(1,5)

$$\Psi(x,t) = A e^{-\lambda|x|} e^{-i\omega t}$$

$\lambda > 0$
$A > 0$
$\omega > 0$

a) Norma

$$|A|^2 \int_{-\infty}^{\infty} dx\, \Psi^*(x,t)\,\Psi(x,t) = 2|A|^2 \int_0^{\infty} dx\, e^{-2\lambda x}$$

$$= 2|A|^2 \left\{ \frac{e^{-2\lambda x}}{-2\lambda} \Big|_0^{\infty} \right\} = 2|A|^2 \left\{ 0 + \frac{1}{2\lambda} \right\}$$

$$= |A^2| \frac{1}{\lambda} = 1, \quad \rightarrow \quad A = \sqrt{\lambda}$$

Vidd λ er L^{-1}, vidd Ψ er $L^{-1/2}$ som er i samsvar med $A = \sqrt{\lambda}$

b) Reikna $\langle x \rangle$ og $\langle x^2 \rangle$

$|\Psi|^2$ er jamn stott $\rightarrow$ $x|\Psi|^2$ er odd stott,

$\rightarrow \langle x \rangle = 0$

$x^2|\Psi|^2$ er jamn stott

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} dx\, x^2 |\Psi|^2 = 2\int_0^{\infty} dx\, x^2 e^{-2\lambda x}\,\lambda$$

$$= 2\lambda \left\{ -\frac{(2\lambda^2 x^2 + 2\lambda x + 1) e^{-2\lambda x}}{4\lambda^3} \Big|_0^{\infty} \right\}$$

$$= 2\lambda \left\{ 0 + \frac{1}{4\lambda^3} \right\} = \frac{1}{2\lambda^2}, \quad \langle x^2 \rangle = \frac{1}{2\lambda^2}$$

$$\Delta x = \sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle} = \sqrt{\frac{1}{2\lambda^2} - 0} = \frac{1}{\sqrt{2}\,\lambda}$$

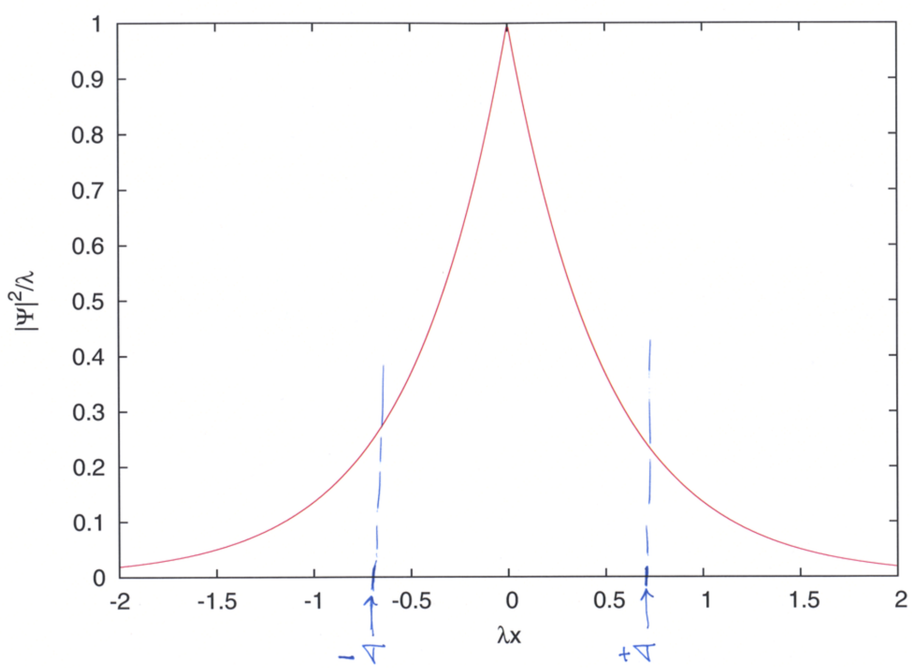
Teikna $|\Psi|^2$ og merkja punktana $(\langle x \rangle + \sigma_x)$ og $(\langle x \rangle - \sigma_x)$

$|\Psi|^2 = \lambda e^{-2\lambda|x|}$ pui er ~~sotlegt~~ ~~det~~

teikna vidderlausu stordina

$$\frac{1}{\lambda}|\Psi|^2 = e^{-2\lambda|x|} = e^{-2|u|} \quad \text{v.s.} \quad u$$

$$u = \lambda x \qquad \sigma_x = \frac{1}{\sqrt{2}\,\lambda} \quad \rightarrow \quad \sigma_u = \frac{1}{\sqrt{2}}$$



(5)

Líkindi þess að finna eindina utan $\pm T$

$$2\int_{T_x}^{\infty} dx\, |\Psi|^2 = 2\lambda \int_{T_x}^{\infty} dx\, e^{-2\lambda x} = 2\int_{1/\sqrt{2}}^{\infty} du\, e^{-2u}$$

$$= 2\left\{ -\frac{e^{-2u}}{2}\Big|_{1/\sqrt{2}}^{\infty} \right\} = \exp\left\{-\frac{2}{\sqrt{2}}\right\} = e^{-\sqrt{2}}$$

$\sim 0{,}243$

(1.7) (1)

finna $d_t\langle p\rangle$

þar sem x er ~~óháð~~ t er

$$d_t\langle p\rangle = d_t\left\{\int \Psi^*(-i\hbar\partial_x\Psi)\,dx\right\}$$

$$= \int dx\left\{(\partial_t\Psi^*)(-i\hbar\partial_x\Psi) + \Psi^*(-i\hbar\partial_x\partial_t\Psi)\right\}$$

$$= \int dx\left\{\left(-\frac{H}{i\hbar}\Psi^*\right)(-i\hbar\partial_x\Psi) + \Psi^*\left(-i\hbar\partial_x\frac{H}{i\hbar}\Psi\right)\right\}$$

Munum að

$$H = -\frac{\hbar^2}{2m}\partial_x^2 + V(x)$$

(2)

$$d_t\langle p\rangle = \int dx\left\{-\frac{\hbar^2}{2m}\left(\partial_x^2\Psi^*\partial_x\Psi - \Psi^*\partial_x^3\Psi\right)\right\}$$

$$+\int dx\left\{V\Psi^*(\partial_x\Psi) - \Psi^*(\partial_x(V\Psi))\right\}$$

$$= I_1 - \int dx\,\Psi^*(\partial_x V)\Psi = I_1 - \langle\partial_x V\rangle$$

skoðum

$$I_1 \sim \int dx\left\{\partial_x^2\Psi^*\partial_x\Psi - \Psi^*\partial_x^3\Psi\right\}$$

$$= \int dx\left\{-\partial_x\Psi^*\partial_x^2\Psi + \partial_x\Psi^*\partial_x^2\Psi\right\} + \left\{\partial_x\Psi^*\partial_x\Psi\Big|_{-\infty}^{\infty}\right\} - \left\{\partial_x\Psi^*\partial_x\Psi\Big|_{-\infty}^{\infty}\right\} = 0$$

(2.5) (1)

Eind í óendanlegum brunni

$$\Psi(x,0) = A\{\psi_1(x) + \psi_2(x)\}$$

a) Staðla $\Psi(x,0)$

$$\int_0^a dx\,|\Psi(x,0)|^2 = |A|^2\int_0^a dx\left\{|\psi_1(x)|^2 + |\psi_2(x)|^2 + \underbrace{2\psi_1(x)\psi_2(x)}_{=0}\right\}$$

Rauntölu föll

$=0$ (heildið)

eru stöðluð

þau eru hornrétt

$$= |A|^2\{1+1\} = 1$$

$$\rightarrow |A|^2 = \frac{1}{2} \quad \text{og} \quad A = \frac{1}{\sqrt{2}}$$

(2)

b) finna $\Psi(x,t)$ og $|\Psi|^2$

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} = n^2\hbar\omega \quad \text{ef} \quad \omega = \frac{\pi^2\hbar}{2ma^2}$$

$$E_1 = \hbar\omega_1 = \hbar\omega, \quad \omega_1 = \omega$$

$$E_2 = \hbar\omega_2 = 4\hbar\omega, \quad \omega_2 = 4\omega$$

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left\{\psi_1(x)e^{-i\omega_1 t} + \psi_2(x)e^{-i\omega_2 t}\right\}$$

$$|\Psi(x,t)|^2 = \frac{1}{2}\left\{\psi_1^2(x) + \psi_2^2(x) + \psi_1(x)\psi_2(x)\left(e^{it(\omega_1-\omega_2)} + e^{-it(\omega_1-\omega_2)}\right)\right\}$$

$$= \frac{1}{2}\left\{\psi_1^2(x) + \psi_2^2(x) + 2\psi_1(x)\psi_2(x)\cos\left((\omega_1-\omega_2)t\right)\right\}$$

(3)

$$|\psi(x,t)|^2 = \frac{1}{a}\left\{\sin^2\left(\pi\frac{x}{a}\right) + \sin^2\left(2\pi\frac{x}{a}\right) + 2\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi x}{a}\right)\cos(3\omega t)\right\}$$

c) Rekna $\langle x\rangle$

$$\langle x\rangle = \int_0^a dx\, \Psi^*(x) \times \Psi(x)$$

$$= \frac{1}{2}\int_0^a dx\left\{\psi_1 e^{+i\omega_1 t} + \psi_2 e^{+i\omega_2 t}\right\} \times \left\{\psi_1 e^{-i\omega_1 t} + \psi_2 e^{-i\omega_2 t}\right\}$$

$$= \int_0^a dx \times |\Psi(x,t)|^2$$

$$= \frac{1}{a}\int_0^a dx\, x\left\{\sin^2\left(\frac{\pi x}{a}\right) + \sin^2\left(\frac{2\pi x}{a}\right) + 2\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi x}{a}\right)\cos(3\omega t)\right\}$$

(4)

$$= a\int_0^1 du\, u\left\{\sin^2(\pi u) + \sin^2(2\pi u) + 2\sin(\pi u)\sin(2\pi u)\cos(3\omega t)\right\}$$

Notum

$$\int_0^1 du\, u\sin^2(\pi u) = \frac{1}{8\pi^2}\left\{1 - 1 + 2\pi^2\right\} = \frac{1}{4}$$

$$\int_0^1 du\, u\sin^2(2\pi u) = \frac{1}{4}$$

$$\int_0^1 du\, u\sin(\pi u)\sin(2\pi u) = -\frac{-1+9}{18\pi^2} - \frac{4}{9\pi^2}$$

$$= \frac{-8-8}{18\pi^2} = -\frac{16}{18\pi^2}$$

$$= -\frac{8}{9\pi^2}$$

(5)

$$\langle x\rangle = a\left\{\frac{1}{2} - \frac{16}{9\pi^2}\cos(3\omega t)\right\}$$

$$= \frac{a}{2}\left\{1 - \frac{32}{9\pi^2}\cos(3\omega t)\right\}$$

$$\simeq \frac{a}{2}\left\{1 - 0{,}36\cdot\cos(3\omega t)\right\}$$

$$\max\{\langle x\rangle\} \simeq a\cdot 0{,}68$$

$$\min\{\langle x\rangle\} \simeq a\cdot 0.32$$

utslaget er $\frac{a}{2}\frac{32}{9\pi^2} \simeq 0{,}18\,a$

d) Reikna $\langle p \rangle$

$$\langle p \rangle = \int_0^a dx\, \Psi^*(x,t)\left\{-i\hbar\partial_x \Psi(x,t)\right\}$$

skoðum fyrst

$$\partial_x \Psi(x,t) = \frac{1}{\sqrt{2}}\left\{\partial_x\psi_1 e^{-i\omega_1 t} + \partial_x\psi_2 e^{-i\omega_2 t}\right\}$$

$$\partial_x\psi_1 \sim \cos\left(\frac{\pi x}{a}\right)\cdot\frac{\pi}{a}$$

$$\partial_x\psi_1 \sim \cos\left(\frac{2\pi x}{a}\right)\cdot\frac{2\pi}{a}$$

$$\langle p \rangle = -i\hbar\frac{1}{a}\int_0^a dx\left\{\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi x}{a}\right)\cdot\frac{\pi}{a} + \sin\left(\frac{2\pi x}{a}\right)\cos\left(\frac{2\pi x}{a}\right)\cdot\frac{2\pi}{a}\right.$$

$$\left. + \frac{2\pi}{a}\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{2\pi x}{a}\right)e^{it(\omega_1-\omega_2)} + \frac{\pi}{a}\sin\left(\frac{2\pi x}{a}\right)\cos\left(\frac{\pi x}{a}\right)e^{-it(\omega_1-\omega_2)}\right\}$$

Notum

$$\int_0^1 du \sin(\pi u)\cos(\pi u) = 0$$

$$\int_0^1 du \sin(2\pi u)\cos(2\pi u) = 0$$

$$\int_0^1 du \sin(\pi u)\cos(2\pi u) = -\frac{1}{3\pi} - \frac{-1+3}{6\pi} = -\frac{2}{3\pi}$$

$$\int_0^1 du \sin(2\pi u)\cos(\pi u) = \frac{2}{3\pi} - \frac{-4}{6\pi} = \frac{4}{3\pi}$$

$$\langle p \rangle = -i\hbar\frac{\pi}{a}\frac{4}{3\pi}\left\{-2i\sin((\omega_1-\omega_2)t)\right\}$$

$$= \frac{\hbar}{a}\frac{8}{3}\sin(3\omega t) \leftarrow \text{Rauntala}$$

e) Mæli orku til helminga fæ ég gildin $E_1 = \hbar\omega$ og $E_2 = 4\hbar\omega$

ψ_1 og ψ_2 hafa sama vægi í $\Psi(x,0)$

finna $\langle H \rangle$

$$\langle H \rangle = \int dx\, \Psi^*(x,t) H \Psi(x,t) = \frac{1}{2}\{E_1 + E_2\}$$

$$= \frac{5\hbar\omega}{2}$$

2.7

Eind í óendanlegum brunni með upphafsb.

$$\Psi(x,0) = \begin{cases} Ax & 0 \le x \le \frac{a}{2} \\ A(a-x), & \frac{a}{2} \le x \le a \end{cases}$$

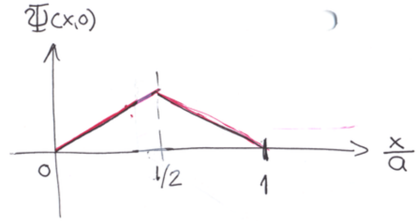
a) Teikna $\Psi(x,0)$ og reikna A

$$\int_0^a dx\, |\Psi(x,0)|^2 = A^2\int_0^{a/2} dx\, x^2 + A^2\int_{a/2}^a dx\,(a-x)^2$$

$$= A^2\left\{a^3\int_0^{1/2} du\, u^2 + a^3\int_{1/2}^1 du\,(1-u)^2\right\}$$

$$= A^2 a^3\left\{\frac{1}{24} + \frac{1}{24}\right\} = A^2 a^3\frac{1}{12} = 1 \rightarrow A = \frac{\sqrt{12}}{\sqrt{a^3}}$$

(2)



b) finna $\Phi(x,t)$

$$\Phi(x,0) = \sum_{n=1}^{\infty} C_n \phi_n(x)$$

$$C_n = \int_0^a dx\, \phi_n^*(x)\, \Phi(x,0)$$

$$C_n = \int_0^{a/2} dx\, \phi_n(x)\, A x + \int_{a/2}^{a} dx\, \phi_n(x)\, A(a-x)$$

$$= \sqrt{\frac{2}{a}}\sqrt{\frac{12}{a^3}} \int_0^{a/2} dx\, \mathrm{Sin}\left(\frac{n\pi x}{a}\right) x + \sqrt{\frac{2}{a}}\sqrt{\frac{12}{a^3}} \int_{a/2}^{a} dx\, \mathrm{Sin}\left(\frac{n\pi x}{a}\right)(a-x)$$

