6.9

$$H = V_0 \begin{pmatrix} 1-\epsilon & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix}$$

V_0 : fasti

$\epsilon << 1$

a) finna róf og ástönd ótruflaða kerfisins þ. $\epsilon = 0$

$$H_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} V_0$$

Hornalínuform $\longrightarrow$

eigingildin eru

$E_1^0 = V_0$, $E_2^0 = V_0$, $E_3^0 = 2V_0$

og eiginástöndin eru

$|1\rangle^0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\quad |2\rangle^0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$|3\rangle^0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

b) finna nákvæm eigingildin á H. Hér er hægt að nota strúktúr

$$H = V_0 \begin{pmatrix} 1-\epsilon & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix}$$

Til þess að fá

$$E_1 = V_0(1-\epsilon)$$

$$E_2 = \frac{V_0}{2}\left\{3 - \sqrt{4\epsilon^2+1}\right\} = \frac{V_0}{2}\left\{3 - \sqrt{1+4\epsilon^2}\right\}$$

$$E_3 = \frac{V_0}{2}\left\{3 + \sqrt{4\epsilon^2+1}\right\} = \frac{V_0}{2}\left\{3 + \sqrt{1+4\epsilon^2}\right\}$$

Liðum

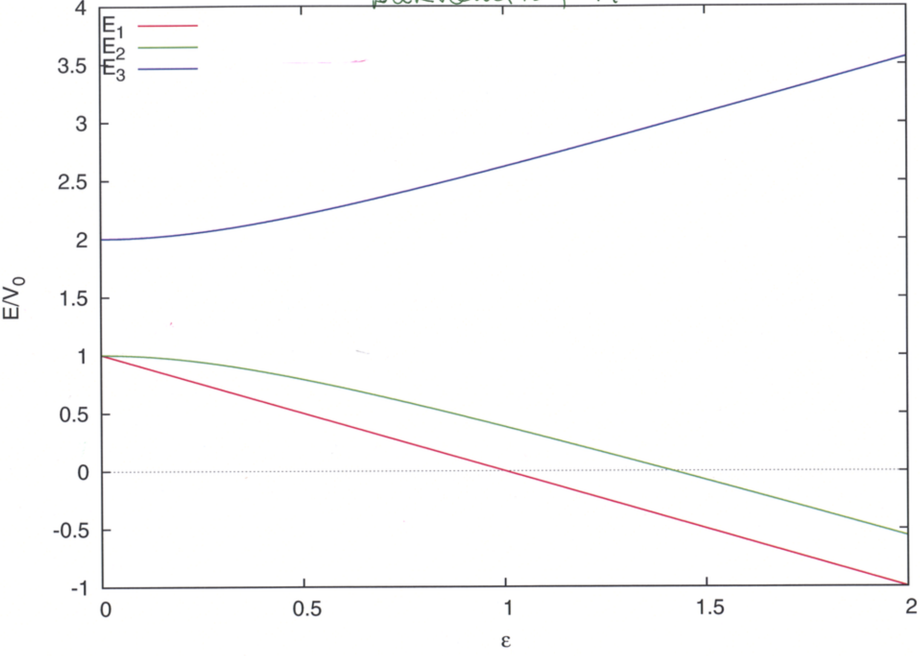
$$E_2 = V_0\left\{1 - \epsilon^2 + \epsilon^4 + \cdots\right\}$$

