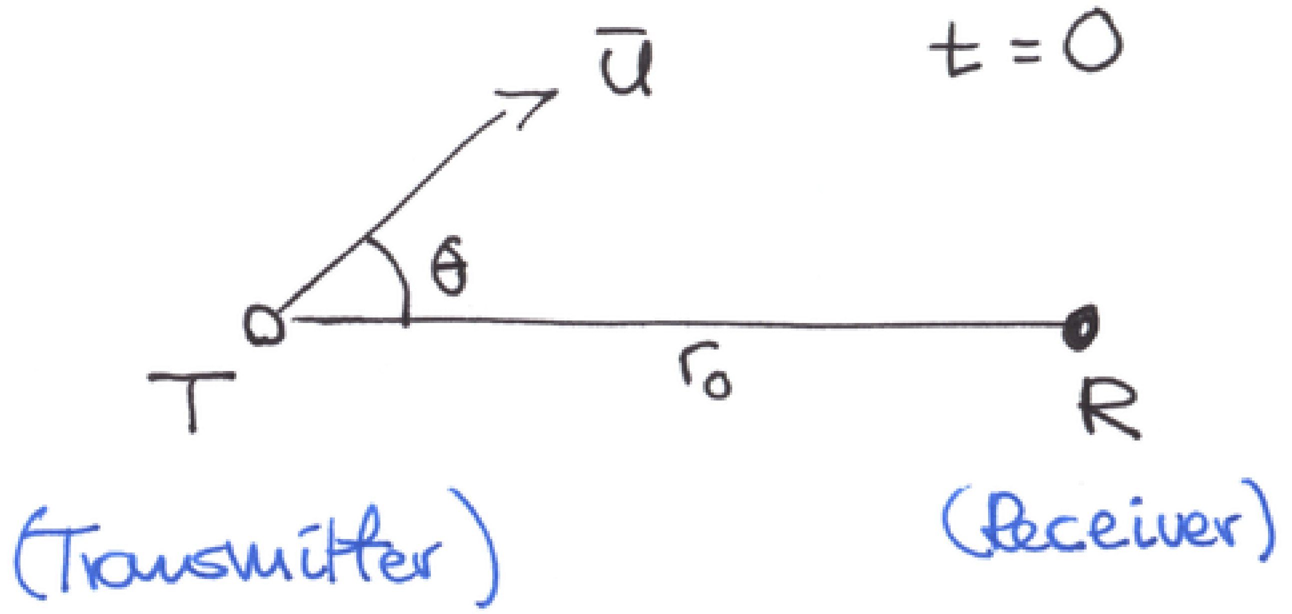
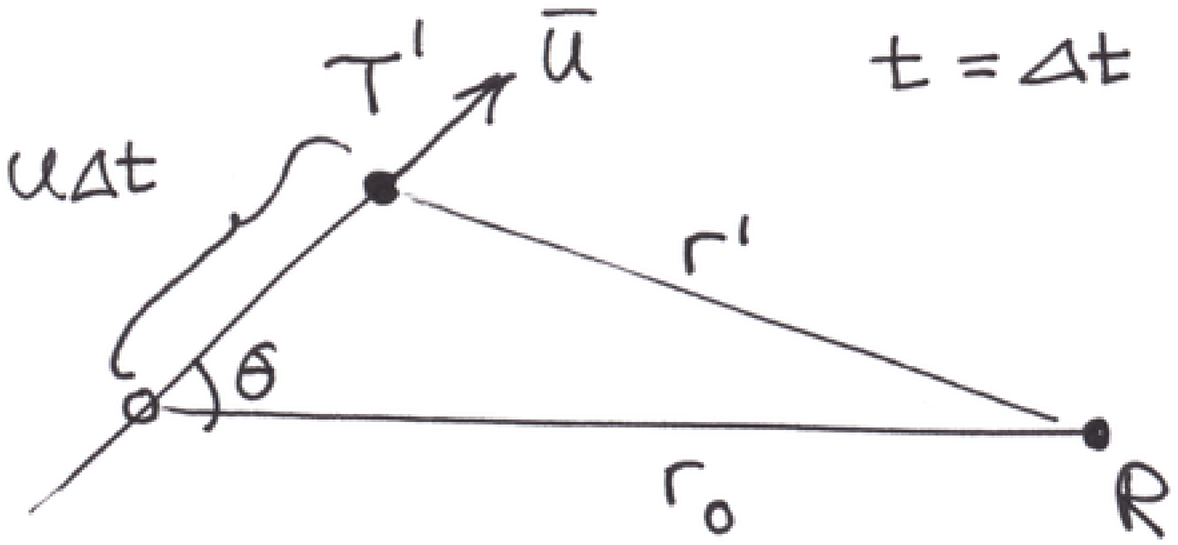
Dopplerhrif





