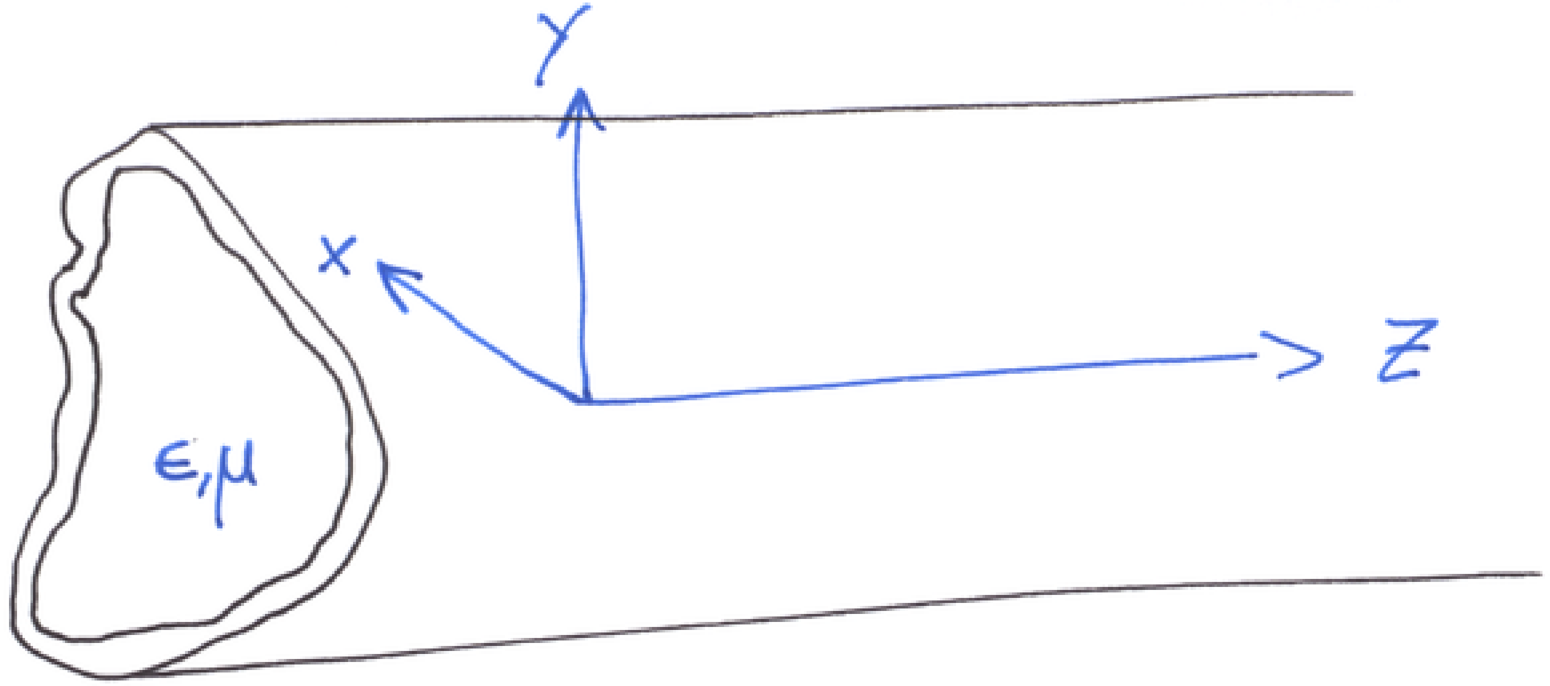
Bylgjuleiðarar



liggur í z-átt með fastan þverskurð

Bylgja berst í z-stefnu með bylgjufasta $\gamma = \alpha + i\beta = (ik_c)$

því bundist við þátti

$$e^{-\gamma z + i\omega t} = e^{-\alpha z + i(\omega t - \beta z)}$$

í sviðunum

Tímaháða rafsviðið er því

$$(*) \quad \bar{E}(x,y,z,t) = \Re\left[\bar{E}^0(x,y)\, e^{i\omega t - \gamma z}\right]$$

bara x og y

Engar hleðslur og straumar í leiðarahólfinu

$\Downarrow$

$$(\nabla^2 + k^2)\bar{E} = 0$$

$$(\nabla^2 + k^2)\bar{H} = 0$$

með $k = \omega\sqrt{\mu\epsilon}$

Vegna útlits $\overline{E}$ og $\overline{H}$ (*)

