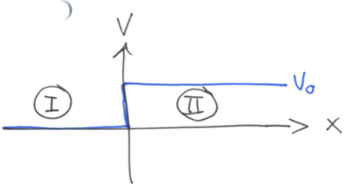
2.34 Mættisþrep

$$V(x) = \begin{cases} 0 & \text{ef } x \leq 0 \\ V_0 & \text{ef } x > 0 \end{cases}$$



a) Reikna endurkast ef $E < V_0$

I

$$k_1 = \frac{\sqrt{2mE}}{\hbar}, \qquad \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

$$\psi(x) = e^{ik_1x} + Be^{-ik_1x}$$

eindin smýgur aðeins inn í vegginn

II

$$\psi(x) = Ce^{-\kappa x}$$

vaxandi lausnin $e^{\kappa x}$ er ekki möguleg

Samfella ψ

$$1 + B = C \quad \text{(i)}$$

Samfella ψ'

$$\psi_{\mathrm{I}}'(0) = \psi_{\mathrm{II}}'(0)$$

$$ik_1 - ik_1 B = -KC \quad \text{(ii)}$$

2 jöfnur, 2 óþekktar stærðir

(i) → (ii)

$$ik_1(1-B) = -K(1+B)$$

$$B(K - ik_1) = -ik_1 - K$$

$$B = \frac{-ik_1 - K}{-ik_1 + K}$$

$$|B|^2 = B^*B = \frac{K^2 - k_1^2}{K^2 - k_1^2} = 1$$

eindin kemur alltaf til baka þó hún smjúgi aðeins inn í vegginn

b) Speglun ef $E > V_0$

$$k_1 = \frac{\sqrt{2mE}}{\hbar}, \quad k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar}$$

I $\quad \psi(x) = e^{ik_1x} + Be^{-ik_1x}$

Aðeins bylgja frá vinstri

II $\quad \psi(x) = Ce^{ik_2x}$

<u>Samfella í ψ</u>

$$1 + B = C \qquad \text{i}$$

<u>Samfella í ψ'</u>

$$ik_1 - ik_1B = ik_2C \qquad \text{ii}$$

i $\rightarrow$ ii

$$ik_1(1-B) = ik_2(1+B)$$

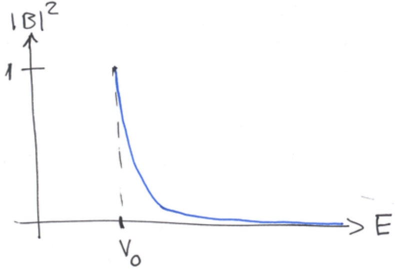
$$B(-ik_1 - ik_2) = -ik_1 + ik_2$$

$$B = -\frac{ik_2 - ik_1}{ik_2 + ik_1} = -\frac{k_2-k_1}{k_2+k_1}$$

$$|B|^2 = \frac{(k_2-k_1)^2}{(k_2+k_1)^2}$$

$$|B|^2 = \frac{\left(\sqrt{E-V_0} - \sqrt{E}\right)^2}{\left(\sqrt{E-V_0} + \sqrt{E}\right)^2}$$

Ef $V_0 = 0 \;\longrightarrow\; |B|^2 = 0$



c) Rifjum upp dæmi 2.19 um líkanda straum þéttleika

þar sást að straumur ástands

$$\Psi_k(x,t) = A \exp\{i(kx - \omega_k t)\}$$

$$J(x,t) = \frac{i\hbar}{2m}\left\{(\partial_x \Psi)^* \Psi - \Psi^* \partial_x \Psi\right\}$$

$$= \frac{i\hbar}{2m}|A|^2\{-ik - ik\} = \frac{\hbar k}{m}|A|^2$$

$E > V_0$ Reiknum C

$$1 + B = C$$

$$ik_1 - ik_1 B = ik_2 C$$

eins og áður

$$B = C-1$$

$$ik_1\left(1-(C-1)\right) = ik_2 C$$

$$ik_1(2-C) = ik_2 C$$

$$2ik_1 = iC(k_2+k_1)$$

$$C = \frac{2k_1}{k_2+k_1}$$

---

straumur einda inn

$$J_{inn} = \frac{\hbar k_1}{m}|A|^2 = \frac{\hbar k_1}{m}$$

endurkast straumur

$$J_R = -\frac{\hbar k_1}{m}|B|^2 = -\frac{\hbar k_1}{m}\frac{(k_2-k_1)^2}{(k_2+k_1)^2}$$