Bylgja frá T klukkan $t=0$ kemur til R kl. $t_1 = \frac{r_0}{c}$

bylgja kl. $t = \Delta t$ frá T' kemur til R kl.

$$t_2 = \Delta t + \frac{r'}{c}$$

$$= \Delta t + \frac{1}{c}\sqrt{r_0^2 + (u\Delta t)^2 - 2r_0(u\Delta t)\cos\theta}$$

$$\approx \Delta t + \frac{r_0}{c}\left(1 - \frac{u\Delta t}{r_0}\cos\theta\right)$$

ef $r_0 \gg u\Delta t$

Tímamunur merkjanna í R er

$$\Delta t' = t_2 - t_1 \simeq \Delta t + \frac{r_0}{c}\left(1 - \frac{u\Delta t}{r_0}\cos\theta\right) - \frac{r_0}{c}$$

$$= \Delta t\left(1 - \frac{u}{c}\cos\theta\right)$$

því er $\Delta t'$ mælt við hljóðnemann ekki sama og Δt mælt við uppsprettuna

Hljóðneminn heyrir tíðnina

$$f' = \frac{1}{\Delta t'} \simeq \frac{1}{\Delta t\left(1 - \frac{u}{c}\cos\theta\right)}$$

$$= \frac{f}{\left(1 - \frac{u}{c}\cos\theta\right)} \simeq f\left(1 + \frac{u}{c}\cos\theta\right)$$

Ef $\left(\frac{u}{c}\right)^2 << 1$

$\boxed{\text{Rauðvik , Blávik}}$

Þverrafsegulbylgjur

Rafsegulbylgja í z-átt var með fasor

$$\bar{E}(z) = \bar{E}_0 e^{-ikz}$$

fyrir almenna stefnu fáum við

$$E(\bar{x}) = \bar{E}_0 e^{-ik_x x - ik_y y - ik_z z}$$

ef

$$k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon$$

eins og jafna Helmholtz krefst.

Skilgreinum bylgjuvigur

$$\bar{k} = \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z = k\hat{a}_n$$