$$\nabla^2 \overline{E} = (\nabla_{xy}^2 + \nabla_z^2)\overline{E}$$

$$= \left(\nabla_{xy}^2 + \frac{\partial^2}{\partial z^2}\right)\overline{E}$$

$$= (\nabla_{xy}^2 + \gamma^2)\overline{E}$$

Helmholtz jöfnurnar ~~verða~~ verða því

$$\left\{\nabla_{xy}^2 + (\gamma^2 + k^2)\right\}\overline{E} = 0$$

$$\left\{\nabla_{xy}^2 + (\gamma^2 + k^2)\right\}\overline{H} = 0$$

En $\overline{E}$ og $\overline{H}$ tengjast líka í gegnum jöfnur Maxwells

$$\overline{\nabla} \times \overline{E} = -i\omega\mu\overline{H}$$

$$\overline{\nabla} \times \overline{H} = i\omega\epsilon\overline{E}$$

$$\Downarrow$$

$$H_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial x} - i\omega\epsilon\frac{\partial E_z^0}{\partial y}\right)$$

$$H_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial y} + i\omega\epsilon\frac{\partial E_z^0}{\partial x}\right) \qquad (**)$$

$$E_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial x} + i\omega\mu\frac{\partial H_z^0}{\partial y}\right)$$

$$E_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial y} - i\omega\mu\frac{\partial H_z^0}{\partial x}\right)$$

$$h^2 = \gamma^2 + k^2$$

* því nægir að leysa jöfnur Helmholtz fyrir E_z^0 og H_z^0 síðan ákvarðast hinir þættirnir frá jöfnum Maxwells (**)

Flokkum bylgjur

TEM: $E_z, H_z = 0$

TM: $H_z = 0$

TE: $E_z = 0$

TEM

$$E_z = 0 \text{ og } H_z = 0$$

(**) → aðeins mögulegar lausnir ef $\gamma_{TEM}^2 + k^2 = 0$

$$\rightarrow \gamma_{TEM} = ik = i\omega\sqrt{\mu\epsilon}$$

$$\rightarrow u_{p(TEM)} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\epsilon}}$$

fasahraði TEM-bylgju er óháður ω

Jöfnur Maxwells gefa

$$Z_{TEM} = \frac{E_x^0}{H_y^0} = \frac{\gamma_{TEM}}{i\omega\epsilon} = \sqrt{\frac{\mu}{\epsilon}} = \eta$$

$$\bar{H} = \frac{1}{Z_{TEM}} \hat{a}_z \times \bar{E}$$

samviðnám

Ampère Maxwell lögmálið

krefst þess að línuheildi
$\overline{H}$ um lokuðu sviðslínurnar
í xy-sléttunni sé jafnt
straumnum og færslustraumnum
í gegnum þær í z-átt

Ekki til hér í holinu

→ $\boxed{\text{TEM-bylgja er ekki til í holinu}}$

TM-bylgjur

$$H_z = 0$$

→ nægir að reikna E_z

$$\left(\nabla_{xy}^2 + h^2\right) E_z^0 = 0 \quad (1)$$

Maxwell jöfnurnar fyrir línu þættina má einfalda sem

$$\boxed{\left(E_T^0\right)_{TM} = -\frac{\gamma}{h^2}\overline{\nabla}_T E_z^0}$$

með

$$\overline{\nabla}_T E_z^0 = \left(\hat{a}_x \frac{\partial}{\partial x} + \hat{a}_y \frac{\partial}{\partial y}\right) E_z^0$$

og $$\boxed{\bar{H} = \frac{1}{Z_{TM}}\left(\hat{a}_z \times \bar{E}\right)}$$

ef $$\boxed{Z_{TM} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{\gamma}{i\omega\epsilon}}$$

Hér er (1) eigingildisjafna fyrir E_z^0 og h^2 eru stöðugt eigingildi sem við finnum þegar jafnan (1) er leyst fyrir það þverskurðarform sem við höfum ákveðið á.