(3)

$$C_n = \sqrt{\frac{2}{a}}\sqrt{\frac{12a^2}{a^3}} \left[\int_0^{1/2} du\, \mathrm{Sin}(n\pi u)\cdot u + \int_{1/2}^{1} du\, \mathrm{Sin}(n\pi u)\cdot(1-u) \right]$$

heildi fyrir Fourier stuðla, þegar þau eru legst fost

$$\Phi(x,t) = \sum_{n=1}^{\infty} C_n \phi_n(x)\, e^{-in^2\omega t} \quad \text{af } \omega = \frac{\pi^2\hbar}{2ma^2}$$

$$\text{og } \phi_n(x) = \sqrt{\frac{2}{a}}\, \mathrm{Sin}\left(\frac{n\pi}{a}\right)$$

$$C_n = \sqrt{24}\, f(n)$$

↑ fall af n sem við byrjum að finna t.d. (GR 2.634.1) eða

$C_n = 0$ fyrir $n = 2, 4, 6, 8$ vegna samhverfu, og einfaldlega

(4)

$$\int_0^{1/2} du\, \mathrm{Sin}(n\pi u)\cdot u = \mathrm{Im}\left\{ \int_0^{1/2} du\, e^{in\pi u}\cdot u \right\} = I_1$$

$$\int_{1/2}^{1} du\, \mathrm{Sin}(n\pi u)(1-u) = \mathrm{Im}\left\{ \int_{1/2}^{1} du\, e^{in\pi u}(1-u) \right\} = I_2$$

$$I_1 = \mathrm{Im}\left\{ -\frac{1}{n^2\pi^2}\left(\frac{(i\pi n - 2)}{2}(i)^n - 1 \right) \right\} = \frac{1}{n^2\pi^2}\mathrm{Im}(i)^n$$

$$I_2 = \mathrm{Im}\left\{ -\frac{1}{n^2\pi^2}\left((-1)^n - \frac{(i\pi n + 2)}{2}(i)^n \right) \right\} = \frac{1}{n^2\pi^2}\mathrm{Im}(i)^n$$

$$I_1 + I_2 = \frac{2}{n^2\pi^2}\mathrm{Im}(i)^n = \frac{2}{n^2\pi^2}(-1)^p \quad \text{af } n = 2p+1$$

(5)

$$C_n = \sqrt{\frac{2}{a}}\sqrt{\frac{12a^2}{a^3}}\, \frac{2}{n^2\pi^2}(-1)^p \quad \text{af } n = 2p+1$$

$$= \sqrt{24}\, \frac{2}{n^2\pi^2}(-1)^p = \sqrt{96}\, \frac{(-1)^p}{n^2\pi^2} \qquad p = 0, 1, 2, \ldots$$

$$\rightarrow \boxed{\Phi(x,t) = \sum_{p=0}^{\infty} \sqrt{96}\, \frac{(-1)^p}{(2p+1)^2\pi^2} \sqrt{\frac{2}{a}}\, \mathrm{Sin}\left(\frac{(2p+1)\pi x}{a}\right) e^{-i(2p+1)^2\omega t}}$$

c) Líkindi fyrir að mæla E_1 er |liðunarstuðulin|^2 við ϕ_1

$$|C_1|^2 = \frac{96}{\pi^4}$$

$p=0$

(6)

d) $$\langle H\rangle = \sum_{p=0}^{\infty} |C_{2p+1}|^2 E_{2p+1} = \frac{96}{\pi^4}\hbar\omega_1 \sum_{p=0}^{\infty} \frac{(2p+1)^2}{(2p+1)^4}$$

$$= \frac{96}{\pi^4}\hbar\omega_1 \sum_{p=0}^{\infty} \frac{1}{(2p+1)^2} \quad , \quad \hbar\omega_1 = \frac{\hbar^2\pi^2}{2ma^2}$$

$$= \frac{96}{\pi^4}\hbar\omega_1 \frac{\pi^2}{8}$$ (GR 0.234. 2)

(1)

2.12 Finna $\langle p\rangle, \langle p^2\rangle, \langle x\rangle, \langle x^2\rangle$ og $\langle T\rangle$ fyrir eiginástönd H.O. {nota virkjana a^+ og a^-}

náttúrulegi lengdarstalinn $a = \sqrt{\frac{\hbar}{m\omega}}$

$$x = \frac{a}{\sqrt{2}}(a_+ + a_-)$$

$$p = \frac{i\hbar}{\sqrt{2}a}(a_+ - a_-)$$

$$\langle x\rangle = \frac{a}{\sqrt{2}}\int_{-\infty}^{\infty} dx\, \psi_n^* (a_+ + a_-)\psi_n = \frac{a}{\sqrt{2}}\int_{-\infty}^{\infty} dx\, \psi_n^* \left\{\sqrt{n+1}\,\psi_{n+1} + \sqrt{n}\,\psi_{n-1}\right\}$$

$$= 0$$

því eiginástöndin eru hornrétt

(2)

$$\langle p\rangle = \frac{\hbar}{\sqrt{2}a}\int dx\, \psi_n^* (a_+ - a_-)\psi_n = 0$$

$$x^2 = \frac{a^2}{2}\left\{a_+^2 + a_+a_- + a_-a_+ + a_-^2\right\}$$ hinir liðirnir hverfa

$$\langle x^2\rangle = \frac{a^2}{2}\int_{-\infty}^{\infty} dx\, \psi_n^* \left[a_+a_- + a_-a_+\right]\psi_n$$

$$= \frac{a^2}{2}\left\{\sqrt{n}\sqrt{n} + \sqrt{n+1}\sqrt{n+1}\right\} = \frac{a^2}{2}\{n+n+1\}$$

$$= a^2\left(n+\tfrac{1}{2}\right)$$

$$\langle p^2\rangle = \frac{\hbar^2}{2a^2}\int dx\, \psi_n^* \left\{+a_+a_- + a_-a_+\right\}\psi$$

$$= +\frac{\hbar^2}{2a^2}\left\{\sqrt{n}\sqrt{n} + \sqrt{n+1}\sqrt{n+1}\right\} = \frac{\hbar^2}{a^2}\left(n+\tfrac{1}{2}\right)$$

(3)

$$\langle T\rangle = \frac{\langle p^2\rangle}{2m} = \frac{\hbar^2}{2ma^2}\left(n+\tfrac{1}{2}\right) = \frac{\hbar^2 m\omega}{2m\hbar}\left(n+\tfrac{1}{2}\right) = \frac{\hbar\omega}{2}\left(n+\tfrac{1}{2}\right)$$

$$\langle T\rangle + \langle V\rangle = \langle H\rangle = \hbar\omega\left(n+\tfrac{1}{2}\right)$$ eins og búast mátti við

$$\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = a\cdot\sqrt{n+\tfrac{1}{2}}$$

$$\sigma_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \frac{\hbar}{a}\sqrt{n+\tfrac{1}{2}}$$

$$\rightarrow \sigma_x\cdot\sigma_p = \hbar\left(n+\tfrac{1}{2}\right) \geq \frac{\hbar}{2}$$ fyrir öll $n = 0,1,2,\dots$

2.21 ①

$$\Psi(x,0) = A e^{-\alpha|x|}, \quad A \text{ og } \alpha \text{ eru jákvæðar rauntölur}$$

Normerum í skiladæmi 1.5, $A = \sqrt{\alpha}$

b) Frjáls vísind er lýst með $\Psi(x,0)$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, \phi(k) e^{i(kx-\omega(k)t)}$$

þ.s. $\omega(k) = \frac{\hbar k^2}{2m}$, Finna $\phi(k)$

$$\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\, \Psi(x,0) e^{-ikx}$$

②

$$\phi(k) = \sqrt{\frac{\alpha}{2\pi}}\int_{-\infty}^{\infty} dx\, e^{-\alpha|x|-ikx} = \sqrt{\frac{\alpha}{2\pi}}\left\{\int_{-\infty}^{0} dx\, e^{\alpha x-ikx} + \int_{0}^{\infty} dx\, e^{-\alpha x - ikx}\right\}$$

því $|x| = \begin{cases} -x & \text{þ. } x<0 \\ x & \text{þ. } x>0 \end{cases}$

$$\phi(k) = \sqrt{\frac{\alpha}{2\pi}}\left\{\left.\frac{e^{\alpha x-ikx}}{\alpha-ik}\right|_{-\infty}^{0} + \left.\frac{e^{-\alpha x-ikx}}{-\alpha-ik}\right|_{0}^{\infty}\right\}$$

$$= \sqrt{\frac{\alpha}{2\pi}}\left\{\frac{1}{\alpha-ik} + \frac{-1}{-\alpha-ik}\right\} = \sqrt{\frac{\alpha}{2\pi}}\left\{\frac{1}{\alpha-ik}+\frac{1}{\alpha+ik}\right\}$$

$$= \sqrt{\frac{\alpha}{2\pi}}\left\{\frac{2\alpha}{\alpha^2+k^2}\right\}$$

③

því fæst

$$\Psi(x,t) = \sqrt{\frac{\alpha}{2\pi}}\,\frac{2\alpha}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, \frac{e^{i(kx-\omega(k)t)}}{\alpha^2+k^2}$$

Markgildi α stórt eða lítið

$$\Psi(x,t) = \frac{\sqrt{\alpha}}{\sqrt{\pi}}\,\frac{\alpha^2}{\alpha^2}\int_{-\infty}^{\infty}\frac{dk}{\alpha}\,\frac{e^{i(\frac{k}{\alpha}(\alpha x)-\omega(k)t)}}{1+(\frac{k}{\alpha})^2}$$

(*)

$$= \sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} du\, \frac{e^{i(u\cdot(\alpha x) - \omega(u)\alpha^2 t)}}{1+u^2}$$

④

Stórt α → þröngt $\Psi(x,0)$

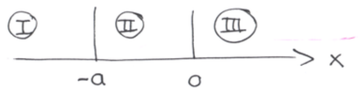
(*) → heildað yfir flatarbylgjur á breiðu k-bili
→ mikil tvístrun

lítið α → breitt $\Psi(x,0)$

(*) → heildað yfir flatarbylgjur á þröngu k-bili
→ lítil tvístrun

(1)

Finna dreifi ástönd mættisins $V(x) = \alpha\{\delta(x) + \delta(x+a)\}$



Bylgjuföllin á svæðum

Gerum ráð fyrir inn-bylgju með $A=1$ frá vinstri, engin inn-bylgja frá hægri

I $\quad \psi(x) = e^{ikx} + Be^{-ikx}$

II $\quad \psi(x) = Ce^{ikx} + De^{-ikx}$

III $\quad \psi(x) = Fe^{ikx}$

$E > 0 \quad , \quad k^2 = \frac{2mE}{\hbar^2}$

(2)

Samfella í $x = -a$

$$e^{-ika} + Be^{+ika} = Ce^{-ika} + De^{+ika} \qquad (1)$$

Samfella í $x = 0$

$$C + D = F \qquad (2)$$

Brot afleiðu í $x = -a$

$$\psi'(-a^+) - \psi'(-a^-) = \frac{2m\alpha}{\hbar^2}\psi(-a)$$

$$ik\{Ce^{-ika} - De^{ika} - e^{-ika} + Be^{ika}\} = \frac{2m\alpha}{\hbar^2}\{e^{-ika} + Be^{ika}\} \qquad (3)$$

(3)

Brot afleiðu í $x = 0$

$$ik\{F - C + D\} = \frac{2m\alpha}{\hbar^2}F \qquad (4)$$

$(2) \to D = F - C$

$(1): \quad e^{-ika} + Be^{ika} = Ce^{-ika} + (F-C)e^{ika}$

$(3): \quad \{Ce^{-ika} - (F-C)e^{ika} - e^{-ika} + Be^{ika}\} = \frac{2m\alpha}{ik\hbar^2}\{e^{-ika} + Be^{ika}\}$

β

$(4): \quad \{F - C + (F-C)\} = \frac{2m\alpha}{\hbar^2 ik}F$

β

(4)

3 jöfnur, endurritum

$(1): \quad e^{ika}B + C(e^{ika} - e^{-ika}) - Fe^{ika} = -e^{-ika}$

$(3): \quad e^{ika}(1-\beta)B + (e^{ika} + e^{-ika})C - Fe^{ika} = e^{-ika}(1+\beta)$

$(4): \quad -2C + F(2-\beta) = 0$

$(4) \to F = \frac{2C}{2-\beta}$

$(4) \to (1): \quad e^{ika}B + 2i\sin(ka)C - \frac{2C}{2-\beta}e^{ika} = -e^{-ika}$

$$\boxed{e^{ika}B + C\left\{2i\sin(ka) - \frac{2e^{ika}}{2-\beta}\right\} = -e^{-ika}} \qquad (i)$$

(5)

$$C = \frac{F}{2}(2-\beta)$$

nota i (1)

$$\boxed{e^{ika}\beta + F\left\{i(2-\beta)\sin(ka) - e^{ika}\right\} = -e^{-ika}} \quad (i)$$

nota i (3)

$$\boxed{e^{ika}(1-\beta)\beta + F\left\{(2-\beta)\cos(ka) - e^{ika}\right\} = e^{-ika}(1+\beta)} \quad (ii)$$

(6)

$$(i) \rightarrow \quad e^{ika}\beta = -e^{-ika} - F\left\{i(2-\beta)\sin(ka) - e^{ika}\right\}$$

nota i (ii)

$$-(1-\beta)e^{-ika} - (1-\beta)F\left\{i(2-\beta)\sin(ka) - e^{ika}\right\}$$

$$+ F\left\{(2-\beta)\cos(ka) - e^{ika}\right\} = e^{-ika}(1+\beta)$$

$$\rightarrow F\left[\left\{(2-\beta)\cos(ka) - e^{ika}\right\} - (1-\beta)\left\{i(2-\beta)\sin(ka) - e^{ika}\right\}\right]$$

$$= e^{-ika}(1+\beta) + (1-\beta)e^{-ika}$$

$$= e^{-ika}\,2$$

(7)

$$F\left[-\beta e^{ika} + (2-\beta)\cos(ka) - i(2-\beta)(1-\beta)\sin(ka)\right] = e^{-ika}\,2$$

$$F\left[-\beta e^{ika} + i(2-\beta)\beta\sin(ka) + (2-\beta)e^{-ika}\right] = e^{-ika}\,2$$

$$F\left[2e^{-ika} - 2\beta\cos(ka) + i(2-\beta)\beta\sin(ka)\right] = e^{-ika}\,2$$

$$F = \frac{e^{-ika}\,2}{2e^{-ika} - 2\beta\cos(ka) + i(2-\beta)\beta\sin(ka)}$$

$$= \frac{e^{-ika}\,2}{2\cos(ka) - 2i\sin(ka) - 2\beta\cos(ka) + i(2-\beta)\beta\sin(ka)}$$

(8)

Setjum $\beta = \frac{2m\alpha}{ik\hbar^2} = i\gamma$ með $\gamma = -\frac{2m\alpha}{k\hbar^2} \in \mathbb{R}$

$$F = \frac{e^{-ika}\,2}{2\cos(ka) - 2i\sin(ka) - 2i\gamma\cos(ka) + i(2-i\gamma)i\gamma\sin(ka)}$$

$$= \frac{e^{-ika}\,2}{2\cos(ka) - 2\gamma\sin(ka) + i\left\{-2\gamma\cos(ka) + (\gamma^2-2)\sin(ka)\right\}}$$

$$|F|^2 = FF^* = \frac{4}{4\left\{\cos(ka) - \beta\sin(ka)\right\}^2 + \left\{(\gamma^2-2)\sin(ka) - 2\gamma\cos(ka)\right\}^2}$$

$$= \frac{1}{\left\{\cos(ka) - \beta\sin(ka)\right\}^2 + \frac{1}{4}\left\{(\gamma^2-2)\sin(ka) - 2\gamma\cos(ka)\right\}^2}$$

9

$$B = -e^{-2ika} - e^{-ika}F\left\{i(2-\beta)\sin(ka) - e^{ika}\right\}$$

$$= -e^{-2ika} - e^{-ika}F\left\{i(2-i\gamma)\sin(ka) - e^{ika}\right\}$$

$$= -e^{-2ika} - \frac{2\left\{i(2-i\gamma)\sin(ka) - e^{ika}\right\}e^{-2ika}}{2\cos(ka) - 2\gamma\sin(ka) + i\left\{(\gamma^2-2)\sin(ka) - 2\gamma\cos(ka)\right\}}$$

Ég ætla að nota í grafik $|F|^2$, og innan gnuplots að reikna $|B|^2$ frá B-inu hér.