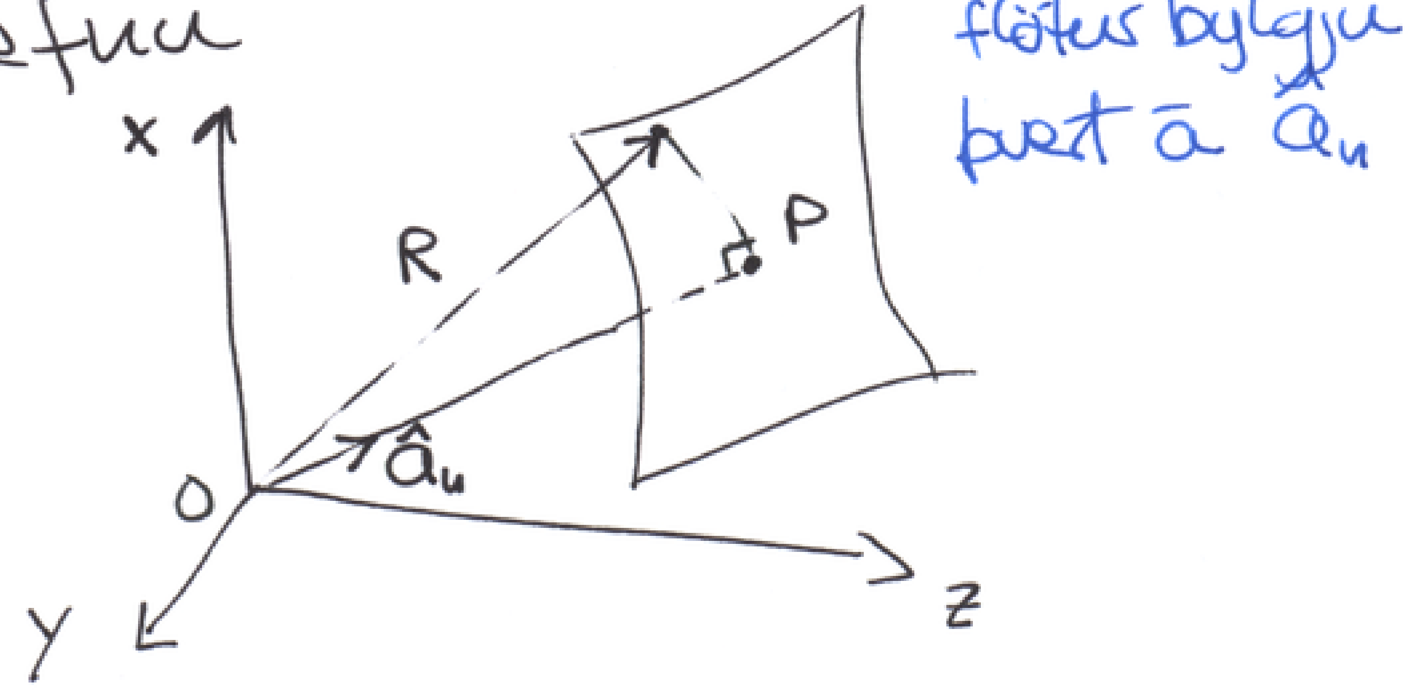
og geislavigur

$$\bar{R} = \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$$

þá fæst

$$\bar{E}(\bar{R}) = \bar{E}_0 e^{-i\bar{k}\cdot\bar{R}} = \bar{E}_0 e^{-ik\hat{a}_n\cdot\bar{R}}$$

$\hat{a}_n$ er einingarvigur í útbreiðslustefnu



$$k_x = \bar{k} \cdot \hat{a}_x = k \underline{\hat{a}_n \cdot \hat{a}_x}$$

og samskonar fyrir $y, z \ldots$

stefnu kosinus fyrir $\hat{a}_n$

$$\hat{a}_n \cdot \bar{R} = \text{fasti} = |\overline{OP}|$$

er jafna slettunnar

þvert á $\hat{a}_n$, með

fastan fasa og útslag

Engin hleðsla á útbreiðslusvæði

$$\rightarrow \bar{\nabla} \cdot \bar{E} = 0$$

$$\rightarrow \bar{E}_0 \cdot \bar{\nabla}\left(e^{-ik\hat{a}_n \cdot \bar{R}}\right) = 0$$

$$\bar{\nabla}\left(e^{-ik\hat{a}_n \cdot \bar{R}}\right) = \left(\hat{a}_x \frac{\partial}{\partial x} + \hat{a}_y \frac{\partial}{\partial y} + a_z \frac{\partial}{\partial y}\right) e^{-i(k_x x + k_y y + k_z z)}$$

$$= -i\left(\hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z\right) e^{-i(k_x x + \cdots + k_z z)}$$