Þegar eigingildin eru fundin má reikna

$$\gamma = \sqrt{h^2 - k^2} = \sqrt{h^2 - \omega^2\mu\epsilon}$$

Til er þröskuldstíðni

$$\omega_c = \frac{h}{\sqrt{\mu\epsilon}}$$

eða $$\boxed{f_c = \frac{h}{2\pi\sqrt{\mu\epsilon}}}$$

þegar $\gamma = 0$

$$\gamma = h\sqrt{1 - \frac{\omega^2\mu\epsilon}{h^2}} = h\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

$f < f_c$

$\longrightarrow \gamma \in \mathbb{R}$

burðarþátturinn er

$$e^{-\gamma z} = e^{-\alpha z}$$

Engin bylgja berst

$\rightarrow$ Dofnunarástand

$$\gamma = \alpha = h\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

$\rightarrow$ f_c er þröskuldstíðni

$f > f_c$

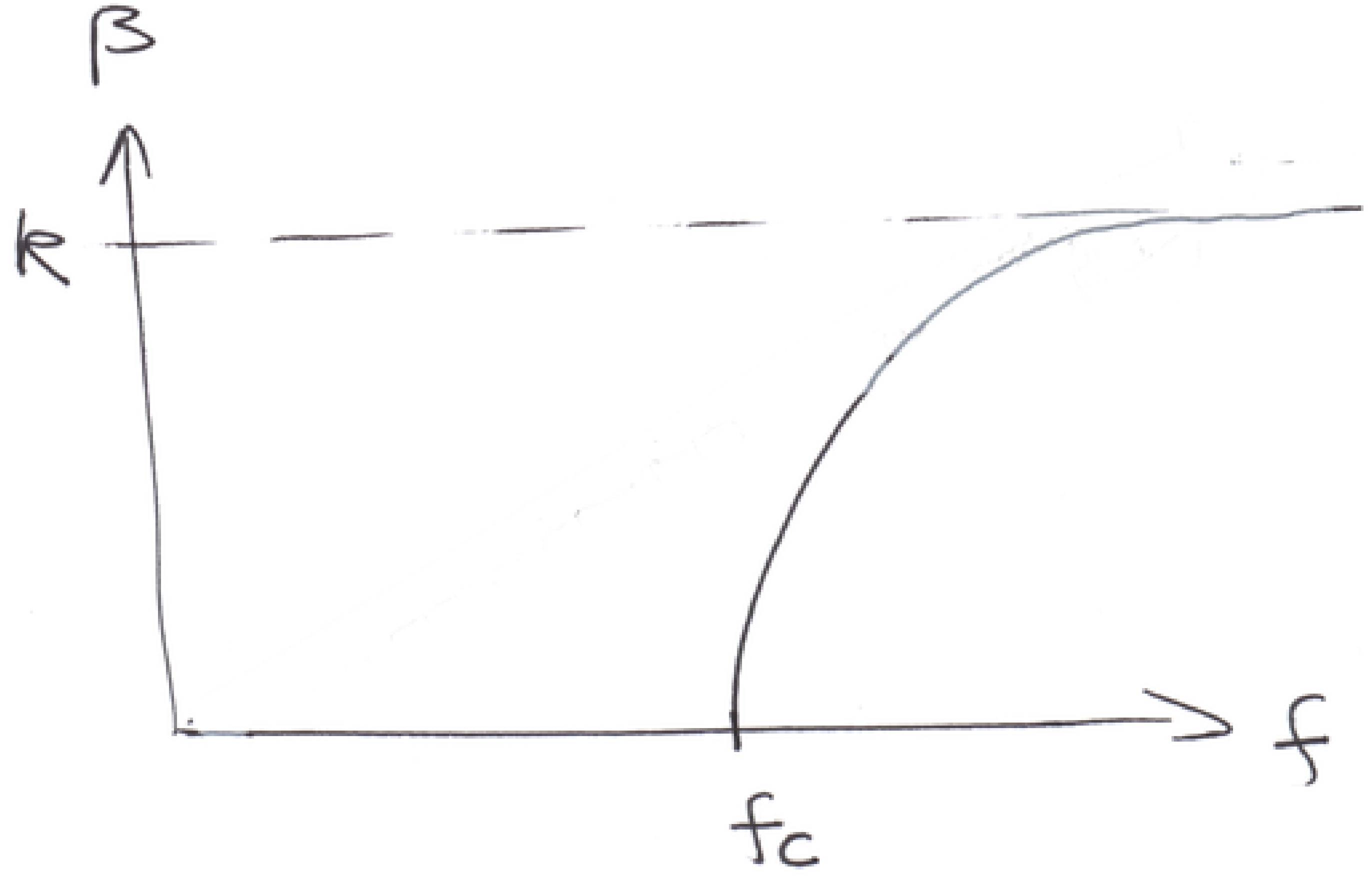
$$\gamma = i\beta = ik\sqrt{1-\left(\frac{h}{k}\right)^2}$$

$$= ik\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

því $k^2 = \omega^2 \mu \epsilon$

Bylgja berst með

$$\beta = k\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$



Bylgjulengd í leiðara

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{k} \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} > \lambda$$

