9.1

Vetnisatóm í rafsviði $\bar{E} = E(t)\hat{z}$

Reikna öll 4 fylkjastök fyrir $H' = eEz$

milli $n=1$ og $n=2$

t.þ. að nota kúluhnit

$$H' = eEz = eE\{r\cos\theta\} = a\cdot eE\frac{r}{a}\sqrt{\frac{4\pi}{3}}\,Y_{10}(\theta,\phi)$$

Bylgjuföllin eru $\psi_{nlm}(\bar{r}) = R_{nl}(r)\,Y_{lm}(\theta,\phi)$

$$\left.\psi_{100} = R_{10}Y_{00}\right\} n=1$$

$$\left.\begin{aligned}\psi_{200} &= R_{20}Y_{00}\\ \psi_{210} &= R_{21}Y_{10}\\ \psi_{21\pm1} &= R_{21}Y_{1\pm1}\end{aligned}\right\} n=2$$

Vegna hornhlutans er eitt stak ekki 0, Y_{lm} eru hornrétt!

$$\langle 100|H'|210\rangle \neq 0$$

2

$$\psi_{100} = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \quad , \quad \psi_{210} = \frac{1}{\sqrt{a^3 \cdot 24}} \left(\frac{r}{a}\right) e^{-\frac{r}{2a}} Y_{10}(\theta,\phi)$$

$$\langle 100 | H' | 210 \rangle = eE \sqrt{\frac{4\pi}{3}} \frac{1}{\sqrt{\pi a^3}} \frac{a}{\sqrt{a^3 \cdot 4 \cdot 6}} \int r^2 dr \left(\frac{r}{a}\right)^2 e^{-\frac{3r}{2a}}$$

$$= eE \frac{a}{\sqrt{3 \cdot 6}} \int_0^\infty du \; u^4 e^{-\frac{3}{2}u} = eE \frac{a}{\sqrt{3 \cdot 6}} \frac{256}{81}$$

$$\simeq 0{,}74494 \cdot eEa$$

þar sem ég hef notað fyrir hornhluta heildisins og man að Y_{00} er fasti

$$\int d\Omega \, Y_{\ell m}(\Omega) Y_{\ell' m'}(\Omega) = \delta_{\ell\ell'} \delta_{m,m'}$$

Getum byrjað með

$$H'_{ba} = \frac{V_{ba}}{2} e^{-i\omega t} \quad , \quad H'_{ab} = \frac{V_{ab}}{2} e^{i\omega t}$$

leysa (9.13) nákvæmlega

fyrir $\quad C_a(0) = 1 \; , \; C_b(0) = 0$

$$\dot{C}_a = -\frac{i}{\hbar} H'_{ab} e^{-i\omega_0 t} C_b = -\frac{i}{2\hbar} V_{ab} e^{+i(\omega-\omega_0)t} C_b$$

$$\dot{C}_b = -\frac{i}{\hbar} H'_{ba} e^{i\omega_0 t} C_a = -\frac{i}{2\hbar} V_{ba} e^{-i(\omega-\omega_0)t} C_a$$

Aðeins hermitískir $\rightarrow$ liðir hafa einfaldast með þessari Rotating wave approximation

Höfum

$$\dot{C}_a = -\frac{i}{2\hbar} V_{ab}\, e^{i\Delta\omega t}\, C_b \qquad (1)$$

með $\Delta\omega = \omega - \omega_0$

$$\dot{C}_b = -\frac{i}{2\hbar} V_{ba}\, e^{-i\Delta\omega t}\, C_a \qquad (2)$$

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setjum saman í eina annarsstigs

$$\dot{(2)} \;\rightarrow\; \ddot{C}_b = -\frac{i}{2\hbar} V_{ba} \left\{ -i\Delta\omega\, C_a + \dot{C}_a \right\} e^{-i\Delta\omega t}$$

$$= \underbrace{\left\{ -\frac{i}{2\hbar} V_{ba}\, e^{-i\Delta\omega t}\, C_a \right\}}_{(2)} (-i\Delta\omega) \underbrace{- \frac{i}{2\hbar} V_{ba}\, \dot{C}_a\, e^{-i\Delta\omega t}}_{(1)}$$

$$= \dot{C}_b\, (-i\Delta\omega) - \frac{|V_{ba}|^2}{4\hbar^2} C_b$$

$$\rightarrow \boxed{\ddot{C}_b + i\Delta\omega\, \dot{C}_b + \frac{|V_{ba}|^2}{4\hbar^2} C_b = 0}$$

línuleg 2. stigs jafna
með stuðlum óháðum
t vegna RW-nálgunar

Reynum lausn $e^{\lambda t}$ með innsetningu t.þ.a þá kemur jöfnu (5)

$$\lambda^2 + i\Delta\omega\lambda + \frac{|V_{ba}|^2}{4\hbar^2} = 0$$

2. stigs jafna með lausn

$$\lambda = \frac{1}{2}\left\{-i\Delta\omega \pm \sqrt{-(\Delta\omega)^2 - \frac{|V_{ba}|^2}{\hbar^2}}\right\} = i\left\{-\frac{\Delta\omega}{2} \pm \omega_r\right\}$$

ef

$$\omega_r = \frac{1}{2}\sqrt{(\Delta\omega)^2 + \frac{|V_{ba}|^2}{\hbar^2}}$$

Rabi tíðnin sem við vorum beðin um að einangra og nota

Almenna lausnin er þá

$$C_b(t) = A\exp\left\{i\left(-\frac{\Delta\omega}{2} + \omega_r\right)t\right\} + B\exp\left\{i\left(-\frac{\Delta\omega}{2} - \omega_r\right)t\right\}$$

$$C_b(t) = \exp\left\{-\frac{i\Delta\omega t}{2}\right\}\left[A e^{i\omega_r t} + B e^{-i\omega_r t}\right]$$