$$= -ik\hat{a}_n e^{-ik\hat{a}_n \cdot \bar{R}}$$

En

$$-ik(\bar{E}_0 \cdot \hat{a}_n) e^{-ik\hat{a}_n \cdot \bar{R}} = 0$$

sem verður aðeins með

$$\hat{a}_n \cdot \bar{E}_0 = 0$$

$\bar{E}_0$ er þvert á útbreiðslu-

stefnu $\hat{a}_n$

Segulsviðið finnum við með

$$\bar{H} = \frac{1}{-i\omega\mu} \bar{\nabla} \times \bar{E}$$

$$\rightarrow \boxed{\bar{H}(\bar{R}) = \frac{1}{\eta} \hat{a}_n \times \bar{E}(\bar{R})}$$

með

$$\eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\epsilon}}$$

þá

$$\boxed{\bar{H}(\bar{R}) = \frac{1}{\eta} (\hat{a}_n \times \bar{E}_0) e^{-ik\hat{a}_n \cdot \bar{R}}}$$

Svo eins og búast mátti við á svæði án ρ og J eru $\bar{E}$ og $\bar{H}$ hornrétt og líka á útbreiðslustefnuna $\hat{a}_n$

Skautun

Skoðum

$$\bar{E}(z) = \hat{a}_x E_1(z) + \hat{a}_y E_2(z)$$

$$= \hat{a}_x E_{10} e^{-ikz} - \hat{a}_y i E_{20} e^{-ikz}$$

Sett saman úr tveimur línulega skautuðum þáttum, annar er 90° á eftir hinum í fasa

Skoðum þessa bylgju í föstum tímapunkti

$$\bar{E}(z,t) = \Re e\left\{\left[\hat{a}_x E_1(z) + \hat{a}_y E_2(z)\right] e^{i\omega t}\right\}$$

$$= \hat{a}_x E_{10} \cos(\omega t - kz) + \hat{a}_y E_{20} \cos(\omega t - kz - \pi/2)$$