enginn línulegur liður í ϵ

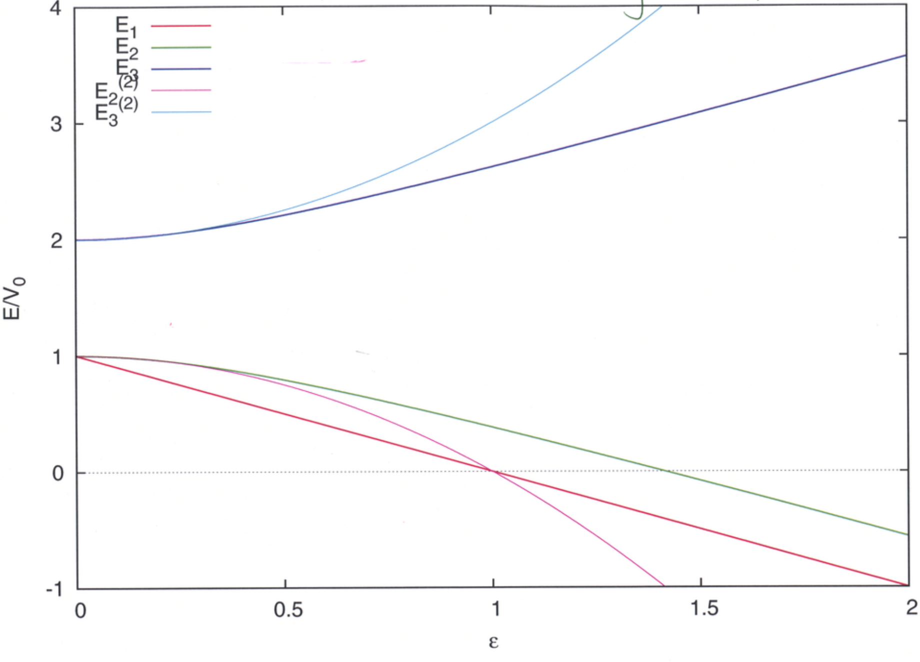
$$E_3 = V_0\left\{2 + \epsilon^2 - \epsilon^4 + \cdots\right\}$$

E_1 hefur bara línul. lið og engan komi

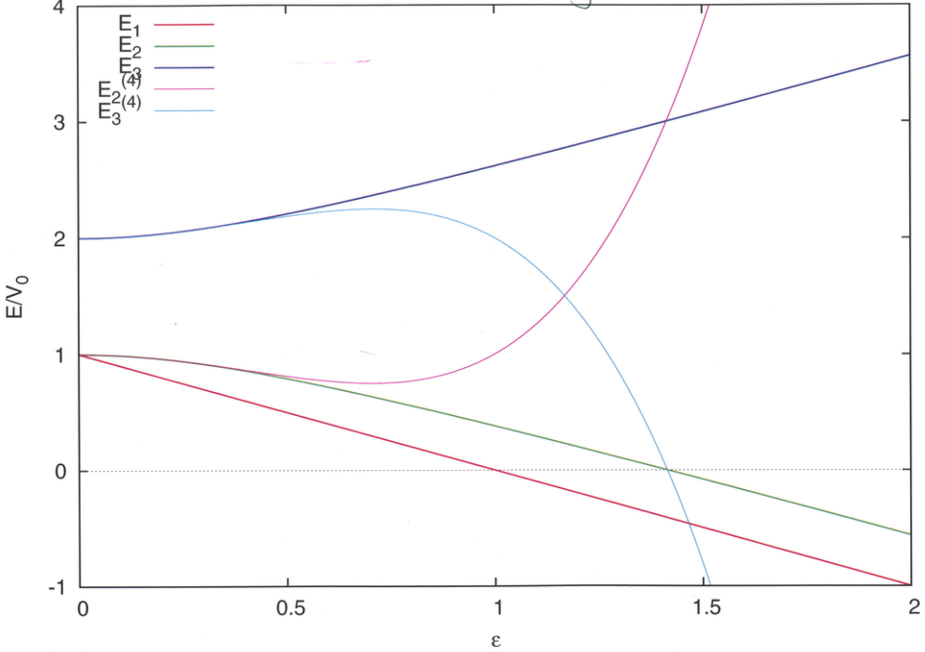
Nākvamtrōf H



Nākvamt + 2. Stigs truHem



Nákvæmt róf + 4. Stigs truflun



c) Notum 1. og 2. stigs truflun til ~~þess~~ að finna nálgun fyrir eigingildið sem er í upphafi einfalt þ.e. E_3

$$E_3^1 = \langle 3 | H' | 3 \rangle = V_0 (0\,0\,1) \begin{pmatrix} -\epsilon & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= V_0 (0\;\epsilon\;0) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