Til þess að uppfylla upphafsskilyrðin er heppilegra að nota hornaföll samantekt

$$C_b(t) = \exp\left\{-\frac{i\Delta\omega}{2}t\right\}\left\{A' \cos(\omega_r t) + B' \sin(\omega_r t)\right\}$$

$$C_b(0) = 0 \longrightarrow A' = 0$$ og

$$C_b(t) = \exp\left\{-\frac{i\Delta\omega t}{2}\right\} B' \sin(\omega_r t)$$

Við þurfum líka að uppfylla að

$$C_a(0) = 1$$

$$\dot{C}_b = -\frac{i\Delta\omega}{2}\exp\left\{-\frac{i\Delta\omega t}{2}\right\} B'\sin(\omega_r t) + \exp\left\{-\frac{i\Delta\omega t}{2}\right\} B'\omega_r\cos(\omega_r t)$$

(2) $\to$ $$C_a = \dot{C}_b\, 2\hbar i\,\frac{1}{V_{ba}}\, e^{i\Delta\omega t}$$

$$\to C_a = B'\frac{2\hbar i}{V_{ba}}\exp\left\{\frac{i\Delta\omega t}{2}\right\}\left\{-i\frac{\Delta\omega}{2}\sin(\omega_r t) + \omega_r\cos(\omega_r t)\right\}$$

$$C_a(0) = 1$$

$$\hookrightarrow B'\frac{2\hbar i}{V_{ba}}\omega_r = 1 \longrightarrow B' = -\frac{iV_{ba}}{2\hbar\omega_r}$$

$$\to C_b(t) = -\frac{iV_{ba}}{2\hbar\omega_r}\, e^{-\frac{i\Delta\omega t}{2}}\sin(\omega_r t)$$

$$C_a(t) = e^{\frac{i\Delta\omega t}{2}}\left\{\cos(\omega_r t) - i\frac{\Delta\omega}{2\omega_r}\sin(\omega_r t)\right\}$$

$\Delta\omega = \omega - \omega_0$

b) $P_{a\to b}(t) = |C_b|^2 = \frac{|V_{ba}|^2}{4\hbar^2\omega_r^2} \sin^2(\omega_r t)$

$$= \frac{|V_{ba}|^2}{\hbar^2(\Delta\omega)^2 + |V_{ba}|^2} \sin^2(\omega_r t) \leq 1$$

verður 1 í hérumbil pegar $\Delta\omega = \omega - \omega_0 = 0$

Varðveisla líkinda

$$|C_b(t)|^2 + |C_a(t)|^2 = \frac{|V_{ba}|^2}{4\hbar^2\omega_r^2}\sin^2(\omega_r t) + \cos^2(\omega_r t)$$

$$+ \frac{(\Delta\omega)^2}{4\omega_r^2}\sin^2(\omega_r t) = \underline{1}$$

$1 \cdot \sin^2(\omega_r t)$

c) fáum við 1. stigs niðurstöðuna fyrir smáa truflun

$$\boxed{P^{\text{truflun}}_{a\to b}(t) = |C_b(t)|^2 \simeq \frac{|V_{ba}|^2}{\hbar^2}\,\frac{\sin^2\left(\frac{\Delta\omega t}{2}\right)}{\Delta\omega}} \qquad (9.28)$$

Athugum, Rabi-tíðnin var ákvörðuð á

$$\omega_r = \frac{1}{2}\sqrt{(\Delta\omega)^2 + \frac{|V_{ab}|^2}{\hbar^2}}$$

$$\to \text{ að } \omega_r \simeq \frac{\Delta\omega}{2} \quad \text{þegar} \quad |V_{ab}|^2 \ll (\hbar\Delta\omega)^2$$

úr b-lið

$$P^{\text{nákvæmt}}_{a\to b}(t) = \frac{|V_{ba}|^2}{\hbar^2(\Delta\omega)^2 + |V_{ba}|^2}\sin^2(\omega_r t)$$

$$\xrightarrow[|V_{ab}|^2 \ll (\hbar\Delta\omega)^2]{} \frac{|V_{ba}|^2}{\hbar^2(\Delta\omega)^2}\sin^2\left(\frac{\Delta\omega t}{2}\right) = P^{\text{truflun}}_{a\to b}$$

d) Hvenær kemst kerfið fyrst í upphafsástand?

þegar $\omega_r t = \pi$ fæst $P_{a \to b} = 0$

og $C_b = 0$, $C_a = 1$

$$\rightarrow t = \frac{\pi}{\omega_r}$$

Munum að þó við höfum fundið nákvæma lausn á afleiðujöfnunni fyrir $C_a(t)$ og $C_b(t)$ sem uppfyllir ýmis skilyrði betur en 1. stigs truflunin þá erum við samt með RW-nálgun sem veldur því að við verðum að vera nærri hermu