Skoðum stefnubreytingu $\bar{E}(z,t)$ þ. $z=0$ en tíminn líður

$$\bar{E}(0,t) = \hat{a}_x E_1(0,t) + \hat{a}_y E_2(0,t)$$

$$= \hat{a}_x E_{10} \cos\omega t + \hat{a}_y E_{20} \sin\omega t$$

$$\sqrt{1 - \left(\frac{E_1(0,t)}{E_{10}}\right)^2}$$

$$\shortparallel$$

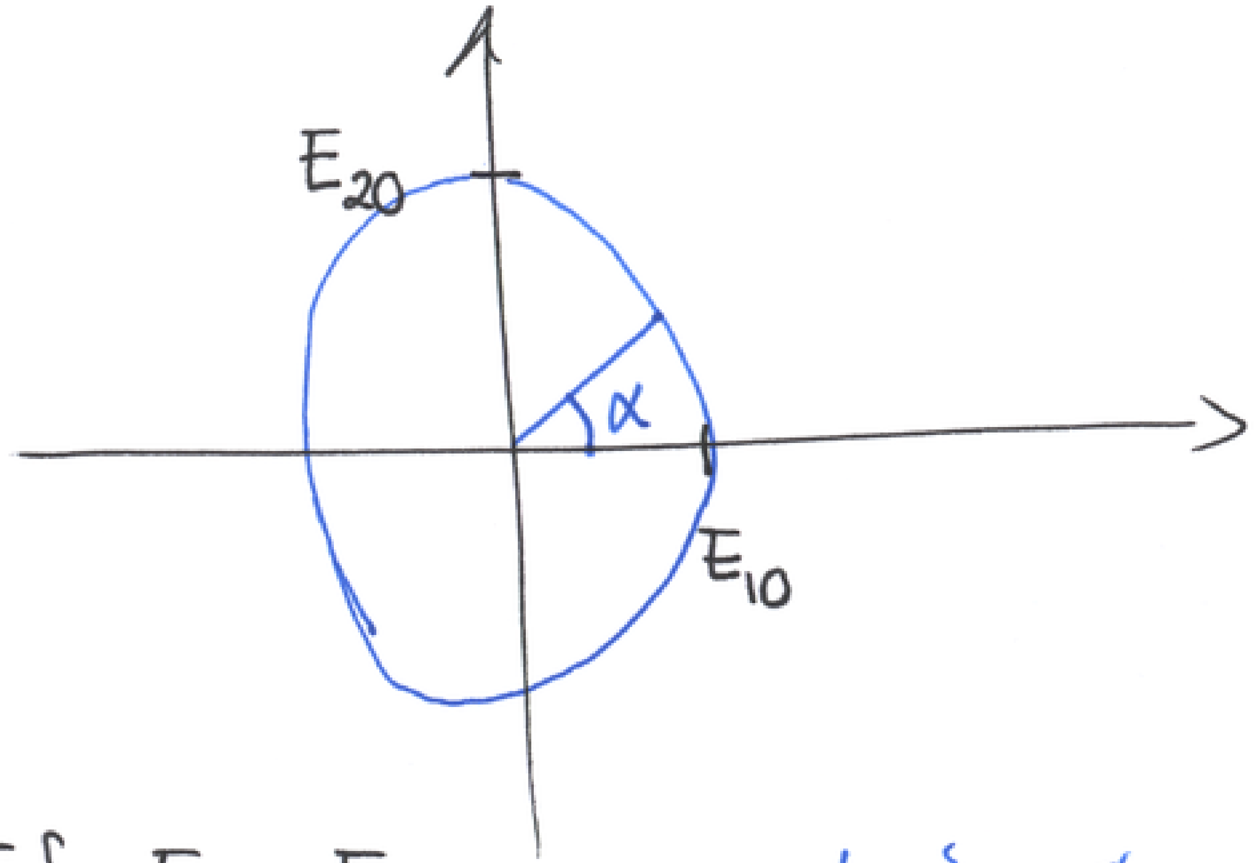
$$\rightarrow \cos\omega t = \frac{E_1(0,t)}{E_{10}} \qquad \sin\omega t = \frac{E_2(0,t)}{E_{20}} = \sqrt{1-\cos^2\omega t}$$

þ.a.

$$\left\{\frac{E_2(0,t)}{E_{20}}\right\}^2 + \left\{\frac{E_1(0,t)}{E_{10}}\right\}^2 = 1$$

jafna sporbaugs (ellipsu)

ellipsuskautun



Ef $E_{20} = E_{10}$ ← hringskautun

$$\alpha = \arctan\left(\frac{E_2(0,t)}{E_1(0,t)}\right) =$$

$$\arctan(\tan \omega t) = \omega t$$

Hægri handar hringsveiflu ellipsu skautun

(jákvæð hringskautun)

Vinstri handar hringskautun

fæst með

$-\omega t$

snúningi

línuleg skautun $\bar{E}(z) = \hat{a}_x E_0 e^{-ikz}$

$$\bar{E}(z) = \bar{E}_{rc}(z) + \bar{E}_{lc}(z)$$

með

$$\bar{E}_{rc}(z) = \frac{E_0}{2}(\hat{a}_x - i\hat{a}_y)e^{-ikz}$$

$$\bar{E}_{lc}(z) = \frac{E_0}{2}(\hat{a}_x + i\hat{a}_y)e^{-ikz}$$

Flatarbylgjur í efni með orkutapi

$$\nabla^2 \overline{E} + k_c^2 \overline{E} = 0$$

$$k_c = \omega\sqrt{\mu\epsilon_c} \in \mathbb{C}$$

Venja að skilgreina

$$\gamma = ik_c = i\omega\sqrt{\mu\epsilon}$$

og ef

$$\epsilon_c = \epsilon - i\frac{\sigma}{\omega}$$

$$\to \gamma = \alpha + i\beta = i\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{i\epsilon\omega}\right)^{1/2}$$

Eða með $\epsilon_c = \epsilon' - i\epsilon''$

fæst

$$\gamma = \alpha + i\beta = i\omega\sqrt{\mu\epsilon'}\left(1 - i\frac{\epsilon''}{\epsilon'}\right)^{1/2}$$