Til þess þarf ég að hugsa um stærðum

Ég vil nota tækifærið til að sýna að gnuplot vinnur mjög einfaldlega með tvinntölur.

10

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 (ka)^2}{2ma^2} = E_1 \cdot (ka)^2$$

$$\beta = \frac{2m\alpha}{ik\hbar^2} = i\gamma \;\to\; \gamma = -\frac{2m\alpha}{\hbar^2 k} = -\frac{2m\alpha a}{\hbar^2 (ka)}$$

$$= -\frac{2ma^2}{\hbar^2 (ka)}\frac{\alpha}{a} = -\left(\frac{\alpha}{aE_1}\right)\frac{1}{ka}$$

víddarlausar stærðir

Ég hugsa mér að E_1 sé gefinn og þarf því að segja til um styrk δ-mættis með $\frac{\alpha}{aE_1}$

Munum að $[\alpha] \sim$ Orka · L

Hér mun fylgja gnu-skrifta sem býr til grafið í gnuplot með skipuninni

"Load skrifta.gnu"

11

```
set term post landscape enhanced solid color "Helvetica" 18
set output 'TR16p0.ps'
#
set xlabel 'ka'
set ylabel 'Probability(ka)'
set title "{/Symbol a}/(aE_1)=16.0"
#
g(x)=-16.0/x
R(x)=(cos(x)-g(x)*sin(x))**2
Q(x)=0.25*(((g(x)**2)-2.0)*sin(x)-2.0*g(x)*cos(x))**2
F2(x)=1.0/(R(x)+Q(x))
#
ci={0.0,1.0}
A1(x)=-exp(-2.0*ci*x)
A2(x)=2.0*(ci*(2.0-ci*g(x))*sin(x)-exp(ci*x))*exp(-2.0*ci*x)
A3(x)=2.0*(cos(x)-g(x)*sin(x))
A4(x)=ci*(((g(x)**2)-2.0)*sin(x)-2.0*g(x)*cos(x))
B(x)=(abs(A1(x)-A2(x)/(A3(x)+A4(x))))**2
#
set samples 4000
plot [0.01:20.0][0:1.1] F2(x) w l title "T" lw 2,\
B(x) w l title "R" lw 2,\
B(x)+F2(x) w l title "T+R" lw 2
```

ps prentun

merking ása og grafs

i

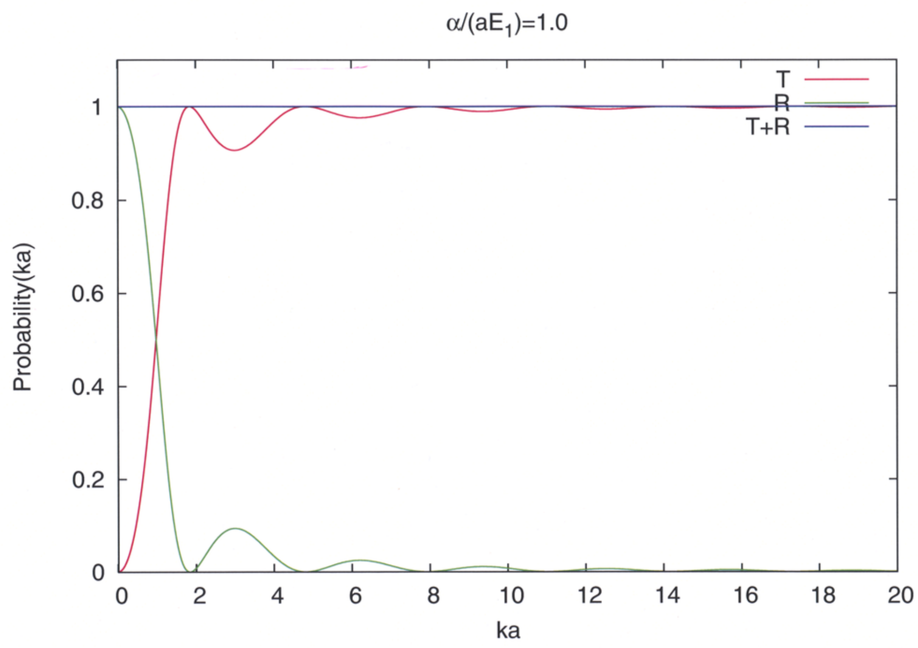
fjölgum punkta

plot

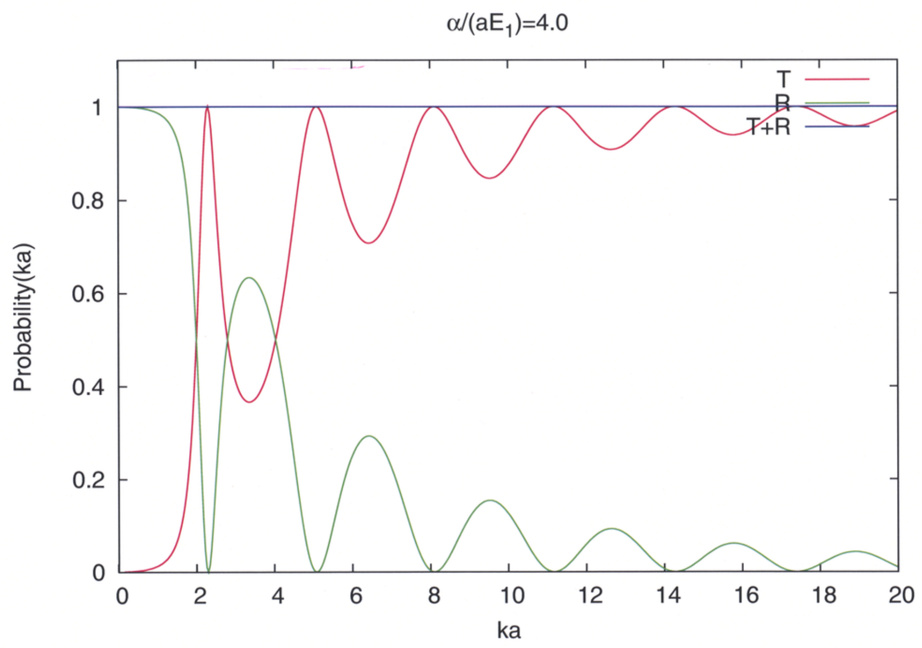
Keyrist með

"gnuplot 2-delta.gnu"

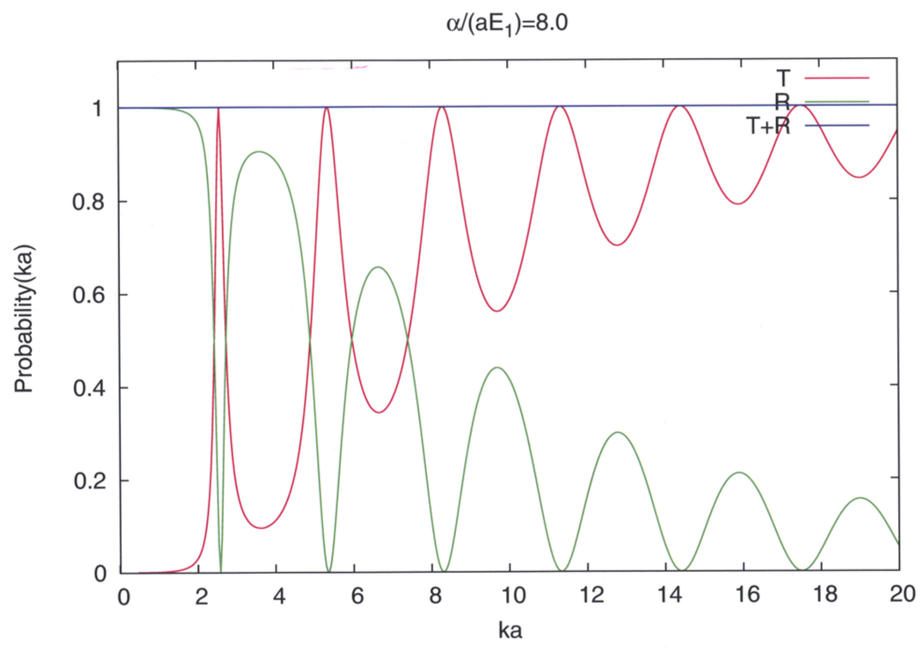
12



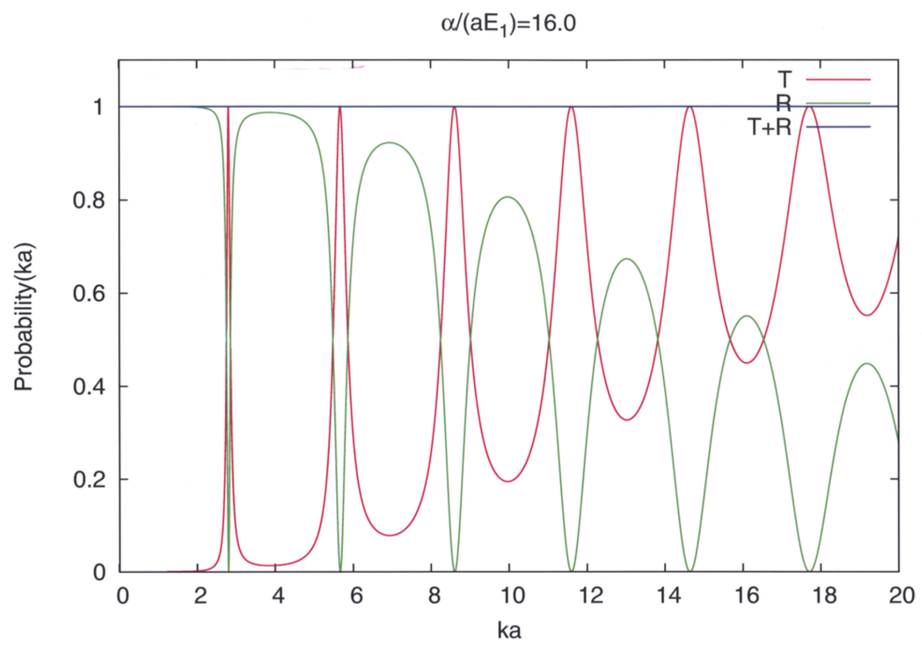
(13)


α/(aE1)=4.0
T
R
T+R
Probability(ka)
ka
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
12
14
16
18
20

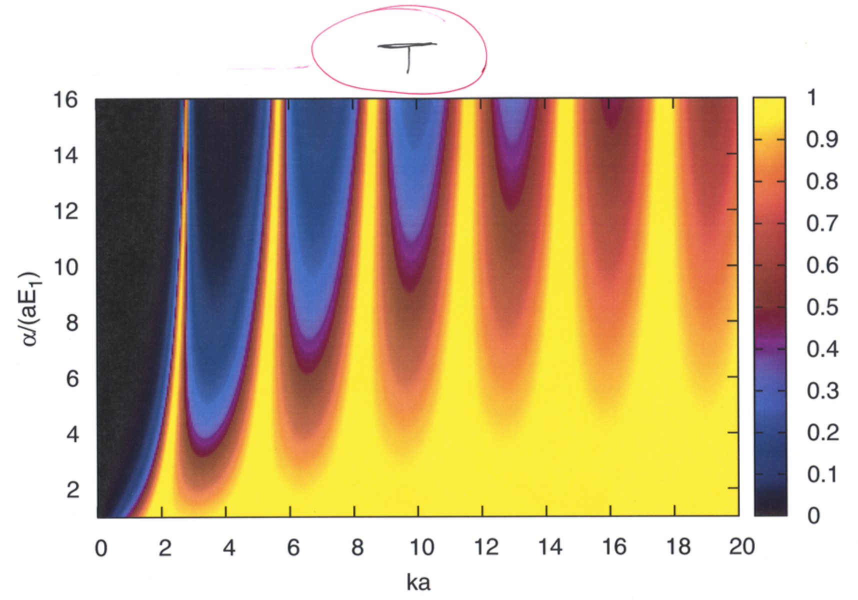
(14)


α/(aE1)=8.0
T
R
T+R
Probability(ka)
ka
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
12
14
16
18
20

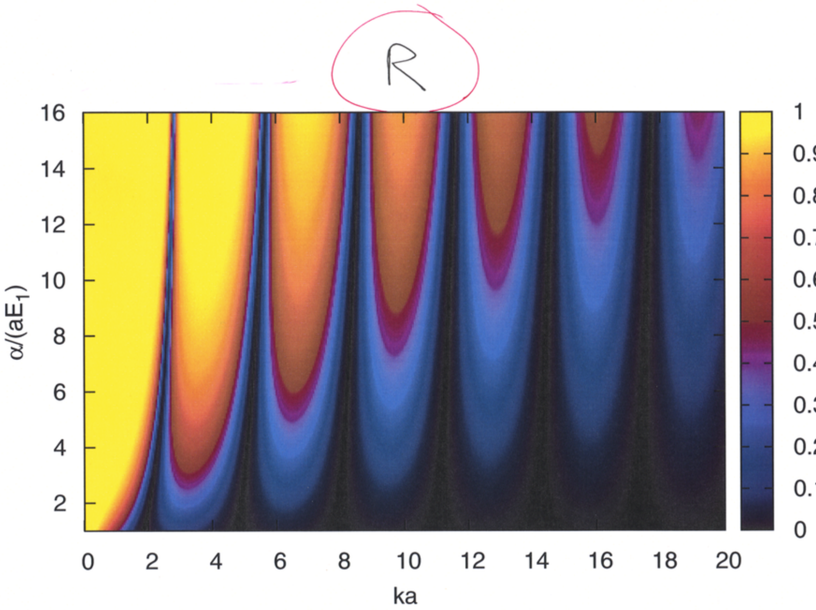
(15)


α/(aE1)=16.0
T
R
T+R
Probability(ka)
ka
0
0.2
0.4
0.6
0.8
1
0
2
4
6
8
10
12
14
16
18
20

(16)


T
α/(aE1)
ka
2
4
6
8
10
12
14
16
0
2
4
6
8
10
12
14
16
18
20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

17



18

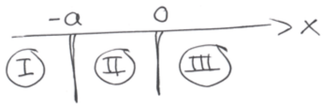
Því sést að auðvelt væri að reikna líka C og D með aðstoð gnuplots (eða octave) og síðan mætti teikna líkindadreifingarnar $|\Psi(x)|^2$ á öllum svæðum og sjá þá um inn og speglunarbylgju á svæði I, hvernig líkindin á II vaxa og minnka með (ka) og hvernig þau eru flöt á svæði III

