6.2 Hreintóna sveifill, ein vídd

$$V(x) = \frac{1}{2} k x^2$$

með orkuróf $E_n = (n+\frac{1}{2})\hbar\omega$

með $n = 0, 1, \ldots$

$$\omega = \sqrt{\frac{k}{m}}$$

Gormstuðlinum er breytt

aðeins $k \to (1+\epsilon)k$

a) finna nákvæma nýja orkurófið

$$\omega' = \sqrt{\frac{k'}{m}} = \sqrt{\frac{(1+\epsilon)k}{m}}$$

$$\omega' = \sqrt{(1+\epsilon)}\sqrt{\frac{k}{m}}$$

$$= \sqrt{1+\epsilon}\,\omega$$

því fæst að hið truflaða rófið er

$$E_n' = (n+\tfrac{1}{2})\hbar\omega'$$

$$= (n+\tfrac{1}{2})\hbar\omega \cdot \sqrt{1+\epsilon}$$

$$= E_n \cdot \sqrt{1+\epsilon}$$

og liðun gefur

$$E_n' \simeq E_n \cdot \left\{1 + \frac{\epsilon}{2} - \frac{\epsilon^2}{8} + \cdots\right\}$$

b) Nota truflana reikning til þess að reikna 1. stig leiðréttingu

$$V'(x) = \frac{1}{2} k' x^2$$

$$= \frac{1}{2} k x^2 (1+\epsilon)$$

Truflunarmættið er því vegna

$$H' = \frac{\epsilon}{2} k x^2$$

ϵ leikur hlutverk truflana-stuðulsins λ.

Við viljum því reikna

$$\langle n | H' | n \rangle = \frac{\epsilon k}{2} \langle n | x^2 | n \rangle$$

munum eftir tröppuvirkjunum

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}} \{ \mp ip + m\omega x \}$$

$$\rightarrow a_+ + a_- = \frac{2m\omega}{\sqrt{2\hbar m\omega}} x$$

$$= \sqrt{\frac{2m\omega}{\hbar}} x$$

$$= \sqrt{2} \frac{x}{a}$$

$a = \sqrt{\frac{\hbar}{m\omega}}$ náttúruleg lengd

$$\rightarrow \quad x = \frac{a}{\sqrt{2}}(a_+ + a_-)$$

$$x^2 = \frac{a^2}{2}(a_+ + a_-)^2$$

$$= \frac{a^2}{2}\left\{a_+a_+ + a_-a_- + a_+a_- + a_-a_+\right\}$$

Notum síðan

$$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$a_-|0\rangle = 0$$

til þess að reikna

$$\langle n|x^2|n\rangle = \frac{a^2}{2}\langle n|\left\{a_+a_- + a_-a_+\right\}|n\rangle$$

$$\langle n | x^2 | n \rangle = \frac{a^2}{2} \langle n | \{ n + n+1 \} | n \rangle = \frac{a^2}{2}(2n+1)$$

$$\rightarrow \langle n | H' | n \rangle = \frac{\epsilon k}{2} \frac{a^2}{2}(2n+1) = \frac{\epsilon k}{2} a^2 \left(n + \frac{1}{2}\right)$$

$$= \frac{\epsilon k}{2} \frac{\hbar}{m\omega}\left(n+\frac{1}{2}\right) = \frac{\epsilon k}{2} \frac{\hbar}{m}\sqrt{\frac{m}{k}}\left(n+\frac{1}{2}\right)$$

$$= \frac{\epsilon}{2} \hbar \sqrt{\frac{k}{m}}\left(n+\frac{1}{2}\right) = \frac{\epsilon}{2}\hbar\omega\left(n+\frac{1}{2}\right)$$

pannig oed

$$E_n^1 = \frac{\epsilon}{2}\hbar\omega\left(n+\frac{1}{2}\right) = \frac{\epsilon}{2} E_n$$

6.3 Tvær eins bósónur í óendanlega djúpum brunni

sem víxlverkast veikt með „snertimætti"

$$V(x_1, x_2) = -aV_0\,\delta(x_1 - x_2)$$

Breidd brunns er a og vídd V_0 er orka

a) Án víxlverkunar, finna grunnástand og fyrsta örvaða ástandið, orku og ástand.

Vinnum með einnar-eindar grunnföllin

$$\psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi}{a}x\right) \qquad n = 1, 2, 3, \ldots$$

og rófið

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

bóseindir → grunnástandið er

$$\psi_g(x_1 x_2) = \psi_1(x_1)\psi_1(x_2) = \frac{2}{a} \sin\left(\frac{\pi}{a} x_1\right) \sin\left(\frac{\pi}{a} x_2\right)$$

samhverft fall þ.a. $\psi_g(x_1 x_2) = \psi_g(x_2 x_1)$

Orkan er $\quad E_g = 2 \cdot E_1 = \dfrac{\pi^2 \hbar^2}{m a^2}$

---

1. örvun fæst með því að lyfta annarri eindinni upp um eitt stig, vitum ekki hvorri!

samhverft

$$\psi_e(x_1 x_2) = \frac{1}{\sqrt{2}} \left\{ \psi_1(x_1)\psi_2(x_2) + \psi_2(x_1)\psi_1(x_2) \right\}$$

$$E_e = E_1 + E_2 = \frac{5 \pi^2 \hbar^2}{2 m a^2}$$

b) 1. Stigs truflun fyrir E_g og E_e

$$\underline{E_g^1} = \langle g | V | g \rangle = -aV_0 \int_0^a dx_1 dx_2 \, \frac{4}{a^2} \sin^2\left(\pi \frac{x_1}{a}\right) \sin^2\left(\pi \frac{x_2}{a}\right) \cdot \delta(x_1 - x_2)$$

$$= -aV_0 \frac{4}{a^2} \int_0^a dx_1 \sin^4\left(\pi \frac{x_1}{a}\right)$$

$$= -aV_0 \frac{4}{a^2} a \int_0^1 du \sin^4(\pi u) = -aV_0 \frac{4}{a^2} a \cdot \frac{12}{32}$$

$$= -V_0 \frac{48}{32} = \underline{-V_0 \frac{3}{2}}$$

$$\underline{E_e^1} = \langle e|V|e\rangle = -aV_0 \int_0^a dx_1 dx_2\, \Phi_e^2(x_1,x_2)\, \delta(x_1-x_2)$$

$$= -aV_0 \int_0^a dx_1\, \Phi_e^2(x_1,x_1)$$

$$= -aV_0 \frac{1}{2} \frac{4a}{a^2} \int_0^1 du \left[\sin(\pi u)\sin(2\pi u)\right]^2 \cdot 4$$

$$= -aV_0 \frac{1}{2} \frac{16a}{a^2} \int_0^1 du\, \sin^2(\pi u) \sin^2(2\pi u)$$

$$= -aV_0 \frac{16a}{2a^2} \frac{12}{48} = \underline{-V_0 \cdot 2}$$

Tökum saman til gamans

$$E_g = \frac{\pi^2 \hbar^2}{m a^2} - \frac{3}{2} V_0$$

$$E_e = \frac{5 \pi^2 \hbar^2}{2 m a^2} - 2 V_0$$

meiri lækkun á e-ástandinu vegna „snerti" aðdráttar mættis