3.39 Sýna að

a)
$$f(x+x_0) = \exp\left\{i\hat{p}\frac{x_0}{\hbar}\right\} f(x)$$
x_0 er föst lengd

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{i\hat{p}x_0}{\hbar}\right)^n}{n!} f(x) = \sum_{n=0}^{\infty} \frac{\left(x_0\partial_x\right)^n}{n!} f(x)$$

$$= \sum_{n=0}^{\infty} \frac{x_0^n}{n!} f^{(n)}(x) = f(x+x_0)$$

$\frac{\hat{p}}{\hbar}$ er vaki hliðrunar í staðarrúminu

b) Ef H er ekki fall af t sýna að

$$\psi(x,t+t_0) = \underbrace{\exp\left\{-i\hat{H}t_0/\hbar\right\}}_{} \psi(x,t)$$

$\hat{H}/\hbar$ er vaki tímahliðrunar

tímaþróunarvirki

uuuuu ∂e $\quad i\hbar \partial_t \Phi = H \Phi$

$$\exp\left\{-i\hat{H}\frac{t_0}{\hbar}\right\}\Phi(x,t) = \sum_{n=0}^{\infty}\frac{\left(-i\hat{H}\frac{t_0}{\hbar}\right)^n}{n!}\Phi(x,t)$$

$$= \sum_{n=0}^{\infty}\frac{(t_0\partial_t)^n}{n!}\Phi(x,t) = \sum_{n=0}^{\infty}\frac{(t_0)^n}{n!}\Phi^{(n)}(x,t) = \Phi(x,t+t_0)$$

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c) Sȳua ∂e

$$\langle Q\rangle_{t+t_0} = \langle \Phi(x,t)|\, e^{i\frac{Ht_0}{\hbar}}\,\hat{Q}(\hat{x},\hat{p},t+t_0)\,e^{-i\frac{Ht_0}{\hbar}}\,|\Phi(x,t)\rangle$$

$$\langle Q\rangle_{t+t_0} = \langle \Phi(x,t+t_0)|\,\hat{Q}(\hat{x},\hat{p},t+t_0)\,|\Phi(x,t+t_0)\rangle$$

$$= \langle \Phi(x,t)|\, e^{i\frac{Ht_0}{\hbar}}\,\hat{Q}(\hat{x},\hat{p},t+t_0)\,e^{-i\frac{Ht_0}{\hbar}}\,|\Phi(x,t)\rangle$$

Setjum $t_0 = dt$

$$\langle Q \rangle_{t+dt} \simeq \langle \psi | \left\{ 1 + \frac{iH}{\hbar} dt \right\} \hat{Q}(\hat{x}, \hat{p}, t+dt) \left\{ 1 - \frac{iH}{\hbar} dt \right\} | \psi \rangle$$

$$= \langle \psi | \hat{Q}(\hat{x}, \hat{p}, t+dt) | \psi \rangle$$

$$+ \frac{i}{\hbar} dt \langle \psi | \left[\hat{H}, \hat{Q}(\hat{x}, \hat{p}, t+dt) \right] | \psi \rangle$$

$$\simeq \langle \psi | \left\{ \hat{Q}(\hat{x}, \hat{p}, t) + \partial_t \hat{Q}(\hat{x}, \hat{p}, t) \cdot dt \right\} | \psi \rangle$$

$$+ \frac{i}{\hbar} dt \langle \psi | \left[\hat{H}, \hat{Q}(\hat{x}, \hat{p}, t) \right] | \psi \rangle + o\left((dt)^2\right)$$

$$\rightarrow \quad d_t \langle \hat{Q} \rangle = \frac{i}{\hbar} \langle [\hat{H}, \hat{Q}] \rangle + \langle \partial_t \hat{Q} \rangle$$