Þetta eru myndir sem ég hef ekki séð í kennslubókum en segja mikið um eðlisfræðina

1

$V(x) = -\alpha\{\delta(x)+\delta(x+a)\}$, α og a eru jákvæðar stærðir

→ tveir brunnar, annar í $x=-a$ og hinn í $x=0$



Beðið er um bundnu ástöndin. Ef þau eru til gildir að $E<0$

Lausnir Schrödingerjöfnunnar á þessum svæðum er því, með $\kappa^2 = -\frac{2mE}{\hbar^2}$, $\kappa > 0$

I $\psi(x) = Ae^{+\kappa x} + Be^{-\kappa x}$

II $\psi(x) = Ce^{+\kappa x} + De^{-\kappa x}$

III $\psi(x) = Fe^{+\kappa x} + Ge^{-\kappa x}$

Bundnar lausnirnar eru normanlegar, því verður að gilda að $B=0$, $F=0$

2

I $\psi(x) = Ae^{\kappa x}$

II $\psi(x) = Ce^{\kappa x} + De^{-\kappa x}$

III $\psi(x) = Ge^{-\kappa x}$

Bylgjuföllin eru samfelld í

$x=-a$

$$Ae^{-\kappa a} = Ce^{-\kappa a} + De^{+\kappa a}$$

$x=0$

$$C + D = G$$

Bylgjuföllin hafa brot í afleiðu í

$x=-a$

$$\psi'(-a^+) - \psi'(-a^-) = -\frac{2m\alpha}{\hbar^2}\psi(a)$$

$$C\kappa e^{-\kappa a} - D\kappa e^{+\kappa a} - A\kappa e^{-\kappa a} = -\frac{2m\alpha}{\hbar^2}Ae^{-\kappa a}$$

$x=0$

$$-\kappa G - C\kappa + D\kappa = -\frac{2m\alpha}{\hbar^2}G$$

4 jöfnur, 4 óþekktar stærðir

$$Ae^{-ka} - Ce^{-ka} - De^{ka} = 0$$

$$C + D - G = 0$$

$$Ce^{-ka} - De^{+ka} - Ae^{-ka} = -\frac{2m\alpha}{\hbar^2 K} A e^{-ka} = -\beta A e^{-ka}$$

$$-G - C + D = -\frac{2m\alpha}{\hbar^2 K} G = -\beta G$$

þ.a. endurritað höfum við

$$Ae^{-ka} - Ce^{-ka} - De^{+ka} = 0$$

$$C + D - G = 0$$

$$Ce^{-ka} - De^{+ka} + A(\beta-1)e^{-ka} = 0$$

$$-C + D + G(\beta-1) = 0$$

$$\begin{pmatrix} e^{-ka} & -e^{-ka} & -e^{+ka} & 0 \\ 0 & 1 & 1 & -1 \\ (\beta-1)e^{-ka} & e^{-ka} & -e^{+ka} & 0 \\ 0 & -1 & 1 & (\beta-1) \end{pmatrix} \begin{pmatrix} A \\ C \\ D \\ G \end{pmatrix} = 0$$

Til þess að lausn sé til þarf ákveðan af fylkinu að vera 0

$$e^{-ka}\left\{-(\beta-1)e^{ka} + e^{ka} - (\beta-1)e^{-ka} - e^{-ka}\right\}$$

$$-e^{ka}\left\{(\beta-1)e^{-ka} - (\beta-1)^2 e^{-ka}\right\} + e^{-ka}\left\{-(\beta-1)e^{-ka} - (\beta-1)^2 e^{-ka}\right\}$$

$$= 0$$

einföldum

$$\det M = -\beta^2 e^{-2ka} + \beta^2 - 4\beta + 4 = 0 \qquad \beta = \frac{2m\alpha}{\hbar^2 K}$$

$$\rightarrow e^{-2ka} = \left(1 - \frac{2}{\beta}\right)^2 \qquad \beta = \frac{2m\alpha a}{\hbar^2 (Ka)}$$

$$e^{-ka} = \pm\left(1 - \frac{2}{\beta}\right) \qquad = \frac{2ma^2 \left(\frac{\alpha}{a}\right)}{\hbar^2} \frac{1}{(Ka)}$$

$$= \left(\frac{\alpha}{E_1 a}\right)\frac{1}{(Ka)}$$

(*) $$e^{-Ka} = \pm\left(1 - 2\left(\frac{E_1 a}{\alpha}\right)(Ka)\right)$$

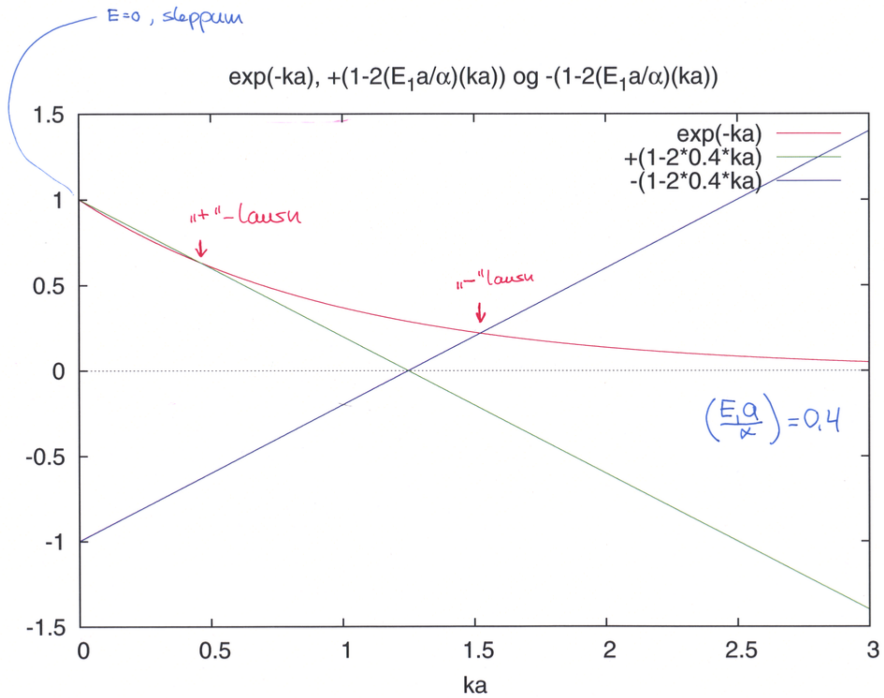
Könnum lausnir

„−"-lausnin er til fyrir öll jákvæð gildi á $\left(\frac{E_1 a}{\alpha}\right)$

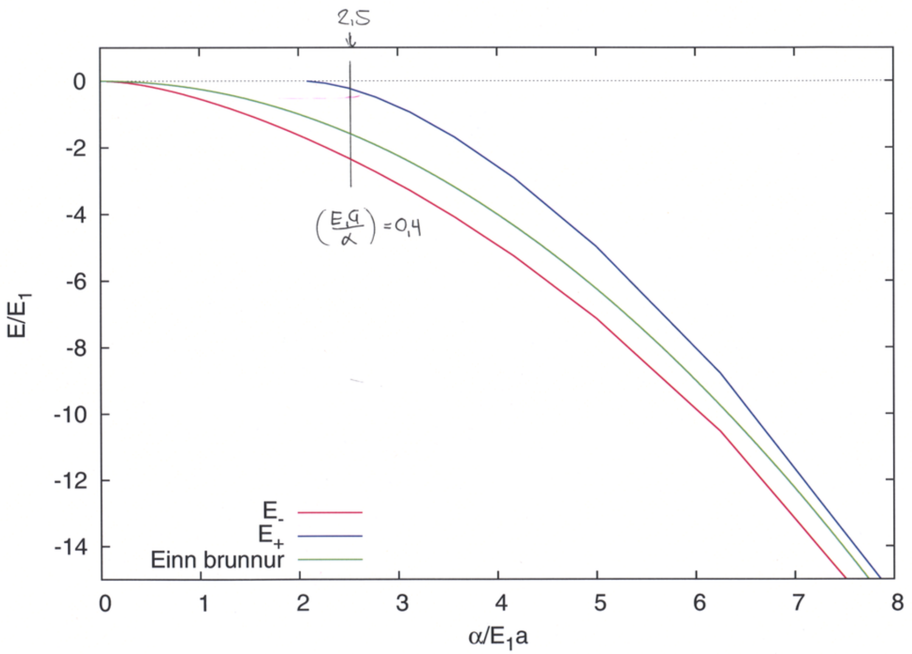
liðum á $e^{-Ka} \sim 1 - (Ka) + \frac{(Ka)^2}{2} + \dots$

sýnir að „+" lausnin er til ef $2\cdot\left(\frac{E_1 a}{\alpha}\right) < 1$

$\rightarrow \frac{E_1 a}{\alpha} < \frac{1}{2}$ eða $\boxed{\alpha > 2E_1 a}$



7



8

Finnum bylgjuföllin

Höfum engar áhyggjur af normun. Útlit bylgjufalla sést vel án hennar. (Föllin eru normanleg hér).

Veljum því $G = 1$, þá eru jöfnurnar

$$Ae^{-\kappa a} - Ce^{-\kappa a} - De^{+\kappa a} = 0$$

$$C + D = 1$$

$$Ce^{-\kappa a} - De^{+\kappa a} + A(\beta - 1)e^{-\kappa a} = 0$$

$$-C + D = 1 - \beta$$

fækkum í 3 jöfnur

þar eru ekki allar óháðar

9

$$\begin{pmatrix} e^{-\kappa a} & -e^{-\kappa a} & -e^{\kappa a} \\ 0 & +1 & +1 \\ 0 & -1 & +1 \end{pmatrix} \cdot \begin{pmatrix} A \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1-\beta \end{pmatrix}$$

Reiknum þegar tvær lausnir eru til, t.d. þ. $\left(\frac{E_1 a}{\alpha}\right) = 0.4$

$E_\pm = -E_1 \cdot (\kappa_\pm a)^2$ tölulega lausn á (*) gefur

$(\kappa_- a) = 1.52266$ sjá mynd á bls 6

$(\kappa_+ a) = 0.464213$ $a\kappa_\pm$ eru núllstöðvar (*)

$$\beta_\pm = \left(\frac{\alpha}{E_1 a}\right) \frac{1}{(\kappa_\pm a)}$$

10

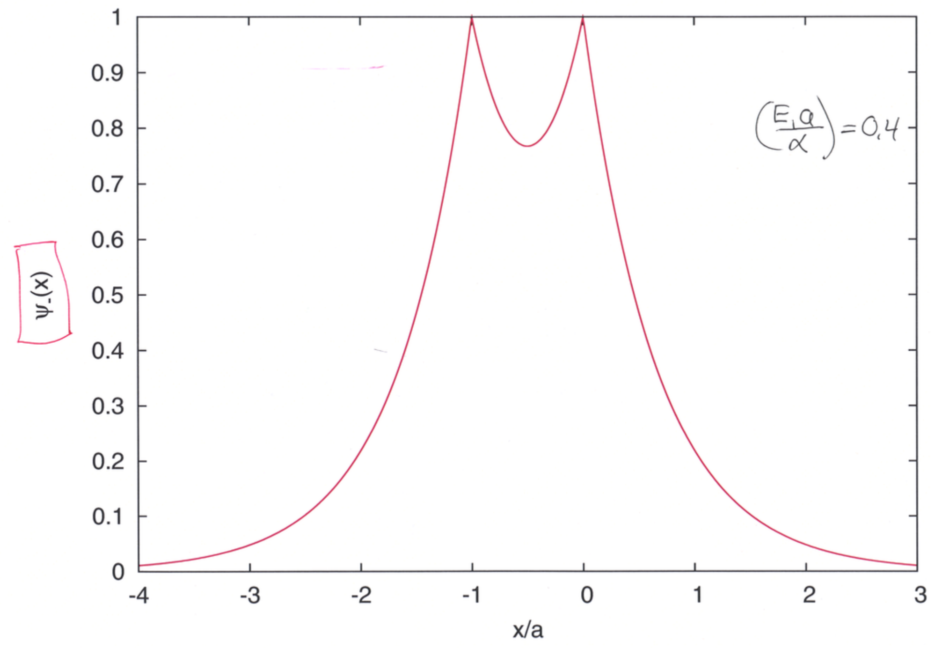
„−"-lausn

$$\left.\begin{aligned} e^{-\kappa a_-} &= 0.21813 \\ e^{\kappa a_-} &= 4.5844 \\ \beta_- &= 1.6419 \end{aligned}\right\} \rightarrow \begin{aligned} A &= 4.5843 \\ C &= 0{,}82094 \\ D &= 0{,}17906 \end{aligned}$$

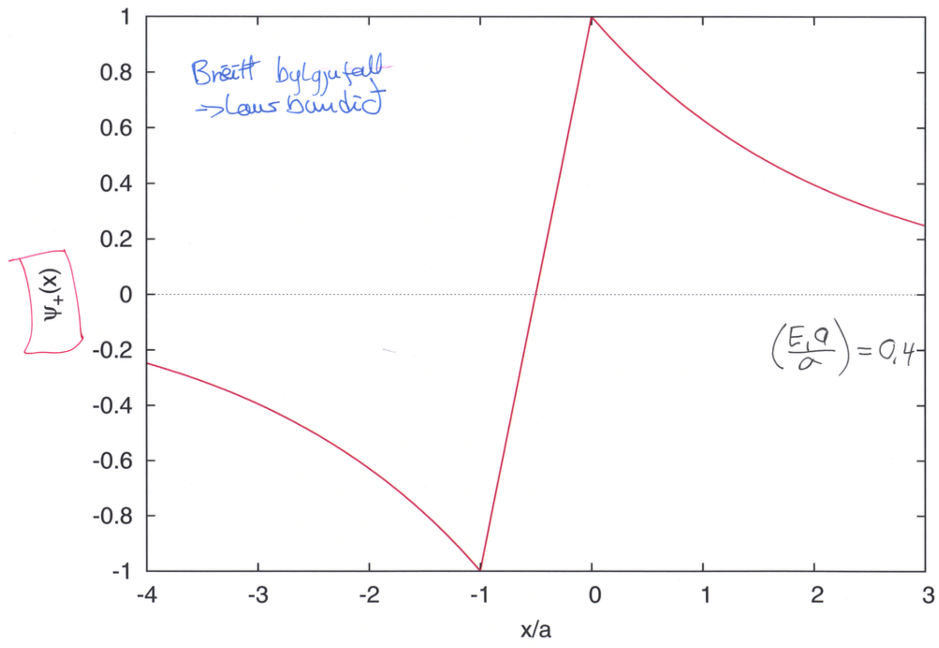
„+"-lausn

$$\left.\begin{aligned} e^{-\kappa_+ a} &= 0{,}62863 \\ e^{\kappa_+ a} &= 1.5908 \\ \beta_+ &= 5.3855 \end{aligned}\right\} \rightarrow \begin{aligned} A &= -1{,}59079 \\ C &= 2.69275 \\ D &= -1{,}69275 \end{aligned}$$

