6-4

a) Óendanlegur brunnur $V(x) = \begin{cases} 0 & \text{ef } 0<x<a \\ \infty & \text{annars} \end{cases}$

$$H' = \alpha\,\delta\left(x-\frac{a}{2}\right) \qquad E_n^0 = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad \psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi x}{a}\right)$$

Reiknum $E_n^2 = \sum_{m\neq n} \frac{|\langle m|H'|n\rangle|^2}{E_n^0 - E_m^0}$ $\qquad E_n^0 = E_1^0 \cdot n^2$

$$\langle m|H'|n\rangle = \alpha \int_0^a dx\, \psi_m^*(x)\,\delta\left(x-\frac{a}{2}\right)\psi_n(x)$$

$$= \frac{2}{a}\alpha \int_0^a dx\, \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{a}\right)\delta\left(x-\frac{a}{2}\right)$$

$$= \frac{2}{a}\alpha\,\sin\left(\frac{m\pi}{2}\right)\sin\left(\frac{n\pi}{2}\right) = \begin{cases} \pm\frac{2\alpha}{a} & \text{ef } n \text{ og } m \text{ odd} \\ 0 & \text{annars} \end{cases}$$

2

$$E_n^2 = \left(\frac{2\alpha}{a}\right)^2 \frac{1}{(E_1^0)^2} \sum_{\substack{m \neq n \\ \text{odd}}} \frac{1}{n^2 - m^2}$$

þessa summu þarf að þátta

$$\frac{1}{n^2 - m^2} = \frac{1}{2n}\left\{\frac{1}{m+n} - \frac{1}{m-n}\right\}$$

Allir liðir munu styttast út í summunni, nema lægsti liðurinn hér

staki liðurinn er $\frac{1}{2n}\left(-\frac{1}{2n}\right)$

$$\rightarrow E_n^2 = \left(\frac{2\alpha}{a}\right)^2 \frac{1}{(E_1^0)^2} \cdot \frac{-1}{4n^2} \quad \text{ef } n = \text{odd, annars } 0$$

$$= \begin{cases} -\left(\dfrac{\alpha}{a\,E_1^0}\right)^2 \dfrac{1}{n^2} & \text{ef } n = \text{odd} \\ 0 & \text{annars} \end{cases}$$

1. stigs liður er jákvæður

2. stigs liður er neikvæður

b) Við höfðum í (6-2) reiknað fyrir hreintóna sveifil hvað gerist þegar $k \to (1+\epsilon)k$

$$E_n^1 \simeq E_n^0 \cdot \left\{ 1 + \frac{\epsilon}{2} - \frac{\epsilon^2}{8} + \cdots \right\}$$

$$E_n^1 = E_n^0 \cdot \sqrt{1+\epsilon}$$

getum við samreynt 2. stigs liðinn?

Munum að

$$H' = \frac{\epsilon}{2} k x^2$$

$$E_n^2 = \sum_{m \neq n} \frac{|\langle m | H' | n \rangle|^2}{E_n^0 - E_m^0}$$

$$a_+ |n\rangle = \sqrt{n+1}\, |n+1\rangle$$

$$a_- |n\rangle = \sqrt{n}\, |n-1\rangle$$

Nøtum after

$$x^2 = \frac{a^2}{2}(a_+ + a_-)^2 = \frac{a^2}{2}\left\{a_+a_+ + a_-a_- + a_+a_- + a_-a_+\right\}$$

og reknum

$$\langle m|H'|n\rangle = \frac{\epsilon k}{2}\langle m|\left\{a_+a_+ + a_-a_- + a_+a_- + a_-a_+\right\}|n\rangle \cdot \frac{a^2}{2}$$

$$= \frac{\epsilon k}{2}\left\{\sqrt{(n+1)(n+2)}\,\langle m|n+2\rangle + \sqrt{n(n-1)}\,\langle m|n-2\rangle\right.$$

$$\left. + n\langle m|n\rangle + (n+1)\langle m|n\rangle\right\} \cdot \frac{a^2}{2}$$

$$= \frac{a^2 \cdot \epsilon k}{2 \cdot 2}\left\{\sqrt{(n+1)(n+2)}\,\delta_{m,n+2} + \sqrt{n(n-1)}\,\delta_{m,n-2}\right.$$