engin 1. stigs truflun, eða línulegar liðir eins og við sáum þ. E_3 var liðað

$$E_3^2 = \sum_{m \neq n} \frac{|\langle m | H' | 3 \rangle|^2}{E_3^0 - E_m^0}$$

þurfum því

$$\langle 1|H'|3\rangle = V_0 (1\,0\,0)\begin{pmatrix} -\epsilon & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (-\epsilon\,0\,0)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} V_0 = 0$$

$$\langle 2|H'|3\rangle = V_0 (0\,1\,0)\begin{pmatrix} -\epsilon & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (0\,0\,\epsilon)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} V_0 = \epsilon V_0$$

$$\rightarrow E_3^2 = \frac{\epsilon^2 V_0^2}{E_3^0 - E_2^0} = \frac{\epsilon^2 V_0^2}{V_0} = \epsilon^2 V_0$$

$$E_3 \approx E_3^0 + \epsilon^2 V_0$$

eins og lausnin á nákvæmu lausninni sagði fyrir um

d) 1. stigs leiðrétting á tvöföldu ástöndunum E_1 og E_2

$$W_{11} = \langle 1|H'|1\rangle = \epsilon V_0 (1\,0\,0)\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \epsilon V_0 (-1\,0\,0)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -\epsilon V_0$$

$$W_{22} = \langle 2|H'|2\rangle = \epsilon V_0 \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \epsilon V_0 = 0$$

$$W_{12} = \langle 1|H'|2\rangle = \epsilon V_0 \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \epsilon V_0 \begin{pmatrix} -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

$$\rightarrow \quad W = \begin{pmatrix} -\epsilon V_0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$E'_{\pm} = \frac{1}{2}\left\{ -\epsilon V_0 \pm \sqrt{\epsilon^2 V_0^2} \right\} = \begin{cases} 0 \\ -\epsilon V_0 \end{cases}$$

Andre ordens gitte verdier på

$$E_+ = E^0_{1 \text{ eller } 2} + 0 = V_0 = E^0_2$$ ingen 1. ordens retting

$$E_- = E^0_{1 \text{ eller } 2} - \epsilon V_0 = V_0(1-\epsilon) = E_1$$ nøyaktig, alle høgre er 0

6.14 Lögsta af stöðu leiðréttingin á hreintóna sveiflinum

$$H_r' = -\frac{p^4}{8m^3c^2}, \qquad a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$$

$$\rightarrow \quad a_+ - a_- = \frac{1}{\sqrt{2\hbar m\omega}}(-ip - ip) = -\sqrt{\frac{2}{\hbar m\omega}}\, ip$$

$$= -\hbar ip\sqrt{\frac{2\hbar}{m\omega}} = -\frac{i}{\hbar}p\sqrt{2}\,a, \qquad a = \sqrt{\frac{\hbar}{m\omega}}$$

$$\rightarrow \quad p = \frac{\hbar}{-ia\sqrt{2}}(a_+ - a_-) = i\hbar\frac{1}{\sqrt{2}\,a}(a_+ - a_-)$$

notum beint

$$E_n^1 = -\frac{1}{8m^3c^2}\langle n|p^4|n\rangle = -\frac{\hbar^4}{4a^4 8m^3c^2}\langle n|(a_+ - a_-)^4|n\rangle$$

$$E_n^1 = -\frac{\hbar^4}{32a^4m^3c^2}\langle n|\Big\{a_+a_+a_-a_- + a_-a_-a_+a_+$$

$$+ a_+a_-a_+a_- + a_-a_+a_-a_+$$

$$+ a_+ a a_- a_+ + a_-a_+a_+a_-\Big\}|n\rangle$$

einungis liðir með jafnan fjölda a_+ og a_- gefa annað en 0

$$E_n^1 = -\frac{\hbar^4}{32a^4m^3c^2}\Big\{n(n-1) + (n+1)(n+2) + n^2$$

$$+ (n+1)^2 + n(n+1) + (n+1)n\Big\}$$

notum

$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$

$$= -\frac{\hbar^4}{32a^4m^3c^2}\Big\{6n^2+6n+3\Big\}$$

$$= -\frac{3\hbar^4m^2\omega^2}{32\hbar^2m^3c^2}\Big\{2n^2+2n+1\Big\} = -\frac{3}{32}\frac{\hbar^2\omega^2}{mc^2}\Big\{2n^2+2n+1$$