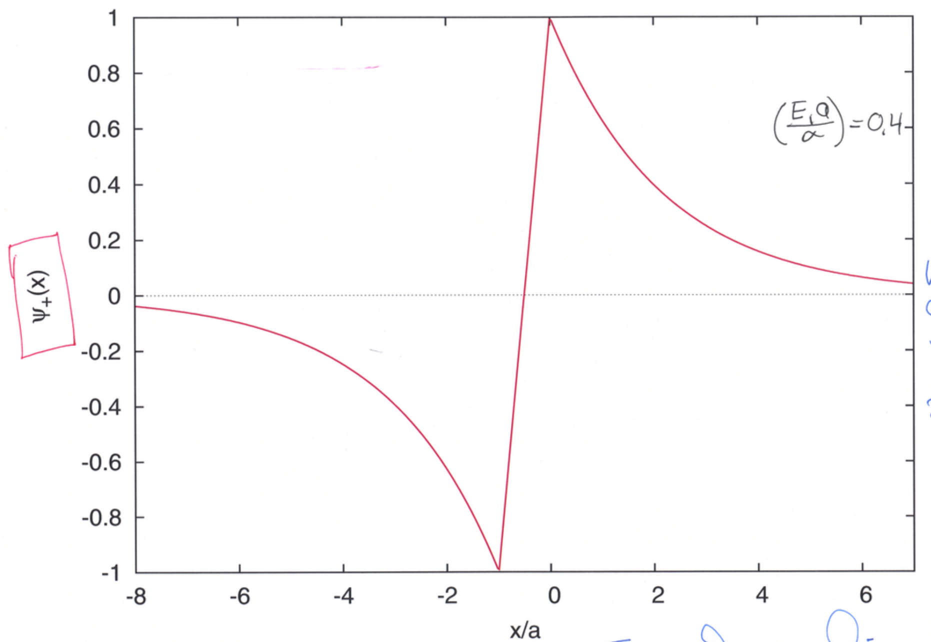
(11)



(12)



(13)



hægari dofnun en fyrir $\psi_-(x)$

← víðara svæði

(3.13) (1)

a) Sýna að $[AB,C] = A[B,C] + [A,C]B$

$$
\begin{aligned}
[AB,C] &= ABC - CAB \\
&= ABC - \underbrace{ACB + ACB}_{=0} - CAB \\
&= A[B,C] + [A,C]B
\end{aligned}
$$

b) Sýna að

$$[x^n, p] = i\hbar\, n\, x^{n-1}$$

$$\left\{ [x^n, p] = x^{n-1}[x,p] + [x^{n-1},p]\,x \right.$$ og svo framvegis, en ég fer aðra leið $\}$

(2)

Ég leyfi mér að vinna í staðarrúmi með bylgjuföllum

$p \to -i\hbar\partial_x$

$$[x^n, p]f = \{x^n(-i\hbar\partial_x)f - (-i\hbar\partial_x x^n f)\}$$
$$= -i\hbar x^n \partial_x f + i\hbar n x^{n-1} f + i\hbar x^n \partial_x f$$
$$= i\hbar n x^{n-1} f$$
$$\to [x^n, p] = i\hbar n x^{n-1}$$

c) sýna að
$$[f(x), p] = i\hbar\partial_x f$$ og því

$$[f(x),p]g(x) = \{f(-i\hbar\partial_x g) - (-i\hbar\partial_x fg)\}$$
$$= -i\hbar f\partial_x g + i\hbar(\partial_x f)g + i\hbar f\partial_x g = i\hbar(\partial_x f)g$$

(3)

2. skiladæmi

Tvístiga kerfi með Hamiltonvirkja

$$H = E\{|1\rangle\langle 1| - |2\rangle\langle 2| + i|1\rangle\langle 2| - i|2\rangle\langle 1|\}$$

þar sem $\{|i\rangle, i=1,2\}$ er fullkominn staðalaður grunnur

Finnið eiginvigra og eigingildi H

Í þessum grunni, $\{|i\rangle, i=1,2\}$ er Hamiltonfylkið

$$H = E\begin{pmatrix} 1 & i \\ -i & -1 \end{pmatrix}$$ Samsvegnist með $\langle i|H|j\rangle$ og Jöfnunni fyrir H

Hér skiptir ekki máli að
$$H = E[\sigma_z - \sigma_y]$$

(4)

Eigingildi H eru

$$E_\pm = \pm E\sqrt{2}$$

með eiginvigra

$$|\pm\rangle = \{|1\rangle \mp i(\sqrt{2}\pm 1)|2\rangle\}\frac{1}{\sqrt{1+(\sqrt{2}\pm 1)^2}}$$

$$= \{|1\rangle \mp i\alpha_\pm |2\rangle\}\frac{1}{\sqrt{1+\alpha_\pm^2}}$$

með

$$\alpha_\pm = \sqrt{2}\pm 1$$

(5)

Hvernig lítur H út í nýja grunninum?

$|\pm\rangle$ eru eigin ástönd H

$$\to \langle +|H|+\rangle = E_+$$
$$\langle -|H|-\rangle = E_-$$

og $\langle \mp|H|\pm\rangle = 0$

$$\to H = \begin{pmatrix} -\sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$$

Ef við númerum
$|+\rangle \to 2$
$|-\rangle \to 1$

Hver eru væntigildi H fyrir ástöndin $|1\rangle$ og $|2\rangle$?

Notum Jöfnuna fyrir H, eða lesum úr útsetningu H í þeim grunni

$$\langle 1|H|1\rangle = E \quad , \quad \langle 2|H|2\rangle = -E$$

1

4.22

a) Hvað er $L_+ Y_{ll}$?

l er hæsta m-gildið

$\rightarrow \quad L_+ Y_{ll} = 0$

b) Nota $L_+ Y_{ll} = 0$

með

$$L_\pm = \pm \hbar e^{\pm i\phi}\left\{\frac{\partial}{\partial\theta} \pm i\cot\theta\frac{\partial}{\partial\phi}\right\}$$

og

$$L_z Y_{ll} = \hbar l Y_{ll}$$

til að ákvarða Y_{ll}

① $$\hbar e^{i\phi}\left\{\frac{\partial}{\partial\theta} + i\cot\theta\frac{\partial}{\partial\phi}\right\} Y_{ll}(\Omega) = 0$$

② $$-i\hbar\frac{\partial}{\partial\phi} Y_{ll}(\Omega) = \hbar l Y_{ll}(\Omega)$$

---

② $\rightarrow \quad i\partial_\phi Y_{ll}(\Omega) = -l Y_{ll}(\Omega)$

notum í ①

$$\hbar e^{i\phi}\left\{\partial_\theta - (\cot\theta)\, l\right\} Y_{ll}(\Omega) = 0$$

$$\rightarrow \left\{\partial_\theta - l\cot\theta\right\} Y_{ll} = 0$$

2

En gleymum ekki að ② gefur

$$i\partial_\phi Y_{ll}(\Omega) + l Y_{ll}(\Omega) = 0$$

$$\rightarrow \quad Y_{ll}(\Omega) = f(\theta)\, e^{il\phi}$$

því er seinni jafnan

$$\left\{d_\theta - l\cot\theta\right\} f(\theta) = 0$$

$$\frac{df}{f} = l\cot\theta\, d\theta$$

með lausn

$$\ln(f) = l\ln(\sin\theta)$$

$$\rightarrow f(\theta) = \sin^l\theta$$

heildarlausnin er því

$$Y_{ll}(\Omega) = \sin^l\theta\, e^{il\phi} A$$

þar sem A er stöðlunarstuðull.

c) finna A

$$\int d\Omega\, |Y_{ll}(\Omega)|^2 = 1$$

$$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta\, \sin^{2l}\theta\, A^2$$

$$= A^2\, 2\pi \int_0^{\pi} d\theta\, \sin^{2l+1}\theta$$

3

$$= A^2\, 4\pi \int_0^{\pi/2} d\theta\, \sin^{2l+1}\theta = A^2\, 4\pi\, \frac{(2l)!!}{(2l+1)!!} = 1$$

(GR 3.621.4)

$$\rightarrow \quad A^2 = \frac{(2l+1)!!}{(2l)!!\; 4\pi} \quad \rightarrow \quad A = \sqrt{\frac{(2l+1)!!}{4\pi\;(2l)!!}}$$

$P_l^l(\cos\theta) = (2l-1)!!\,\sin^l\theta$ þ.v. séð frá (4.32)

$$Y_{ll}(\Omega) = \epsilon\sqrt{\frac{(2l+1)}{4\pi}\frac{1}{(2l)!}}\; e^{il\theta}\,(2l-1)!!\,\sin^l\theta$$

$$= \epsilon\sqrt{\frac{(2l+1)(2l-1)!!\;(2l-1)!!}{4\pi\;(2l)!}}\; e^{il\theta}\sin^l\theta$$

$$= \epsilon\sqrt{\frac{(2l+1)!!}{4\pi\;(2l)!!}}\; e^{il\theta}\sin^l\theta$$

Ég hendi í burtu 2 heildunarstuðlum án skýringa

Með þeim verða eiginföllin ekki korrétt

4

H-atóm

Rafeind er í ástandi

$$|\mu\rangle = \left\{4|100\rangle + 3|211\rangle - |210\rangle + \sqrt{10}\,|21\,{-1}\rangle\right\}\frac{1}{6}$$

þar sem $|nlm\rangle$ eru eigin ástönd H-atóms (rafeindar í H-...)

$$\langle\mu|\mu\rangle = \left\{16 + 9 + 1 + 10\right\}\frac{1}{36} = 1$$

Svo ástandið er staðlað

---

a) Finna væntigildi orku rafeindarinnar

$H|nlm\rangle = E_n|nlm\rangle$ þ.a. $E_n = -Ry\frac{1}{n^2}$

$$\langle\mu|H|\mu\rangle = \frac{1}{36}\left\{16E_1 + 9E_2 + E_2 + 10E_2\right\} = \frac{1}{36}\left\{16E_1 + 20E_2\right\}$$

(5)

$$\rightarrow \langle\mu|H|\mu\rangle = -\frac{R_y}{36}\left\{16\cdot\frac{1}{1} + 20\cdot\frac{1}{4}\right\} = -\frac{R_y}{36}\{21\}$$

$$= -R_y\cdot\frac{21}{36} = -R_y\cdot\frac{7}{12}$$

b) Væntigildi L^2 ?

$|nlm\rangle$ eru eiginástönd L^2 með eigingildi $\hbar^2 l(l+1)$

$$L^2\cdot|nlm\rangle = \hbar^2 l(l+1)|nlm\rangle$$

$$\langle\mu|L^2|\mu\rangle = \frac{\hbar^2}{36}\{16\cdot 0 + 20\cdot 1(1+1)\}$$

$$= \frac{\hbar^2}{36}\{40\} = \hbar^2\cdot\frac{10}{9}$$

(6)

c) Væntigildi L_z

$$L_z|nlm\rangle = \hbar m|nlm\rangle$$

$$\rightarrow \langle\mu|L_z|\mu\rangle = \frac{\hbar}{36}\{16\cdot 0 + 9\cdot 1 + 1\cdot 0 - 10\cdot 1\}$$

$$= \frac{\hbar}{36}(-1) = -\hbar\frac{1}{36}$$

(4.17)

(1)

Í stað mólikúla

$$V(r) = -\frac{e^2}{4\pi\epsilon_0 r}$$

skoðum við

a)

$$V(r) = -\frac{GMm}{r}$$

M: massi sólar

m: massi jarðar

þ.a. í jöfnum fyrir H-atómið þurfum við að setja

$$GMm \leftarrow \frac{e^2}{4\pi\epsilon_0}$$

b) fyrir H-atóm er Bohr geislinn

$$a_H = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \left(\frac{4\pi\epsilon_0}{e^2}\right)\frac{\hbar^2}{m_e}$$

fyrir J-S kerfið fæst þá

$$a_G = \frac{\hbar^2}{GMm^2}, \quad G = 6.673\cdot 10^{-11}\frac{m^3}{kg\,s^2}$$

$$= \frac{(1.055\cdot 10^{-34}\,Js)^2}{6.673\cdot 10^{-11}\frac{m^3}{kg\,s^2}\cdot 2\cdot 10^{30}kg\,(6\cdot 10^{24}kg)^2}$$

$$= 2.32\cdot 10^{-138}\,m$$

(2)

c) fyrir H-atóm var

$$E_n = -R_y^H\cdot\frac{1}{n^2}$$

$$R_y^H = \left[\frac{m_e}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]$$

$$\rightarrow R_y^G = \frac{m}{2\hbar^2}(GMm)^2$$

$$E_n^G = -R_y^G\frac{1}{n^2} = -\frac{m}{2\hbar^2}(GMm)^2\frac{1}{n^2}$$

Orka jarðar á hringhreyfingu

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r_0} = \text{fasti}$$

notum $F = ma$ t.þ.a. finna v

$$m\left(\frac{v^2}{r_0}\right) = \frac{GMm}{r_0^2} \rightarrow v^2 = \frac{GM}{r_0}$$

$$\rightarrow E = \frac{GMm}{2r_0} - \frac{GMm}{r_0}$$

$$= -\frac{GMm}{2r_0}$$

setjum jafnt

$$E_n^G = E$$

$$\frac{m}{2\hbar^2}(GMm)^2\frac{1}{n^2} = \frac{GMm}{2r_0}$$

3

$$\rightarrow \frac{m}{2\hbar^2} GMm \frac{1}{n^2} = \frac{1}{2r_0}$$

$$\rightarrow n^2 = \frac{r_0 GMm^2}{\hbar^2} = \frac{r_0}{a_G}$$

$$\rightarrow n = \sqrt{\frac{r_0}{a_G}}$$

$$\rightarrow n \sim \sqrt{\frac{150\cdot 10^{9}\,m}{2.32\cdot 10^{-138}\,m}}$$

$$\sim 2.5\cdot 10^{74}$$

d) Jörðin geislar $n \rightarrow n-1$

hve mikil orka losnar

$$\Delta E^G = \left|E^G_{n-1} - E^G_n\right|$$

$$= +R_y^G\left\{\frac{1}{(n-1)^2} - \frac{1}{n^2}\right\}$$

$$= +R_y^G \frac{1}{n^2}\left\{\frac{n^2}{(n-1)^2} - 1\right\}$$

$$= |E_n|\cdot\left\{\frac{1}{(1-\frac{1}{n})^2} - 1\right\}$$

$$\approx |E_n|\cdot\left\{1 + \frac{2}{n} - 1\right\}$$

$$= |E_n|\cdot\frac{2}{n}$$

$$\Delta E^G = 2\cdot R_y^G \frac{1}{n^3}$$

4

$$R_y^G = \frac{m}{2\hbar^2}(GMm)^2 = \frac{(6\cdot 10^{24}\,kg)^3\left(6.673\cdot 10^{-11}\frac{m^3}{kg\,s^2}\right)^2(2\cdot 10^{30}\,kg)^2}{2\cdot(1.055\cdot 10^{-34}\,Js)^2}$$

$$\approx 1.73\cdot 10^{182}\,J$$

$$\rightarrow \Delta E^G \approx 2\cdot R_y^G \frac{1}{n^3} \sim 2.1\cdot 10^{-41}\,J.$$

Hvaða bylgjulengd myndi þetta samsvara fyrir þyngdareind?

$$\Delta E^G = \hbar\omega = h\frac{\omega}{2\pi} = h\nu$$

$$= h\frac{c}{\lambda}$$

$$\rightarrow \lambda = \frac{hc}{\Delta E^G} = \frac{(1.055\cdot 10^{-34}\,Js\cdot 2\pi)\,3\cdot 10^{8}\,m}{2.1\cdot 10^{-41}\,J} \approx 9.48\cdot 10^{15}\,m$$

$\approx$ 1 ljósár

5

4.55 Rafeind í vetni í

$$R_{21}\left\{\sqrt{\tfrac{1}{3}}\,Y_{10}\chi_+ + \sqrt{\tfrac{2}{3}}\,Y_{11}\chi_-\right\}$$

a) Mæliniðurstöður L^2?

Báðir þættir ástandsins eru eiginástönd L^2 með eigingildi $\hbar^2 l(l+1) = 2\hbar^2$, líkindin á þeirri mæliniðurstöðu eru því 1.

b) Mæliniðurstöður L_z?

