarrow TM_{01}$-háttur er lægstur hér

Lesa sjálf um TE-bylgjur
í sívalningsbylgjustokki

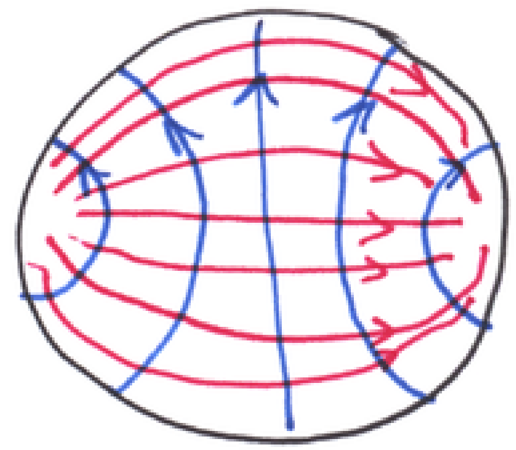


TM$_{01}$

Það kemur í ljós að $J_n'(ka)=0$
hefur lægstu rót fyrir $n=1$

$\rightarrow$ TE$_{11}$ er lægsti hátturinn

og er ríkjandi háttur í
hringlaga bylgjuleiðurum



TE$_{11}$

Bessel föll

$$u'' + \frac{1}{z} u' + \left(1 - \frac{\nu^2}{z^2}\right) u = 0$$

Lausnir eru líka til fyrir raungild ν og jafnvel tvinngild.

hefur lausnir

$$u = A_n J_n(z) + B Y_n(z)$$

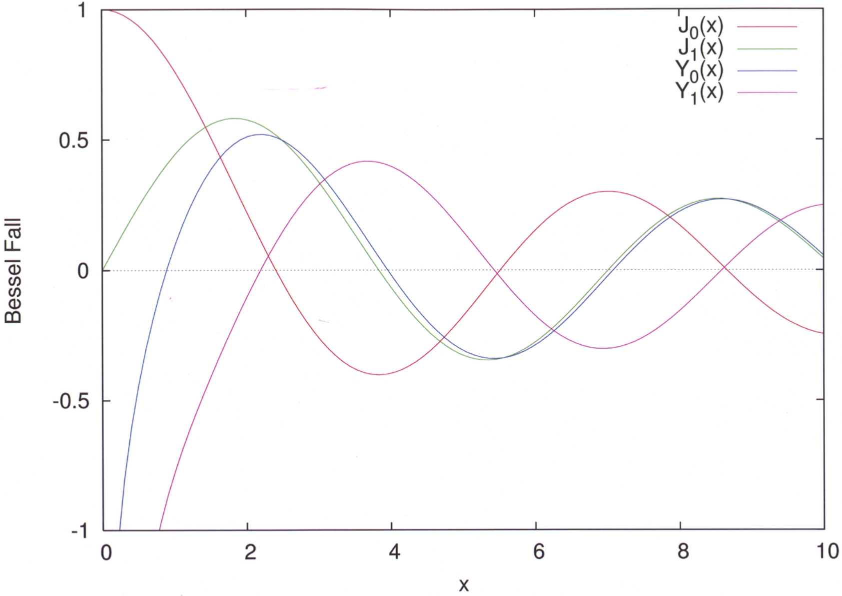
sérstöðup. í $z=0$

fyrir $\nu = n = 0, 1, 2, \ldots$

$$J_n'(z) = \frac{1}{2}\left\{ J_{n-1}(z) - J_{n+1}(z) \right\} \qquad n = 1, 2, \ldots$$

$$J_{n-1}(z) + J_{n+1}(z) = \frac{2n}{z} J_n(z)$$

$$J_0'(z) = -J_1(z)$$



$J_0(x)$
$J_1(x)$
$Y_0(x)$
$Y_1(x)$
Bessel Fall
x
1
0.5
0
-0.5
-1
0
2
4
6
8
10