α og β eru raun og þverhlutar γ

án taps er

$$\boxed{\begin{array}{l} \sigma = 0, \quad \epsilon'' = 0, \quad \epsilon = \epsilon' \\ \alpha = 0, \quad \beta = k = \omega\sqrt{\mu\epsilon} \end{array}}$$

Jafna Helmholtz er hér

$$\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0$$

og fyrir slétta bylgju í z-stefnu línulega skautaða í x-átt

$$\bar{E} = \hat{a}_x E_x = \hat{a}_x E_0 e^{-\gamma z}$$

$$= \hat{a}_x E_0 e^{-\alpha z} e^{-i\beta z}$$

α, β eru báðar jákvæðar stærðir (kemur í ljós)

α : dofnunarfasti

β : fasa fasti

Rafsvari með litlu tapi

$\epsilon' \gg \epsilon''$ eða $\frac{\sigma}{\omega\epsilon} \ll 1$

$$\gamma = \alpha + i\beta \simeq i\omega\sqrt{\mu\epsilon'}\left(1 - \frac{i\epsilon''}{2\epsilon'} + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right)$$

$$\rightarrow \alpha \simeq \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}$$ ← línulegt m.ω

$$\beta \simeq \omega\sqrt{\mu\epsilon'}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right]$$

nálgun

$$\eta = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu}{\epsilon'}}\left(1 - i\frac{\epsilon''}{\epsilon'}\right)^{-1/2} \simeq \sqrt{\frac{\mu}{\epsilon'}}\left(1 + i\frac{\epsilon''}{2\epsilon'}\right)$$

hlutfall E_x og H_y hér $\rightarrow$ $\boxed{\text{rafsviðið og segulsviðið eru ekki í fasa eins og í efni án taps}}$

fasa hraðinn er núna

$$u_p = \frac{\omega}{\beta} \simeq \frac{1}{\sqrt{\mu\epsilon'}}\left[1 - \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right]$$ ← minnkaðar fasahraði vegna taps

α-þáttur k_c berst ekki í efninu

Góður leiðari

$$\frac{\sigma}{\omega\epsilon} \gg 1$$

$$\gamma = i\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{i\omega\epsilon}\right)^{1/2}$$

$$\simeq i\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{i\omega\epsilon}}$$

$$= \sqrt{i}\sqrt{\omega\mu\sigma} = \frac{1+i}{\sqrt{2}}\sqrt{\omega\mu\sigma}$$

$$\to \gamma = \alpha + i\beta \simeq (1+i)\sqrt{\pi f\mu\sigma}$$

$$\to \alpha = \beta = \sqrt{\pi f\mu\sigma}$$

$$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} \simeq \sqrt{\frac{i\omega\mu}{\sigma}}$$

$$= (1+i)\sqrt{\frac{\pi f\mu}{\sigma}} = (1+i)\frac{\alpha}{\sigma}$$

$\to$ Segulsviðið er á eftir rafsviðinu í fasa um $\frac{\pi}{4}$

fasahraðinn

$$u_p = \frac{\omega}{\beta} \simeq \sqrt{\frac{2\omega}{\mu\sigma}}$$

í réttu hlutfalli við $\sqrt{f}$ og $\frac{1}{\sqrt{\sigma}}$

fyrir góðan leiðara eins og kopar fæst að

$$u_p \approx 720 \text{ m/s}$$

fyrir $f = 3 \text{ MHz}$

Dofnunin er líka stórt $\alpha = \beta$, bylgjan dofnar niður í e^{-1} $\rightarrow$ skilgreinir lengd

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$= \frac{1}{\beta} = \frac{\lambda}{2\pi}$$

Skindýpt (ekki skinn....)

$\rightarrow$ fyrir örbylgjur er skindýptin orðin lítil

Sjá töflu 8-1

Gull $\delta = 0{,}0025$ mm fyrir $f = 1 \text{ GHz}$