4.56 Sýna

$$f(\phi+\varphi) = \exp\left\{iL_z\varphi/\hbar\right\} f(\phi)$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{iL_z\varphi}{\hbar}\right)^n}{n!} f(\phi) = \sum_{n=0}^{\infty} \frac{(\varphi\partial_\varphi)^n}{n!} f(\phi)$$

$$= \sum_{n=0}^{\infty} \frac{\varphi^n}{n!} f^{(n)}(\phi) = f(\phi+\varphi)$$

$\frac{L_z}{\hbar}$ er vaki snúnings um z-ás $\{$ ϕ er „miðbaugshornið" $\}$

Almennir snúningur fæst með

$$\exp\left\{i\hat{L}\cdot\hat{n}\frac{\varphi}{\hbar}\right\}$$

fyrir spuna

$$\chi' = \exp\left\{\frac{i(\bar{\sigma}\cdot\hat{n})\varphi}{2}\right\}\chi$$

b) Búa til (2×2) fylki fyrir snúning um 180° um x-ás

$$\bar{\sigma}\cdot\hat{a}_x = \sigma_x$$

Reiknum því $\exp\left\{\frac{i\pi}{2}\sigma_x\right\}$, $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\exp\left\{\frac{i\pi}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right\} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$\exp\left\{\frac{i\pi}{2}\sigma_x\right\}\chi_+ = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i\chi_-$$

$$\exp\left\{\frac{i\pi}{2}\sigma_x\right\}\chi_- = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} i \\ 0 \end{pmatrix} = i\chi_+$$

c) $\underline{90^\circ \text{ um } y\text{-ás}}$

$$\exp\left\{\frac{i\pi}{4}\sigma_y\right\} = \exp\left\{\frac{i\pi}{4}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right\} = \exp\left[\frac{\pi}{4}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right]$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$= \chi_-^{(x)}$$

eins og var sýnt í spuna kaflanum (4.151) í bók

$$\rightarrow \exp\left\{\frac{i\pi}{4}\sigma_y\right\}\chi_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(\chi_+ - \chi_-)$$

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d) 360 gráður um z-ás

$$\exp\{i\pi\sigma_z\} = \exp\left\{\begin{pmatrix} i\pi & 0 \\ 0 & -i\pi \end{pmatrix}\right\} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\rightarrow \exp\{i\pi\sigma_z\}\chi_+ = -\chi_+ \quad , \quad \exp\{i\pi\sigma_z\}\chi_- = -\chi_-$$

360° snúningur um z-ás skiptir um formerki

Þetta er oft túlkað þ.a. 2π - snúningur geti formerkjabreytingu
$\rightarrow$ þurfi 4π snúning til þess að fá sama ástand,
en gleymum ekki að fasastuðull (heildar) skiptir ekki
máli.

e) $$\exp\left\{\frac{i\varphi}{2}(\bar{\sigma}\cdot\hat{n})\right\} = \sum_{n=0}^{\infty}\frac{\left(\frac{i\varphi}{2}\right)^n(\bar{\sigma}\cdot\hat{n})^n}{n!}$$

Hér ætla ég að beyta nú að vita

$$(\bar{\sigma}\cdot\bar{A})(\bar{\sigma}\cdot\bar{B}) = \bar{A}\cdot\bar{B} + i\bar{\sigma}\cdot(\bar{A}\times\bar{B})$$

$$\rightarrow (\bar{\sigma}\cdot\hat{n})^2 = \hat{n}^2 = I$$

$$\rightarrow (\bar{\sigma}\cdot\hat{n})^n = \begin{cases} I & \text{ef } n \text{ er jöfn} \\ \bar{\sigma}\cdot\hat{n} & \text{ef } n \text{ er oddatala} \end{cases}$$

$$\rightarrow \sum_{n=0}^{\infty} \frac{\left(\frac{i\varphi}{2}\right)^n (\bar{\sigma}\cdot\hat{n})^n}{n!} = \cos\left(\frac{\varphi}{2}\right)\cdot I + i(\bar{\sigma}\cdot\hat{n})\sin\left(\frac{\varphi}{2}\right)$$