$$\left. + n\,\delta_{m,n} + (n+1)\,\delta_{m,n}\right\}$$

en mennum ðe i summuni er $m \neq n$

$$E_n^{2} = \frac{\epsilon^2 k^2 \cdot a^4}{4\hbar\omega\cdot 4}\sum_{m\neq n}\frac{\left|\sqrt{(n+1)(n+2)}\,\delta_{m,n+2}+\sqrt{n(n-1)}\,\delta_{m,n-2}\right|^2}{(n+\frac{1}{2})-(m+\frac{1}{2})}$$

nota

$$= \epsilon^2\hbar\omega\,\frac{1}{16}\sum_{m\neq n}\frac{(n+1)(n+2)\,\delta_{m,n+2}+n(n-1)\,\delta_{m,n-2}}{n-m}$$

$$= \epsilon^2\hbar\omega\,\frac{1}{16}\left\{\frac{(n+1)(n+2)}{n-(n+2)}+\frac{n(n-1)}{n-(n-2)}\right\}$$

$$= \epsilon^2\hbar\omega\,\frac{1}{16}\left\{-\frac{1}{2}(n+1)(n+2)+\frac{1}{2}n(n-1)\right\}$$

$$= \epsilon^2\hbar\omega\,\frac{1}{32}\left\{-\cancel{n^2}-3n-2+\cancel{n^2}-n\right\} = -\epsilon^2\frac{1}{8}\hbar\omega\left(n+\frac{1}{2}\right)$$

$$= -\epsilon^2\frac{1}{8}E_n^0$$

eins og áður

6-30

3D - hreintóna sveifill (einsleitur)

$$H' = \lambda x^2 y z$$

a) Reikna E_0^1 1. stigs truflun grunnástands

Grunnástandið er eins og 3 hreintóna sveiflar, óháðir, í 3 höfuð stefnum í grunnástandi

$$\rightarrow E_0 = \hbar\omega \cdot \frac{3}{2}$$

$$E_0^1 = \langle 0|H'|0\rangle = \lambda\langle 0|x^2|0\rangle \underbrace{\langle 0|y|0\rangle}_{=0} \underbrace{\langle 0|z|0\rangle}_{=0} = 0$$

b) þrefalda lögsta örvaða ástandið

$|1\rangle = |1,0,0\rangle$ — p.s. ástöndin eru $|n_x, n_y, n_z\rangle$

$|2\rangle = |0,1,0\rangle$

$|3\rangle = |0,0,1\rangle$

$\langle 3|H'|3\rangle = \lambda \langle 0|x^2|0\rangle \underline{\langle 0|y|0\rangle} \langle 1|z|1\rangle = 0$

$\langle 3|H'|2\rangle = \lambda \langle 0|x^2|0\rangle \langle 0|y|1\rangle \langle 1|z|0\rangle \neq 0$

$\langle 3|H'|1\rangle = \lambda \langle 0|x^2|1\rangle \langle 0|y|0\rangle \langle 1|z|0\rangle = 0$

Öll önnur nema $\langle 2|H'|3\rangle$ gefa líka 0

$$\langle 3|H'|2\rangle = \lambda \langle 0|x^2|0\rangle \langle 0|y|1\rangle \langle 1|z|0\rangle$$

$$= \underbrace{\lambda \langle 0|x^2|0\rangle \, |\langle 0|x|1\rangle|^2}$$

ef við notum að þetta er einsleitur hreintóna sveifill og við notum fylkjastökin fyrir einvíðan hreintóna sveifil

Munum að

$$\langle 0|x^2|0\rangle = \frac{a^2}{2}$$

$$\langle 0|x|1\rangle = \frac{a}{\sqrt{2}}\langle 0|(a_+ + a_-)|1\rangle = \frac{a}{\sqrt{2}}\langle 0|a_-|1\rangle$$

$$= \frac{a}{\sqrt{2}}\langle 0|0\rangle \cdot 1 = \frac{a}{\sqrt{2}}$$

$$\langle 3|H'|2\rangle = \lambda \frac{a^2}{2} \cdot \frac{a^2}{2} = \lambda \frac{a^4}{4}$$

þess vegna verður fylkið

$$W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{\lambda a^4}{4}$$

með eigingildi $0, \pm \frac{\lambda a^4}{4}$

þriklofnun