Báðir þættir ástandsins eru eiginástönd L_z, en með mismunandi eigingildi. Við fáum því

$\hbar\cdot 0$ með líkindum $\frac{1}{3}$

$\hbar$ með líkindum $\frac{2}{3}$

heildarlíkur eru því 1

6

c) S^2?

Báðir spuna þættir ástandsins eru eiginástönd S^2 með sama eigingildi $\rightarrow$ mæling gefur

$$\hbar^2\,\tfrac{1}{2}\left(\tfrac{1}{2}+1\right) = \frac{3\hbar^2}{4}$$ með líkum 1

d) S_z

Báðir þættir eru eiginástönd S_z, en með mismunandi eigingildum, því fæst

$+\frac{\hbar}{2}$ með líkum $\frac{1}{3}$

$-\frac{\hbar}{2}$ með líkum $\frac{2}{3}$

(7)

e) J^2

Hér vandast málið, ástöndin eru ekki eigin ástönd J^2.
Við þurfum því að liða ástandið í þeim

$Y_{10}\chi_+$ stendur fyrir $|1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle$

$Y_{11}\chi_-$ — ‖ — $|1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle$

táknum $|l,m\rangle|s,m_s\rangle$

Í báðum tilfellum fæst að $m+m_s=\frac{1}{2}$, því í samanburði við jöfnu (4.184) fæst aðeins að $j=\frac{3}{2}$ eða $\frac{1}{2}$

Við þurfum því að nota (4.186) t.þ.a finna

$$|l,m\rangle|s,m_s\rangle = \sum_j C^{l,s,j}_{m,m_s,m_j}\,|j,m_j\rangle$$

(8)

$$|1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle = \sum_j C^{1,\frac{1}{2},j}_{0,\frac{1}{2},m_j}\,|j,m_j\rangle$$

$$=\sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2},\tfrac{1}{2}\rangle-\sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle$$

samkvæmt töflu 4.8

$$|1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle = \sum_j C^{1,\frac{1}{2},j}_{1,-\frac{1}{2},m_j}\,|j,m_j\rangle$$

$$=\sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2},\tfrac{1}{2}\rangle+\sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle$$

og heildar hverfiþunga hluti ástandsins er því í $\{|j,m_j\rangle\}$-grunni

$$\left[\sqrt{\tfrac{1}{3}}\,|1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle+\sqrt{\tfrac{2}{3}}\,|1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle\right]$$

$$=\sqrt{\tfrac{1}{3}}\left\{\sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2},\tfrac{1}{2}\rangle-\sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right\}+\sqrt{\tfrac{2}{3}}\left\{\sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2},\tfrac{1}{2}\rangle+\sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle\right\}$$

(9)

$$=\frac{2\sqrt{2}}{3}\,|\overset{j}{\tfrac{3}{2}},\overset{m_j}{\tfrac{1}{2}}\rangle+\frac{1}{3}\,|\overset{j}{\tfrac{1}{2}},\overset{m_j}{\tfrac{1}{2}}\rangle$$

Ástandið er því liðað í eiginástöndum J^2 og J_z og við sjáum að mæling gefur

gildi $\frac{3}{2}\left(\frac{5}{2}\right)\hbar^2$ með líkum $\frac{4\cdot 2}{9}=\frac{8}{9}$

$\frac{1}{2}\left(\frac{3}{2}\right)\hbar^2$ með líkum $\frac{1}{9}$

f) mæling á J_z gefur

$\frac{\hbar}{2}$ með líkum 1

báðir þættirnir eru eiginástönd J_z með sama eigingildi

(10)

g) Ástandið var $R_{21}\left\{\sqrt{\tfrac{1}{3}}\,Y_{10}\chi_+ + \sqrt{\tfrac{2}{3}}\,Y_{11}\chi_-\right\}$

líkindaþéttleikinn fyrir því að finna eindina er

$$|R_{21}|^2\left\{\tfrac{1}{3}|Y_{10}(\Omega)|^2+\tfrac{2}{3}|Y_{11}(\Omega)|^2\right\}$$

þar sem $\chi_+^*\chi_-=0$ og $\chi_-^*\chi_+=0$

$\chi_+\chi_+=1$ og $\chi_-\chi_-=1$

$$\frac{1}{24}\,\frac{1}{a^3}\left(\frac{r}{a}\right)^2 e^{-\frac{r}{a}}\left\{\frac{1}{4\pi}\cos^2\theta+\frac{2}{8\pi}\sin^2\theta\right\}$$

$$=\frac{1}{96\pi}\,\frac{1}{a^3}\left(\frac{r}{a}\right)^2 e^{-\frac{r}{a}}$$

11

h) líkurnar á að mæla fjarlægð r og S_z með $+\frac{\hbar}{2}$

$$|R_{21}|^2 \frac{1}{3} \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, d\theta \, |Y_{10}|^2$$

$$= \frac{1}{24} \frac{1}{a^3} \left(\frac{r}{a}\right)^2 e^{-\frac{r}{a}} \frac{2\pi}{3} \int_0^{\pi} d\theta \, \sin\theta \cos^2\theta \, \frac{3}{4\pi}$$

$$= \frac{1}{48} \frac{1}{a^3} \left(\frac{r}{a}\right)^2 e^{-\frac{r}{a}} \boxed{\int_0^{\pi} d\theta \, \sin\theta \cos^2\theta}$$

$$= \frac{1}{48} \frac{1}{a^3} \left(\frac{r}{a}\right)^2 e^{-\frac{r}{a}} \frac{2}{3} = \frac{1}{72 a^3} \left(\frac{r}{a}\right)^2 e^{-\frac{r}{a}}$$

6.2 Hreintóna sveifill, ein vídd 1

$$V(x) = \frac{1}{2} k x^2$$

með orkuróf $E_n = (n+\frac{1}{2})\hbar\omega$

með $n = 0, 1, \ldots\ldots$

$$\omega = \sqrt{\frac{k}{m}}$$

Gormstuðullinn er breytt þannig $k \to (1+\epsilon)k$

a) finna nákvæma nýja orkurófið

$$\omega' = \sqrt{\frac{k'}{m}} = \sqrt{\frac{(1+\epsilon)k}{m}}$$

$$\omega' = \sqrt{(1+\epsilon)} \sqrt{\frac{k}{m}} = \sqrt{1+\epsilon}\,\omega$$

því fæst að truflaða rófið er

$$E_n' = (n+\tfrac{1}{2})\hbar\omega' = (n+\tfrac{1}{2})\hbar\omega \cdot \sqrt{1+\epsilon} = E_n \cdot \sqrt{1+\epsilon}$$

og liðun gefur

$$E_n' \simeq E_n \cdot \left\{1 + \frac{\epsilon}{2} - \frac{\epsilon^2}{8} + \ldots\ldots\right\}$$

2

b) Nota truflanareikning til þess að reikna 1. stigs leiðréttingu

$$V'(x) = \frac{1}{2} k' x^2 = \frac{1}{2} k x^2 (1+\epsilon)$$

Truflunarmættið er þess vegna $H' = \frac{\epsilon}{2} k x^2$

ϵ leikur hlutverk truflana-stuðulsins λ.

Við viljum því reikna

$$\langle n|H'|n\rangle = \frac{\epsilon k}{2} \langle n|x^2|n\rangle$$

munum eftir tröppuvirkjunum

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}} \{\mp ip + m\omega x\}$$

$$\to a_+ + a_- = \frac{2m\omega}{\sqrt{2\hbar m\omega}} x = \sqrt{\frac{2m\omega}{\hbar}}\, x = \sqrt{2}\, \frac{x}{a}$$

$a = \sqrt{\frac{\hbar}{m\omega}}$ náttúruleg lengd

3

$$\to x = \frac{a}{\sqrt{2}} (a_+ + a_-)$$

$$x^2 = \frac{a^2}{2} (a_+ + a_-)^2 = \frac{a^2}{2} \{a_+a_+ + a_-a_- + a_+a_- + a_-a_+\}$$

Notum síðan

$$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$a_-|0\rangle = 0$$

til þess að reikna

$$\langle n|x^2|n\rangle = \frac{a^2}{2} \langle n| \{a_+a_- + a_-a_+\} |n\rangle$$

(4)

$$\langle n|x^2|n\rangle = \frac{a^2}{2}\langle n|\{n + n+1\}|n\rangle = \frac{a^2}{2}(2n+1)$$

$$\to \langle n|H'|n\rangle = \frac{\epsilon k}{2}\frac{a^2}{2}(2n+1) = \frac{\epsilon k}{2}a^2\left(n+\tfrac{1}{2}\right)$$

$$= \frac{\epsilon k}{2}\frac{\hbar}{m\omega}\left(n+\tfrac{1}{2}\right) = \frac{\epsilon k}{2}\frac{\hbar}{m}\sqrt{\frac{m}{k}}\left(n+\tfrac{1}{2}\right)$$

$$= \frac{\epsilon}{2}\hbar\sqrt{\frac{k}{m}}\left(n+\tfrac{1}{2}\right) = \frac{\epsilon}{2}\hbar\omega\left(n+\tfrac{1}{2}\right)$$

þannig að

$$E_n^1 = \frac{\epsilon}{2}\hbar\omega\left(n+\tfrac{1}{2}\right) = \frac{\epsilon}{2}E_n$$

(5)

6.3 Tvær eins bósónur í óendanlega djúpum brunni

þar víxlverkast veikt með „snertimætti"

$$V(x_1,x_2) = -aV_0\,\delta(x_1-x_2)$$

Breidd brunns er a og vídd V_0 er orka

a) Án víxlverkunar, finna grunnástand og fyrsta örvaða ástandið, orku og ástand.

Vinnum með einner-einder grunnföllin

$$\psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi}{a}x\right) \qquad n = 1, 2, 3, \ldots$$

og rófið

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

(6)

bóseindir $\to$ grunnástandið er

$$\psi_g(x_1x_2) = \psi_1(x_1)\psi_1(x_2) = \frac{2}{a}\sin\left(\frac{\pi}{a}x_1\right)\sin\left(\frac{\pi}{a}x_2\right)$$

samhverft fall þ.a. $\psi_g(x_1x_2) = \psi_g(x_2x_1)$

Orkan er $E_g = 2\cdot E_1 = \dfrac{\pi^2\hbar^2}{ma^2}$

1. örvun fæst með því að lyfta annarri eindinni upp um eitt stig, vitum ekki hvorri!

samhverft

$$\psi_e(x_1x_2) = \frac{1}{\sqrt{2}}\left\{\psi_1(x_1)\psi_2(x_2) + \psi_2(x_1)\psi_1(x_2)\right\}$$

$$E_e = E_1 + E_2 = \frac{5\pi^2\hbar^2}{2ma^2}$$

(7)

b) 1. stigs truflun fyrir E_g og E_e

$$E_g^1 = \langle g|V|g\rangle = -aV_0\int_0^a dx_1dx_2\,\frac{4}{a^2}\sin^2\left(\pi\frac{x_1}{a}\right)\sin^2\left(\pi\frac{x_2}{a}\right)\cdot\delta(x_1-x_2)$$

$$= -aV_0\frac{4}{a^2}\int_0^a dx_1\,\sin^4\left(\pi\frac{x_1}{a}\right)$$

$$= -aV_0\frac{4}{a^2}a\int_0^1 du\,\sin^4(\pi u) = -aV_0\frac{4}{a^2}a\cdot\frac{12}{32}$$

$$= -V_0\frac{48}{32} = -V_0\frac{3}{2}$$

(8)

$$\underline{E_e^1} = \langle e|V|e\rangle = -aV_0\int_0^a dx_1 dx_2\, \Phi_e^2(x_1,x_2)\,\delta(x_1-x_2)$$

$$= -aV_0\int_0^a dx_1\, \Phi_e^2(x_1,x_1)$$

$$= -aV_0\frac{1}{2}\frac{4a}{a^2}\int_0^1 du\left[\sin(\pi u)\sin(2\pi u)\right]^2\cdot 4$$

$$= -aV_0\frac{1}{2}\frac{16a}{a^2}\int_0^1 du\,\sin^2(\pi u)\sin^2(2\pi u)$$

$$= -aV_0\frac{16a}{2a^2}\frac{12}{48} = \underline{-V_0\cdot 2}$$

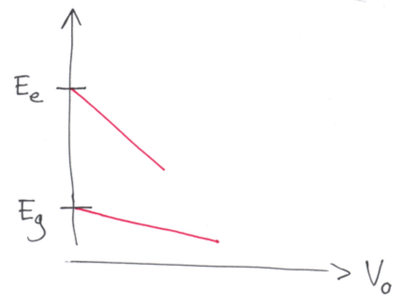
(9)

Tökum saman til gamans

$$E_g = \frac{\pi^2\hbar^2}{ma^2} - \frac{3}{2}V_0$$

$$E_e = \frac{5\pi^2\hbar^2}{2ma^2} - 2V_0$$

meiri lækkun á e-ástandinu vegna „snerti" eðlisháttar mættis



(1)

6.5 Veikt rafsvið lagt á hreintóna sveifil

$$H' = -qEx$$

náttúruleg lengd $a = \sqrt{\frac{\hbar}{m\omega}}$

a) Sýna að fyrsta stigs truflun hverfi

Notum úr dæmi 6.2

$$x = \frac{a}{\sqrt{2}}(a_+ + a_-), \quad E_n^1 = \langle n|H'|n\rangle = -qE\langle n|x|n\rangle$$

$$= -qE\langle n|(a_+ + a_-)|n\rangle\frac{a}{\sqrt{2}}$$

$= 0$ því $a_\pm$ hækka eða lækka n

og $\langle n|n\pm 1\rangle = 0$

$$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$$

(2)

Reikna 2. stigs truflun ástandanna

$$E_n^2 = \sum_{m\neq n}\frac{|\langle m|H'|n\rangle|^2}{E_n^0 - E_m^0} = q^2E^2\sum_{m\neq n}\frac{|\langle m|x|n\rangle|^2}{\hbar\omega(n-m)}$$

$$\langle m|x|n\rangle = \frac{a}{\sqrt{2}}\langle m|(a_+ + a_-)|n\rangle = \frac{a}{\sqrt{2}}\left\{\langle m|\sqrt{n+1}|n+1\rangle + \langle m|\sqrt{n}|n-1\rangle\right\}$$

$$= \frac{a}{\sqrt{2}}\left\{\sqrt{n+1}\,\delta_{m,n+1} + \sqrt{n}\,\delta_{m,n-1}\right\}$$

$$E_n^2 = \frac{q^2E^2a^2}{2\hbar\omega}\sum_{m\neq n}\frac{|\sqrt{n+1}\,\delta_{m,n+1} + \sqrt{n}\,\delta_{m,n-1}|^2}{n-m}$$

$$= \frac{q^2E^2a^2}{2\hbar\omega}\left\{\frac{n+1}{n-(n+1)} + \frac{n}{n-(n-1)}\right\}$$

$$E_n^2 = \frac{q^2E^2a^2}{2\hbar\omega}\left\{\frac{n+1}{-1} + \frac{n}{+1}\right\} = -\frac{q^2E^2a^2}{2\hbar\omega} = -\frac{q^2E^2\hbar}{2\hbar\omega m\omega}$$

$$= -\frac{q^2E^2}{2m\omega^2}$$

og þessvegna

$$E_n = \hbar\omega\left(n+\tfrac12\right) - \frac{q^2E^2}{2m\omega^2} = \hbar\omega\left\{\left(n+\tfrac12\right) - \frac{q^2E^2}{2m\omega^2\hbar\omega}\right\}$$

$$= \hbar\omega\left\{\left(n+\tfrac12\right) - \frac{q^2E^2a^2}{2(\hbar\omega)^2}\right\}$$