því $\lambda = \frac{2\pi}{k} = \frac{1}{f\sqrt{\mu\epsilon}} = \frac{u}{f}$

og $u = \frac{1}{\sqrt{\mu\epsilon}}$

Með þröskulds bylgjulengdinni

$$\lambda_c = \frac{u}{f_c}$$

má fá

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

Grúpuhraði

$$u_g = \frac{1}{d\beta/d\omega} = u\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \frac{\lambda}{\lambda_g} u < u$$

Fasahraði

$$u_p = \frac{\omega}{\beta} = \frac{u}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\lambda_g}{\lambda} u > u$$

$$\boxed{u_g u_p = u^2}$$

Bylgjur tvístrast í þessum leiðara

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$$Z_{TM} = \frac{\gamma}{i\omega\epsilon}$$

$$= \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

En fyrir $f < f_c$

fæst

$$Z_{TM} = -i\frac{h}{\omega\epsilon}\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

TE-bylgjur

$$E_z = 0$$

Nú er eigingildisjafnan

$$\boxed{\nabla_{xy}^2 H_z + h^2 H_z = 0}$$

Maxwell gefur

$$\boxed{(H_T^0)_{TE} = -\frac{\gamma}{h^2}\overline{\nabla}_T H_z^0}$$

og

$$\boxed{\overline{E} = -Z_{TE}(\hat{a}_z \times \overline{H})}$$

með

$$\boxed{Z_{TE} = \frac{i\omega\mu}{\gamma}}$$

fyrir $f > f_c$

fast aftur