útstraumur

$$J_T = \frac{\hbar k_2}{m}|C|^2 = \frac{\hbar k_2}{m}\frac{4k_1^2}{(k_2+k_1)^2}$$

Ef líkindi R eru $|B|^2$

þá eru líkindi $T = |C|^2 \frac{k_2}{k_1}$

$$T = \frac{4k_1^2}{(k_2+k_1)^2}\cdot\frac{k_2}{k_1}$$

$$= \frac{4k_1k_2}{(k_2+k_1)^2}$$

$$T = |C|^2\sqrt{\frac{E-V_0}{E}}$$

vegna straumvarðveislu

d)

$$\left.\begin{aligned} T &= \frac{4k_1k_2}{(k_2+k_1)^2} \\ R &= \frac{(k_2-k_1)^2}{(k_2+k_1)^2} \end{aligned}\right\} \rightarrow T+R=1$$

Vordweila Streams

$$J_{in} + J_R = J_T \qquad \frac{\hbar k_1}{m} - \frac{\hbar k_1}{m}|B|^2 = \frac{\hbar k_2}{m}|C|^2$$

$$\frac{\hbar k_1}{m} - \frac{\hbar k_1}{m}\frac{(k_2-k_1)^2}{(k_2+k_1)^2} = \frac{\hbar k_2}{m}\frac{4k_1^2}{(k_2+k_1)^2}$$

$$\frac{\hbar k_1}{m}\left[1 - \frac{(k_2-k_1)^2}{(k_2+k_1)^2} = \frac{4k_2k_1}{(k_2+k_1)^2}\right]$$

(2.44)

$$V(x) = \begin{cases} \alpha\delta(x) & \text{fyrir } -a < x < +a \\ \infty & \text{-||- } |x| \geq a \end{cases}$$

Møttið er samhvørft → loysnir er annaðhvørt odd- ella javnstøður og verða at uppfylla

$$d_x \psi(0^+) - d_x \psi(0^-) = + \frac{2m\alpha}{\hbar^2} \psi(0)$$

vegna δ-veggsins í brunninum

<u>odd-støður loysnir</u>

Fyrir loysn sum er oddstøð gildir at $\psi(0) = 0$ → δ-veggurin hevur ongin ávirkan og loysnin eru oddstøðu loysnir óendanligs brunns við breidd $2a$

oddstæðar lausnir eru þær með orkuna

$$E_n^{odd} = \frac{(2n)^2 \pi^2 \hbar^2}{2m(2a)^2}, \quad n = 1, 2, 3, \ldots$$

$$\psi_n^{odd}(x) = \sqrt{\frac{2}{2a}} \sin\left(\frac{2n\pi x}{2a}\right) = \sqrt{\frac{1}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

<u>jafnstæðar lausnir</u>

Ef δ-veggurinn væri ekki, þ.a $\alpha = 0$

$$E_n^{even} = \frac{(2n-1)^2 \pi^2 \hbar^2}{2m(2a)^2}, \quad n = 1, 2, 3, \ldots$$

$$\psi^{even}(x) = \sqrt{\frac{2}{2a}} \cos\left(\frac{(2n-1)\pi x}{2a}\right) = \sqrt{\frac{1}{a}} \cos\left(\frac{(2n-1)\pi x}{2a}\right)$$

en ef $\alpha \neq 0$ jafnstætt lausn

$$\psi_n(x) = \begin{cases} -A\,\mathrm{Sin}(k_n(x+a)) & -a \leq x < 0 \\ +A\,\mathrm{Sin}(k_n(x-a)) & 0 < x \leq +a \end{cases}$$

Samfella í ψ er sjálfkrafa

$$-A\,\mathrm{Sin}(k_n a) = A\,\mathrm{Sin}(-k_n a)$$

Brot í afleiðu

$$+A k_n \mathrm{Cos}(k_n a) + A k_n \mathrm{Cos}(k_n a) = +\frac{2m\alpha}{\hbar^2} A\,\mathrm{Sin}(-k_n a)$$

$$k_n \mathrm{Cos}(k_n a) = -\frac{2m\alpha}{\hbar^2}\mathrm{Sin}(k_n a)$$

$$k_n \mathrm{Cot}(k_n a) = -\frac{2m\alpha}{\hbar^2}$$

jafna (óbein) sem ákvarðar k_n

4

umskrifum sem

$$\cot(k_n a) + \frac{2m\alpha}{\hbar^2 k_n} = 0$$

$$\cot(k_n a) + \frac{2m\alpha a}{\hbar^2 (k_n a)} = 0$$

$$\cot(k_n a) + \frac{2ma^2}{\hbar^2}\frac{\left(\frac{\alpha}{a}\right)}{(k_n a)} = 0$$