↑ Stark hliðrun orkurófs

b) Finna nákvæmu lausnina

Hamilton virkinn var

$$\frac{p^2}{2m} + \frac12 m\omega^2 x^2$$

með rafsviðinu verður hann

$$\frac{p^2}{2m} + \frac12 m\omega^2 x^2 - qEx$$

Getum við umritað H yfir í hreintóna sveifil aftur

$$\frac{p^2}{2m} + \frac12 m\omega^2\left(x^2 - \frac{qEx2}{m\omega^2}\right)$$

$$\rightarrow \frac{p^2}{2m} + \frac12 m\omega^2\left(x - \frac{qE}{m\omega^2}\right)^2 - \frac{q^2E^2}{m^2\omega^4}\frac{m\omega^2}{2}$$

$$= \frac{p^2}{2m} + \frac12 m\omega^2\left(x - \frac{qE}{m\omega^2}\right)^2 - \frac{q^2E^2}{2m\omega^2}$$

$$= \frac{p^2}{2m} + \frac12 m\omega^2(x-x_0)^2 - \frac{q^2E^2}{2m\omega^2}$$

$$x_0 = \frac{qE}{m\omega^2}$$

hliðrun hreintóna sveifils með lækkaða orku

Í föstu rafsviði er lausn hreintóna sveifilsins eins og lausnin fyrir hliðraðan hreintónasveifil með lækkaða orku

2. stigs nálgunin er líka nákvæm lausn

Stark-hrif hreintóna sveifils

(6.7) Eind með massa m á bili með lengd L (lotubundið 1D, t.d. hringur)

a) Eiginföll og róf

$$H = \frac{p^2}{2m}, \quad \psi\left(-\tfrac L2\right) = \psi\left(\tfrac L2\right)$$

$$-\frac{\hbar^2}{2m}d_x^2\psi = E\psi$$

Reynum lausn með

$$\psi = Ae^{i\alpha x}$$

jaðarskilyrði

$$Ae^{-i\alpha\frac{L}{2}} = Ae^{+i\alpha\frac{L}{2}}$$

eða $1 = e^{i\alpha L}$

$$\rightarrow \alpha = \frac{2\pi n}{L}, \quad n \in \mathbb{Z}$$

$$\psi = Ae^{2\pi i n\frac{x}{L}}$$

finnum A

$$1 = |A|^2\int_{-L/2}^{+L/2} dx\,|\psi|^2 = |A|^2 L$$

þá fæst t.d. $A = \frac{1}{\sqrt{L}}$

og orkan fæst með innsetningu í jöfnu Schrödingers

$$E_n = \frac{\hbar^2}{2m}\frac{4\pi^2 n^2}{L^2}$$

$$= \underbrace{\left(\frac{\hbar^2}{2mL^2}\right)}_{\text{með vídd orku}} 4\pi^2 n^2$$

Öll ástönd eru tvöföld nema $n = 0$

Köllum þau $|n\rangle$

b) Bætum við truflun

$$H' = -V_0 e^{-\frac{x^2}{a^2}}, \quad a \ll L$$

Finna 1. stigs truflun rófsins

$$E_n^1 = \langle n|H'|n\rangle$$

$$= -V_0 \frac{1}{L}\int_{-L/2}^{+L/2} dx\, |\Phi_n|^2 e^{-\frac{x^2}{a^2}}$$

$$= -\frac{V_0}{L}\int_{-L/2}^{+L/2} dx\, e^{-\frac{x^2}{a^2}} = -V_0\frac{a}{L}\int_{-\frac{L}{2a}}^{+\frac{L}{2a}} \frac{dx}{a} e^{-\frac{x^2}{a^2}}$$

$$= -V_0 \frac{a}{L}\int_{-\frac{L}{2a}}^{+\frac{L}{2a}} du\, e^{-u^2}$$

$a \ll L \to \frac{L}{a} \to \infty$ og

$$E_n^1 \approx -V_0\frac{a}{L}\int_{-\infty}^{\infty} du\, e^{-u^2} = -V_0\sqrt{\pi}\frac{a}{L}$$

Sem er nothæft fyrir einfalda ástandið $n = 0$

$$\boxed{E_0^1 \simeq -V_0\sqrt{\pi}\frac{a}{L}}$$

fyrir tvöföldu pörin $n \neq 0$ verðum við að nota truflanareikning fy– tvöföld ástönd með

$$W_{nn} = W_{aa} = W_{bb} = -V_0\sqrt{\pi}\frac{a}{L}$$

$$W_{n,-n} = W_{ab} = \langle n|V|-n\rangle = -V_0\frac{1}{L}\int_{-\frac{L}{2}}^{+\frac{L}{2}} dx\, e^{-\frac{x^2}{a^2}} e^{4\pi n i\frac{x}{L}}$$

$$= -V_0\frac{a}{L}\int_{-\infty}^{\infty} du\, e^{-u^2}\exp\left\{4\pi n i u\frac{a}{L}\right\} = -V_0\frac{a}{L}\sqrt{\pi}\, e^{-(2\pi n\frac{a}{L})^2}$$

$$E_\pm^1 = \frac{1}{2}\left\{\underbrace{W_{aa}+W_{bb}}_{2W_{aa}} \pm \sqrt{\underbrace{(W_{aa}-W_{bb})^2}_{0} + 4|W_{ab}|^2}\right\}$$

$$= W_{aa} \pm |W_{ab}| = \boxed{-V_0\sqrt{\pi}\left(\frac{a}{L}\right)\left[1 \mp e^{-(2\pi n\frac{a}{L})^2}\right]}$$

Tvöföldu orkustigin klofna $\quad n \neq 0$

c) Hvaða samantekt $|+|n|\rangle$ og $|-|n|\rangle$ er góð samantekt fyrir venjulegan 1. stigs truflunarreikn.

Aðeins 2 ástönd, svo ég giska á

$$|+\rangle = \frac{1}{\sqrt{2}}\left\{|+|n|\rangle + |-|n|\rangle\right\} \to \Phi_+ = \sqrt{\frac{2}{L}}\cos\left(2\pi n\frac{x}{L}\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left\{|+|n|\rangle - |-|n|\rangle\right\} \to \Phi_- = i\sqrt{\frac{2}{L}}\sin\left(2\pi n\frac{x}{L}\right)$$

Reynum

$$E_+^1 = \langle +|H'|+\rangle = -V_0\frac{2}{L}\int_{-L/2}^{L/2} dx\, e^{-\frac{x^2}{a^2}}\cos^2\left(2\pi n\frac{x}{L}\right)$$

$$\to -V_0\frac{2a}{L}\int_{-\infty}^{\infty} du\, e^{-u^2}\cos^2\left(2\pi n u\frac{a}{L}\right)$$

(11)

$$= -V_0 \frac{2a}{L}\frac{\sqrt{\pi}}{2}\left\{\exp\left(-\left(\frac{2\pi n a}{L}\right)^2\right)+1\right\} = -V_0\sqrt{\pi}\left(\frac{a}{L}\right)\left\{1+e^{-(2\pi n \frac{a}{L})^2}\right\}$$

$$E_-^1 = \langle -|H'|-\rangle = -V_0 \frac{2}{L}\int_{-L/2}^{L/2} dx\, e^{-\frac{x^2}{a^2}}\sin^2\left(2\pi n \frac{x}{L}\right)$$

$$\simeq -V_0\frac{2a}{L}\int_{-\infty}^{\infty} du\, e^{-u^2}\sin^2\left(2\pi n u \frac{a}{L}\right) = -V_0\sqrt{\pi}\left(\frac{a}{L}\right)\left\{1-e^{-(2\pi n \frac{a}{L})^2}\right\}$$

Þannig að við þekkjum hér aftur orku stigin þ.a

$|+\rangle$ hefur orkuna sem við fundum áður E_-^1

$|-\rangle$ — // — E_+^1

(12)

d) $|+\rangle$ er jafnstætt fall

$|-\rangle$ er oddstætt fall

Speglunarvirkin P þ.a. $P\psi(x) = \psi(-x)$
hefur mismunandi eigingildi fyrir $|+\rangle$ og $|-\rangle$

P víxlast við H^0 og H'

Hér datt líka í hug L_z ef ég læt $2\pi\frac{x}{L} = \phi$, en
sá virki víxlast ekki við H'

(1)

6.9

$$H = V_0\begin{pmatrix} 1-\epsilon & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix}$$

V_0 : fasti

$\epsilon << 1$

a) Finna róf og ástönd óturflaða kerfisins þ. $\epsilon = 0$

$$H_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} V_0$$

Hornalínuform $\rightarrow$ eigingildin eru

$E_1^0 = V_0$, $E_2^0 = V_0$, $E_3^0 = 2V_0$

og eigin ástöndin eru

$|1\rangle^0 = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $|2\rangle^0 = \begin{pmatrix}0\\1\\0\end{pmatrix}$

$|3\rangle^0 = \begin{pmatrix}0\\0\\1\end{pmatrix}$

b) Finna nákvæmu eigingildin á H. Hér er hægt að nota strúktúr

$$H = V_0\begin{pmatrix} \boxed{1-\epsilon} & 0 & 0 \\ 0 & \boxed{\begin{matrix}1 & \epsilon \\ \epsilon & 2\end{matrix}} \end{pmatrix}$$

(2)

Til þess að fá

$$E_1 = V_0(1-\epsilon)$$

$$E_2 = \frac{V_0}{2}\left\{3-\sqrt{4\epsilon^2+1}\right\} = \frac{V_0}{2}\left\{3-\sqrt{1+4\epsilon^2}\right\}$$

$$E_3 = \frac{V_0}{2}\left\{3+\sqrt{4\epsilon^2+1}\right\} = \frac{V_0}{2}\left\{3+\sqrt{1+4\epsilon^2}\right\}$$

Liðum

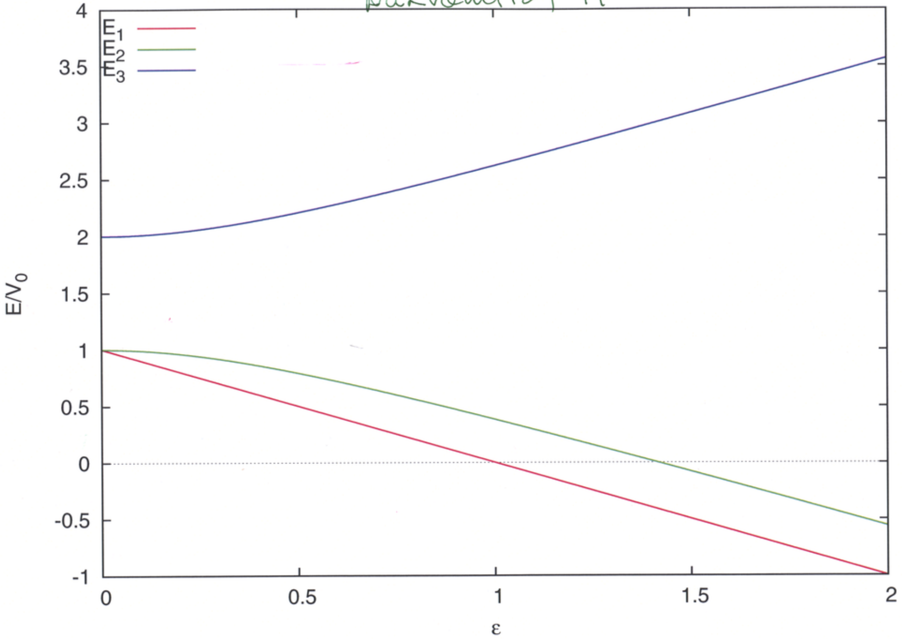
$$E_2 = V_0\left\{1-\epsilon^2+\epsilon^4+\dots\right\}$$

$$E_3 = V_0\left\{2+\epsilon^2-\epsilon^4+\dots\right\}$$

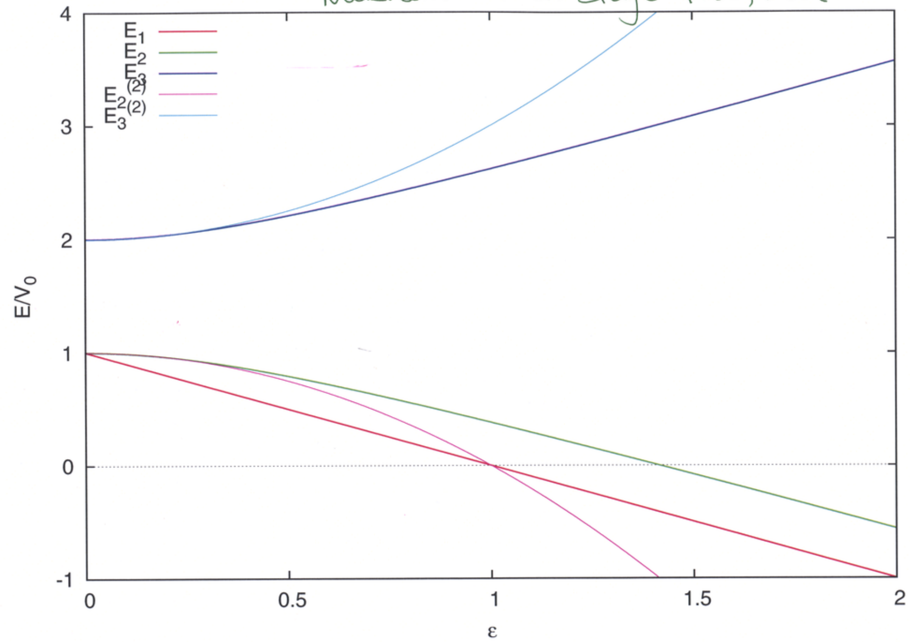
engin línulegur liður í ϵ

E_1 hefur bara línul. lið og engan komi

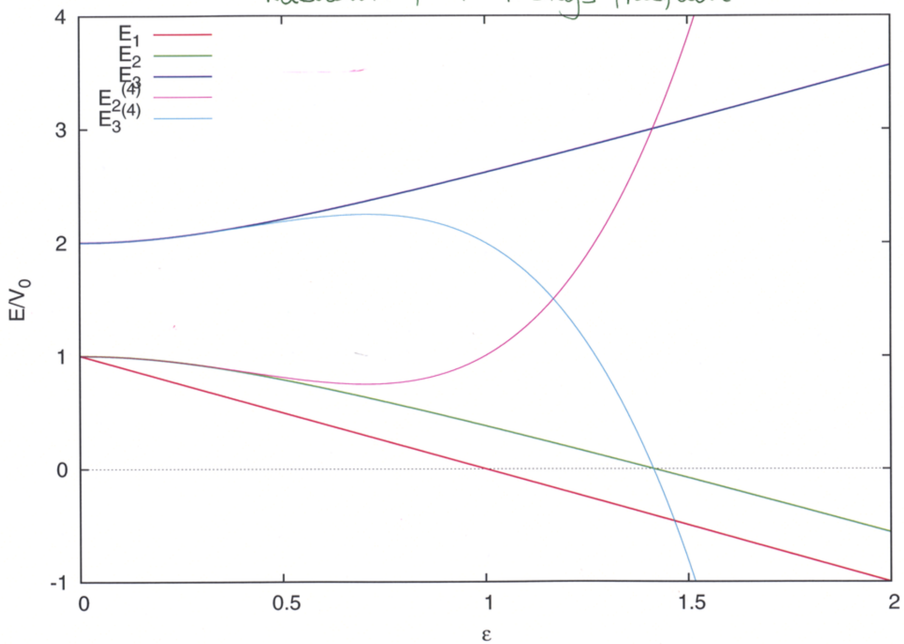
Nákvæmt róf H



Nákvæmt + 2. Stigs truflun



Nákvæmt róf + 4. Stigs truflun



c) Notum 1. og 2. Stigs truflun til ~~þess~~ að finna nálgun fyrir eigingildið sem er í upphafi einfalt þ.e. E_3

$$E_3^1 = \langle 3|H'|3\rangle = V_0 (0\,0\,1)\begin{pmatrix} -\epsilon & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= V_0 (0\,\epsilon\,0)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

engin 1. stigs truflun, eða línulegur liður eins og við sáum þ. E_3 var liðað

$$E_3^2 = \sum_{m\neq n} \frac{|\langle m|H'|3\rangle|^2}{E_3^0 - E_m^0}$$

þurfum því

$$\langle 1|H'|3\rangle = V_0(1\,0\,0)\begin{pmatrix}-\epsilon & 0 & 0\\ 0 & 0 & \epsilon\\ 0 & \epsilon & 0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix} = (-\epsilon\,0\,0)\begin{pmatrix}0\\0\\1\end{pmatrix}V_0 = 0$$

$$\langle 2|H'|3\rangle = V_0(0\,1\,0)\begin{pmatrix}-\epsilon & 0 & 0\\ 0 & 0 & \epsilon\\ 0 & \epsilon & 0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix} = (0\,0\,\epsilon)\begin{pmatrix}0\\0\\1\end{pmatrix}V_0 = \epsilon V_0$$

$$\to E_3^2 = \frac{\epsilon^2 V_0^2}{E_3^0 - E_2^0} = \frac{\epsilon^2 V_0^2}{V_0} = \epsilon^2 V_0$$