$$\gamma = ik\sqrt{1-\left(\frac{f_c}{f}\right)^2} = i\beta$$

bylgjulausn með

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(f_c/f\right)^2}}$$

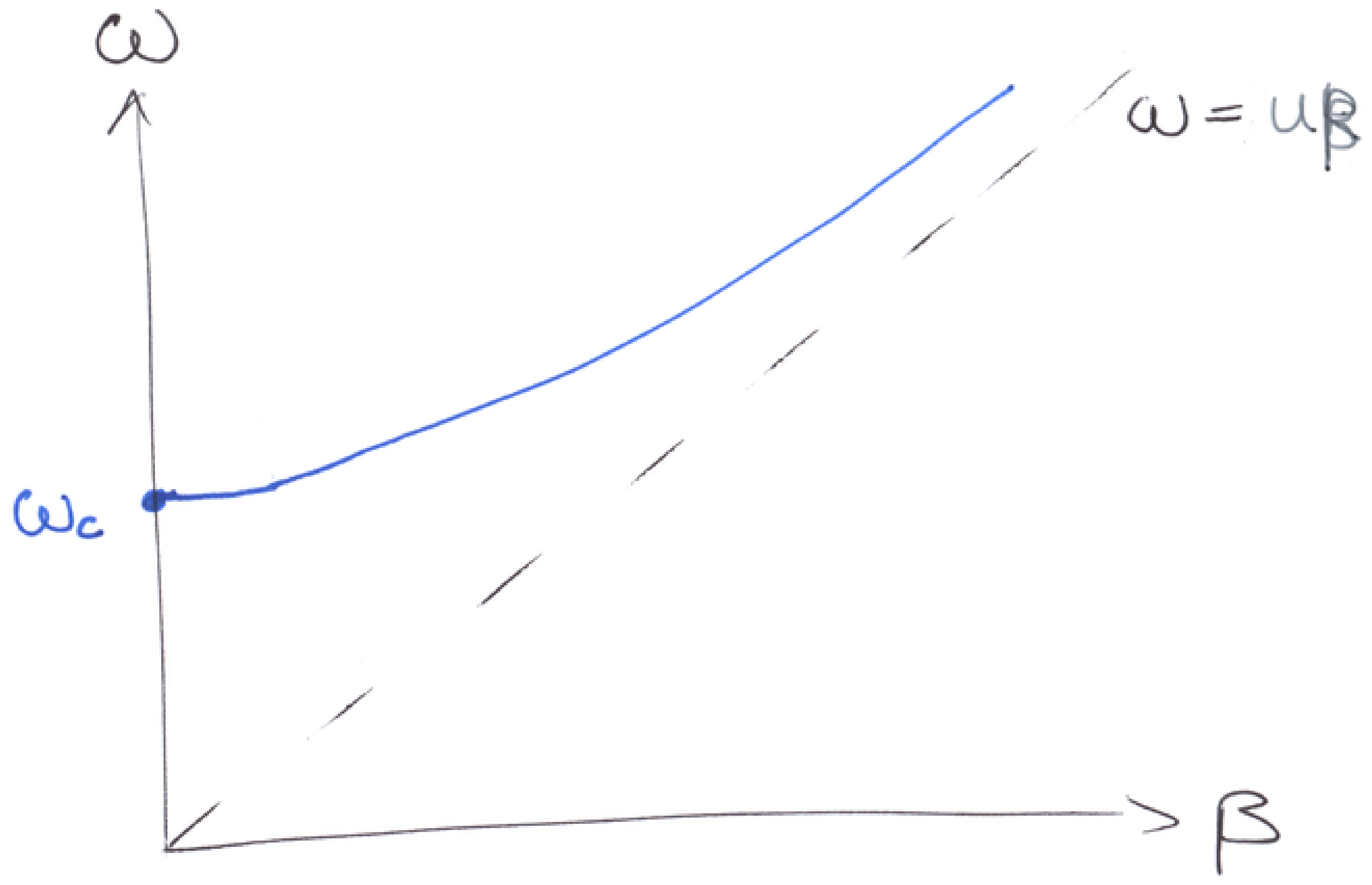
fyrir $f < f_c$

$$\gamma = \alpha = h\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

Deyfingarlausn með

$$Z_{TE} = i\frac{\omega\mu}{h\sqrt{1-\left(f/f_c\right)^2}}$$

Tvístrun: f



$$\beta = k\sqrt{1-\left(\frac{f_c}{f}\right)^2}, \quad k^2 = \frac{\omega^2}{u^2}$$

$$\to \omega^2 = \omega_c^2 + \beta^2 u^2$$

$$\omega = \sqrt{\omega_c^2 + \beta^2 u^2}$$

TM bylgjur milli samsíða leiðara

$$H_z = 0$$

$$\left(\frac{d^2}{dy^2} + h^2\right) E_z^0(y) = 0$$

Jaðarskilyrði

$$E_z^0(0) = 0 \,, \quad E_z(b) = 0$$

$$\Rightarrow \quad E_z^0(y) = A_n \sin\left(\frac{n\pi y}{b}\right)$$

Maxwell gefur

$$H_x^0(y) = \frac{i\omega\epsilon}{h} A_n \cos\left(\frac{n\pi y}{b}\right)$$

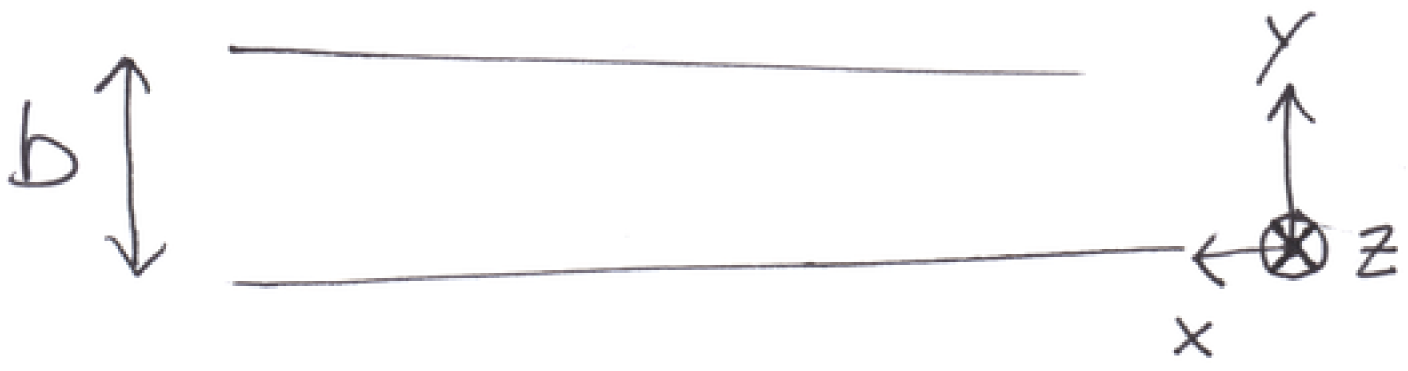
$$E_y^0(y) = -\frac{\gamma}{h} A_n \cos\left(\frac{n\pi y}{b}\right)$$

$$\gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon} \qquad h = \frac{n\pi}{b}$$

með þröskuldstíðni

$$f_c = \frac{n}{2b\sqrt{\mu\epsilon}}$$

einsleitt Kerfi í x-átt



TM_0 er TEM háttur ($f_c = 0$) og ríkjandi háttur → lægsti þröskuldur