$$\cot(k_n a) + \left(\frac{\alpha}{E_1 a}\right)\frac{1}{(k_n a)} = 0$$

Allt víddarlausar stærðir í sviganum

$$\cot(k_n a) + \beta \frac{1}{k_n a} = 0$$

---

p. $\beta \to 0$

$\alpha \ll E_1 a$

$$\cot(k_n a) \sim 0$$

$$\to k_n a = \frac{\pi}{2} n, \quad n = 1, 2, 3$$

$$E \simeq \frac{\hbar^2 (k_n a)^2}{2ma^2} = \frac{\hbar^2 \pi^2 n^2}{2m(2a)^2}$$

Lausnin fyrir engan vegg

---

p. $\beta \to \infty$

þá en rótunnar

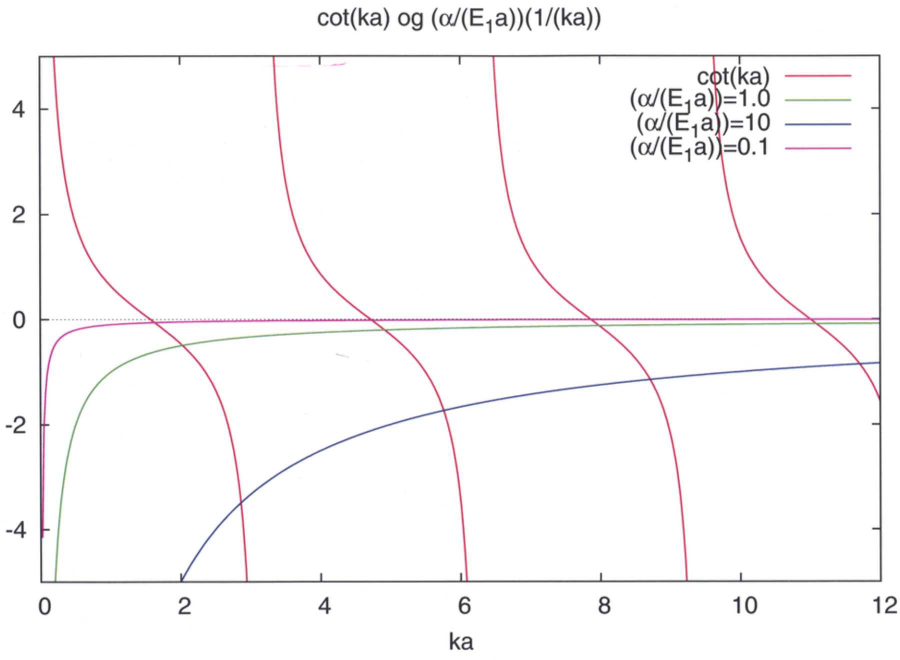
$$k_n a = n\pi, \quad n = 1, 2, 3$$

$$E \simeq \frac{\hbar^2 (k_n a)^2}{2ma^2} = \frac{\hbar^2 \pi^2 n^2}{2ma^2}$$

$$= \frac{\hbar^2 \pi^2 (2n)^2}{2m(2a)^2} \qquad n = 1, 2, 3, \ldots$$

Sama lausn og fyrir oddstöðu

(5)



cot(ka) og (α/(E1a))(1/(ka))
cot(ka)
(α/(E1a))=1.0
(α/(E1a))=10
(α/(E1a))=0.1
4
2
0
-2
-4
0
2
4
6
8
10
12
ka

Sjá S. Flügge bls. 38

Fig. 4

$$\Omega = \frac{2m\alpha}{\hbar^2}$$

$$\Omega a = \frac{2m\left(\frac{\alpha}{a}\right)a^2}{\hbar^2}$$

$$= \left(\frac{\alpha}{E_1 a}\right)$$

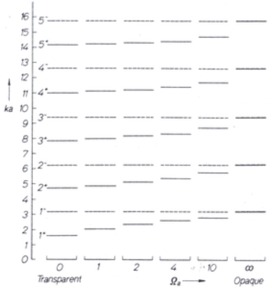


Fig. 3. Level positions for different values of wall opacity. Full lines even, broken lines odd parity

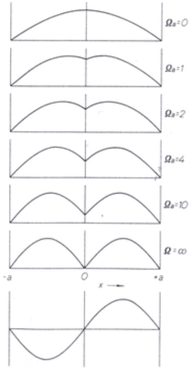


Fig. 4. Lowest eigenfunction for different wall opacities. Above 1^+, below 1^-, as limiting cases