2.6

$$\Psi(x,0) = A\left\{\psi_1(x) + e^{i\phi}\psi_2(x)\right\}$$

Reikna

$$\Psi(x,t) = A\left\{\psi_1(x)e^{-i\omega_1 t} + \psi_2(x)e^{-i\omega_2 t + i\phi}\right\}$$

Normunin er óbreytt $A = \frac{1}{\sqrt{2}}$

$$|\Psi(x,t)|^2 = \frac{1}{2}\left\{\psi_1^2(x) + \psi_2^2(x) + \psi_1(x)\psi_2(x)\left(e^{it(\omega_1-\omega_2)+i\phi} + e^{-it(\omega_1-\omega_2)-i\phi}\right)\right\}$$

$$= \frac{1}{2}\left\{\psi_1^2(x) + \psi_2^2(x) + 2\psi_1(x)\psi_2(x)\cos((\omega_1-\omega_2)t+\phi)\right\}$$

$$= \frac{1}{2}\left\{\psi_1^2(x) + \psi_2^2(x) + 2\psi_1(x)\psi_2(x)\cos(3\omega t - \phi)\right\}$$

$$\langle x \rangle = a\left\{\frac{1}{2} - \frac{16}{9\pi^2}\cos(3\omega t - \phi)\right\}$$

$\phi = \pi$ gefur $\cos(3\omega t - \pi) = -\cos(3\omega t)$ π úr fasa

$\phi = \frac{\pi}{2}$ –||– $\cos(3\omega t - \frac{\pi}{2}) = -\sin(3\omega t)$ $\frac{\pi}{2}$ úr fasa

2.14

H.O. með tíðni ω, eind í grunnástandi

Allt í einu verður $\omega' = 2\omega$

Hver eru líkindi fyrir því að mæla $E = \frac{\hbar\omega}{2}$?

— \|\| — $E = \hbar\omega$?

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fyrir óbreytt ω er

$$\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}\, a}} H_n\left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2} \quad , \quad a = \sqrt{\frac{\hbar}{m\omega}}$$

breytt $\psi_0'(x) = \frac{1}{\sqrt{\sqrt{\pi}\, a'}} e^{-\frac{1}{2}\left(\frac{x}{a'}\right)^2}$ $\quad a' = \sqrt{\frac{\hbar}{m 2\omega}} = \frac{a}{\sqrt{2}}$

$$= \frac{\sqrt{\sqrt{2}}}{\sqrt{\sqrt{\pi}\, a}} e^{-\left(\frac{x}{a}\right)^2}$$

Nú þyrfti að liða $\psi_0'(x)$ í gamla grunninum $\{\psi_n\}$

$$\psi_0'(x) = \sum_{n=0}^{\infty} C_n \psi_n(x)$$

en við erum bara beðin um að finna C_0 og C_1

$$C_0 = \int dx\, \psi_0^*(x)\, \psi_0'(x) = \frac{\sqrt{\sqrt{2}}}{\sqrt{\pi}\, a} \int_{-\infty}^{\infty} dx\, e^{-\frac{3}{2}\left(\frac{x}{a}\right)^2}$$

$$= \frac{\sqrt{\sqrt{2}}}{\sqrt{\pi}} \int_{-\infty}^{\infty} du\, e^{-\frac{3}{2}u^2} = \frac{\sqrt{\sqrt{2}}}{\sqrt{\pi}} \frac{\sqrt{2}\sqrt{\pi}}{\sqrt{3}} = \sqrt{\frac{2\sqrt{2}}{3}}$$

$$C_1 = 0 = \sqrt{\frac{2\sqrt{2}}{\sqrt{\pi}}} \int_{-\infty}^{\infty} du\; u\, e^{-\frac{3}{2}u^2}$$

(oddstætt)

ψ_0' er ekki eiginástand H, en ψ_0' er liðað í eiginástöndum H, við getum aðeins mælt orku sem eigingildi H með líkindum $|C_n|^2$

$$|C_0|^2 = \frac{2\sqrt{2}}{3} \simeq 0{,}94281\ldots$$

$$|C_1|^2 = 0$$

2.15 ①

Hver eru líkindi ~~þess~~ að finna H.O. í grunnástandi utan sígildu markanna

$$V = \frac{1}{2} m\omega^2 x^2 \quad , \quad E_0 = \frac{\hbar\omega}{2}$$

$$\rightarrow \quad x_{cl} = \pm\sqrt{\frac{\hbar}{m\omega}} = \underline{\pm a}$$

$|\psi_0|^2$ er jafnstætt, því eru líkindin

$$2\int_a^\infty dx\, |\psi_0|^2 = \frac{2}{\sqrt{\pi}} \int_a^\infty \frac{dx}{a}\, e^{-\left(\frac{x}{a}\right)^2} = \frac{2}{\sqrt{\pi}} \int_1^\infty du\, e^{-u^2}$$

$$\underbrace{= \frac{2}{\sqrt{\pi}\cdot 2} \underbrace{\Gamma\left(\tfrac{1}{2}, 1\right)}} = 0{,}07865$$

Incomplete gamma

$$\Gamma(a,x) = \int_x^\infty dt\, t^{a-1} e^{-t}$$