$E_3 \approx E_3^0 + \epsilon^2 V_0$

eins og liðinum á nákvæmu lausninni sagði fyrir um

d) 1. stigs leiðrétting á tvöföldu ástöndunum E_1 og E_2

$$W_{11} = \langle 1|H'|1\rangle = \epsilon V_0(1\,0\,0)\begin{pmatrix}-1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = \epsilon V_0(-1\,0\,0)\begin{pmatrix}1\\0\\0\end{pmatrix} = -\epsilon V_0$$

$$W_{22} = \langle 2|H'|2\rangle = \epsilon V_0(0\,1\,0)\begin{pmatrix}-1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix} = (0\,0\,1)\begin{pmatrix}0\\1\\0\end{pmatrix}\epsilon V_0 = 0$$

$$W_{12} = \langle 1|H'|2\rangle = \epsilon V_0(1\,0\,0)\begin{pmatrix}-1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix} = \epsilon V_0(-1\,0\,0)\begin{pmatrix}0\\0\\1\end{pmatrix} = 0$$

$$\to W = \begin{pmatrix}-\epsilon V_0 & 0\\ 0 & 0\end{pmatrix}$$

$$E^1_\pm = \frac{1}{2}\left\{-\epsilon V_0 \pm \sqrt{\epsilon^2 V_0^2}\right\} = \begin{cases}0\\ -\epsilon V_0\end{cases}$$

Annað orku gildið verður þá

$$E_+ = E^0_{1\&2} + 0 = V_0 = E_2^0$$

engin 1. stigs leiðrétting

$$E_- = E^0_{1\&2} - \epsilon V_0 = V_0(1-\epsilon) = E_1$$

nákvæmt, aller hærri en 0

6.14 lægsta afstæða leiðréttingin á hrein tóna sveiflunum

$$H'_r = -\frac{p^4}{8m^3c^2}, \qquad a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$$

$$\to a_+ - a_- = \frac{1}{\sqrt{2\hbar m\omega}}(-ip - ip) = -\sqrt{\frac{2}{\hbar m\omega}}\,ip$$

$$= -\frac{i}{\hbar}p\sqrt{\frac{2\hbar}{m\omega}} = -\frac{i}{\hbar}p\sqrt{2}\,a, \qquad a = \sqrt{\frac{\hbar}{m\omega}}$$

$$\to p = \frac{\hbar}{-ia\sqrt{2}}(a_+ - a_-) = i\hbar\frac{1}{\sqrt{2}a}(a_+ - a_-)$$

notum beint

$$E_n^1 = -\frac{1}{8m^3c^2}\langle n|p^4|n\rangle = -\frac{\hbar^4}{4a^4 8m^3c^2}\langle n|(a_+ - a_-)^4|n\rangle$$

$$E_n^1 = -\frac{\hbar^4}{32a^4m^3c^2}\langle n|\{a_+a_+a_-a_- + a_-a_-a_+a_+ + a_+a_-a_+a_- + a_-a_+a_-a_+ + a_+a_-a_-a_+ + a_-a_+a_+a_-\}|n\rangle$$

einungis liðir með jafnan fjölda a_+ og a_- geta annað en 0

$$E_n^1 = -\frac{\hbar^4}{32a^4m^3c^2}\{n(n-1) + (n+1)(n+2) + n^2 + (n+1)^2 + n(n+1) + (n+1)n\}$$

notum

$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$

$$= -\frac{\hbar^4}{32a^4m^3c^2}\{6n^2 + 6n + 3\}$$

$$= -\frac{3\hbar^4 m^2\omega^2}{32\hbar^2 m^3c^2}\{2n^2 + 2n + 1\} = -\frac{3}{32}\frac{\hbar^2\omega^2}{mc^2}\{2n^2+2n+1\}$$

(9.1) ①

Vetnisatóm í rafsviði $\vec{E} = E(t)\hat{z}$

Reikna öll 4 fylkjastök fyrir $H' = eEz$

milli $n=1$ og $n=2$

t.þ. að nota kúluhnit

$$H' = eEz = eE\{r\cos\theta\} = a\,eE\frac{r}{a}\sqrt{\frac{4\pi}{3}}\,Y_{10}(\theta,\phi)$$

Bylgjuföllin eru $\psi_{n\ell m}(\vec{r}) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$

$$\psi_{100} = R_{10}Y_{00} \quad \} n=1$$

$$\left.\begin{aligned}\psi_{200} &= R_{20}Y_{00}\\ \psi_{210} &= R_{21}Y_{10}\\ \psi_{21\pm1} &= R_{21}Y_{1\pm1}\end{aligned}\right\} n=2$$

Vegna hornhlutans er eitt stak ekki 0, $Y_{\ell m}$ eru hornrétt!

$$\langle 100|H'|210\rangle \neq 0$$

②

$$\psi_{100} = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}, \quad \psi_{210} = \frac{1}{\sqrt{a^3\cdot 24}}\left(\frac{r}{a}\right)e^{-\frac{r}{2a}}Y_{10}(\theta,\phi)$$

$$\langle 100|H'|210\rangle = eE\sqrt{\frac{4\pi}{3}}\frac{1}{\sqrt{\pi a^3}}\frac{a}{\sqrt{a^3\cdot 4\cdot 6}}\int r^2dr\left(\frac{r}{a}\right)^2e^{-\frac{3r}{2a}}$$

$$= eE\frac{a}{\sqrt{3\cdot 6}}\int_0^\infty du\, u^4e^{-\frac{3}{2}u} = eE\frac{a}{\sqrt{3\cdot 6}}\frac{256}{81}$$

$$\simeq 0{,}74494\cdot eEa$$

þar sem ég hef notað fyrir hornhlutaheildisins og man að Y_{00} er fasti

$$\int d\Omega\, Y_{\ell m}(\Omega)Y^*_{\ell' m'}(\Omega) = \delta_{\ell\ell'}\delta_{mm'}$$

(9-7) ③

Gerum byrjað með

$$H'_{ba} = \frac{V_{ba}}{2}e^{-i\omega t}, \quad H'_{ab} = \frac{V_{ab}}{2}e^{i\omega t}$$

leysa (9.13) nákvæmlega

fyrir $C_a(0)=1$, $C_b(0)=0$

$$\dot{C}_a = -\frac{i}{\hbar}H'_{ab}e^{-i\omega_0 t}C_b = -\frac{i}{2\hbar}V_{ab}e^{+i(\omega-\omega_0)t}C_b$$

$$\dot{C}_b = -\frac{i}{\hbar}H'_{ba}e^{i\omega_0 t}C_a = -\frac{i}{2\hbar}V_{ba}e^{-i(\omega-\omega_0)t}C_a$$

Aðeins hermiliður → Liðir hafa einfaldast með þessari Rotating wave approximation

④

Höfum

$$\dot{C}_a = -\frac{i}{2\hbar}V_{ab}e^{i\Delta\omega t}C_b \quad ①$$

$$\dot{C}_b = -\frac{i}{2\hbar}V_{ba}e^{-i\Delta\omega t}C_a \quad ②$$

með $\Delta\omega = \omega - \omega_0$

setjum saman í eina annars stigs

$$\dot{②} \to \quad \ddot{C}_b = -\frac{i}{2\hbar}V_{ba}\left\{-i\Delta\omega C_a + \dot{C}_a\right\}e^{-i\Delta\omega t}$$

$$= \underbrace{\left\{-\frac{i}{2\hbar}V_{ba}e^{-i\Delta\omega t}C_a\right\}}_{②}(-i\Delta\omega) - \frac{i}{2\hbar}V_{ba}\underbrace{\dot{C}_a e^{-i\Delta\omega t}}_{①}$$

$$= \dot{C}_b(-i\Delta\omega) - \frac{|V_{ba}|^2}{4\hbar^2}C_b$$

$$\to \boxed{\ddot{C}_b + i\Delta\omega\dot{C}_b + \frac{|V_{ba}|^2}{4\hbar^2}C_b = 0}$$

línuleg 2. stigs jafna með stuðlum óháðum t vegna RW-nálgunar

⑤

Reynum lausn $e^{\lambda t}$ með innsetningu t.þ.a fá kennijöfnu

$$\lambda^2 + i\Delta\omega\lambda + \frac{|V_{ba}|^2}{4\hbar^2} = 0$$

2. stigs jafna með lausn

$$\lambda = \frac{1}{2}\left\{-i\Delta\omega \pm \sqrt{-(\Delta\omega)^2 - \frac{|V_{ba}|^2}{\hbar^2}}\right\} = i\left\{-\frac{\Delta\omega}{2} \pm \omega_r\right\}$$

ef

$$\omega_r = \frac{1}{2}\sqrt{(\Delta\omega)^2 + \frac{|V_{ba}|^2}{\hbar^2}}$$

Rabi tíðnin sem við vorum beðin um að einangra og nota

Allmenna lausnin er þá

$$C_b(t) = A\exp\left\{i\left(-\frac{\Delta\omega}{2} + \omega_r\right)t\right\} + B\exp\left\{i\left(-\frac{\Delta\omega}{2} - \omega_r\right)t\right\}$$

⑥

$$C_b(t) = \exp\left\{-\frac{i\Delta\omega t}{2}\right\}\left[Ae^{i\omega_r t} + Be^{-i\omega_r t}\right]$$

Til þess að uppfylla upphafsskilyrðin er heppilegra að nota hornafalla samantekt

$$C_b(t) = \exp\left\{-\frac{i\Delta\omega}{2}t\right\}\left\{A'\cos(\omega_r t) + B'\sin(\omega_r t)\right\}$$

$$C_b(0) = 0 \rightarrow A' = 0 \quad \text{og}$$

$$C_b(t) = \exp\left\{-\frac{i\Delta\omega t}{2}\right\} B'\sin(\omega_r t)$$

Við þurfum líka að uppfylla að

$$C_a(0) = 1$$

⑦

$$\dot{C}_b = -\frac{i\Delta\omega}{2}\exp\left\{-\frac{i\Delta\omega t}{2}\right\}B'\sin(\omega_r t) + \exp\left\{-\frac{i\Delta\omega t}{2}\right\}B'\omega_r\cos(\omega_r t)$$

munum ② $\rightarrow C_a = \dot{C}_b\, 2\hbar i \frac{1}{V_{ba}} e^{i\Delta\omega t}$

$$\rightarrow C_a = B'\frac{2\hbar i}{V_{ba}}\exp\left\{\frac{i\Delta\omega t}{2}\right\}\left\{-i\frac{\Delta\omega}{2}\sin(\omega_r t) + \omega_r\cos(\omega_r t)\right\}$$

$$C_a(0) = 1$$

$$\hookrightarrow B'\frac{2\hbar i}{V_{ba}}\omega_r = 1 \longrightarrow B' = -\frac{iV_{ba}}{2\hbar\omega_r}$$

$$\rightarrow \boxed{\begin{aligned} C_b(t) &= -\frac{iV_{ba}}{2\hbar\omega_r}e^{-\frac{i\Delta\omega t}{2}}\sin(\omega_r t) \\ C_a(t) &= e^{\frac{i\Delta\omega t}{2}}\left\{\cos(\omega_r t) - i\frac{\Delta\omega}{2\omega_r}\sin(\omega_r t)\right\}\end{aligned}}$$

$\Delta\omega = \omega - \omega_0$

⑧

b)
$$P_{a\to b}(t) = |C_b|^2 = \frac{|V_{ba}|^2}{4\hbar^2\omega_r^2}\sin^2(\omega_r t)$$

$$= \frac{|V_{ba}|^2}{\hbar^2(\Delta\omega)^2 + |V_{ba}|^2}\sin^2(\omega_r t) \leq 1$$

verður 1 í hérum þegar $\Delta\omega = \omega - \omega_0 = 0$

Sannreyna líkindi

$$|C_b(t)|^2 + |C_a(t)|^2 = \frac{|V_{ba}|^2}{4\hbar^2\omega_r^2}\sin^2(\omega_r t) + \cos^2(\omega_r t) + \frac{(\Delta\omega)^2}{4\omega_r^2}\sin^2(\omega_r t) = \underline{1}$$

$1 \cdot \sin^2(\omega_r t)$

(9)

c) fáum við 1. stigs nálgunina fyrir smáa truflun

$$\boxed{P^{\text{truflun}}_{a\to b}(t) = |C_b(t)|^2 \simeq \frac{|V_{ba}|^2}{\hbar^2}\frac{\sin^2\left(\frac{\Delta\omega t}{2}\right)}{\Delta\omega}} \quad (9.28)$$

Athugum, Rabi-tíðnin var útleidd á

$$\omega_r = \frac{1}{2}\sqrt{(\Delta\omega)^2 + \frac{|V_{ab}|^2}{\hbar^2}}$$

$$\to \text{ að } \omega_r \simeq \frac{\Delta\omega}{2} \quad \text{þegar} \quad |V_{ab}|^2 \ll (\hbar\Delta\omega)^2$$

úr b-lið

$$P^{\text{nákvæmt}}_{a\to b}(t) = \frac{|V_{ba}|^2}{\hbar^2(\Delta\omega)^2 + |V_{ba}|^2}\sin^2(\omega_r t)$$

$$\xrightarrow[|V_{ab}|^2 \ll (\hbar\Delta\omega)^2]{} \frac{|V_{ba}|^2}{\hbar^2(\Delta\omega)^2}\sin^2\left(\frac{\Delta\omega t}{2}\right) = P^{\text{truflun}}_{a\to b}$$

(10)

d) Hvenær kemst kerfið fyrst í upphafsástand?

þegar $\omega_r t = \pi$ fæst $P_{a\to b} = 0$

og $C_b = 0$, $C_a = 1$

$$\to t = \frac{\pi}{\omega_r}$$

Munum að þó við höfum fundið nákvæma lausn á afleiðujöfnunni fyrir $C_a(t)$ og $C_b(t)$ sem uppfyllir ýmis skilyrði betur en 1. stigs truflunin þá erum við samt með RW-nálgunina veldur því að við verðum að vera nærri hermu