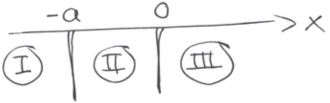
$V(x) = -\alpha\{\delta(x) + \delta(x+a)\}$, α og a eru jákvæðar stærðir

$\to$ tveir brunnar, annar í $x = -a$ og hinn í $x = 0$



erum bundnu ástöndin. Ef þau eru til gildir að $E < 0$

Lausnir Schrödingerjöfnunnar á þessum svæðum eru því, með $\kappa^2 = -\frac{2mE}{\hbar^2}$, $\kappa > 0$

I $\psi(x) = Ae^{+\kappa x} + Be^{-\kappa x}$

II $\psi(x) = Ce^{+\kappa x} + De^{-\kappa x}$

III $\psi(x) = Fe^{+\kappa x} + Ge^{-\kappa x}$

Bundnar lausnirnar normanlegar, því verður að gilda að

$B = 0$, $F = 0$

I $\psi(x) = Ae^{\kappa x}$

II $\psi(x) = Ce^{\kappa x} + De^{-\kappa x}$

III $\psi(x) = Ge^{-\kappa x}$

Bylgjufallið er samfellt í

$x = -a$

$$Ae^{-\kappa a} = Ce^{-\kappa a} + De^{+\kappa a}$$

$x = 0$

$$C + D = G$$

Bylgjufallið hefur brot í afleiðu í

$x = -a$

$$\psi'(-a^+) - \psi'(-a^-) = -\frac{2m\alpha}{\hbar^2}\psi(a)$$

$$C\kappa e^{-\kappa a} - D\kappa e^{+\kappa a} - A\kappa e^{-\kappa a} = -\frac{2m\alpha}{\hbar^2}Ae^{-\kappa a}$$

$x = 0$

$$-\kappa G - C\kappa + D\kappa = -\frac{2m\alpha}{\hbar^2}G$$

4 jöfnur, 4 óþekktar stærðir

$$Ae^{-ka} - Ce^{-ka} - De^{ka} = 0$$

$$C + D - G = 0$$

$$Ce^{-ka} - De^{+ka} - Ae^{-ka} = -\frac{2m\alpha}{\hbar^2 K} A e^{-ka} = -\beta A e^{-ka}$$

$$-G - C + D = -\frac{2m\alpha}{\hbar^2 K} G = -\beta G$$

þ.a. endurritað höfum við

$$Ae^{-ka} - Ce^{-ka} - De^{+ka} = 0$$

$$C + D - G = 0$$

$$Ce^{-ka} - De^{+ka} + A(\beta - 1)e^{-ka} = 0$$

$$-C + D + G(\beta - 1) = 0$$

$$\begin{pmatrix} e^{-\kappa a} & -e^{-\kappa a} & -e^{+\kappa a} & 0 \\ 0 & 1 & 1 & -1 \\ (\beta-1)e^{-\kappa a} & e^{-\kappa a} & -e^{+\kappa a} & 0 \\ 0 & -1 & 1 & (\beta-1) \end{pmatrix} \begin{pmatrix} A \\ C \\ D \\ G \end{pmatrix} = 0$$

Til þess að lausn sé til þarf ákveðan at fylkinu að vera 0

$$e^{-\kappa a}\left[-(\beta-1)e^{\kappa a} + e^{\kappa a} - (\beta-1)e^{-\kappa a} - e^{-\kappa a}\right]$$

$$-e^{\kappa a}\left[(\beta-1)e^{-\kappa a} - (\beta-1)^2 e^{-\kappa a}\right] + e^{-\kappa a}\left[-(\beta-1)e^{-\kappa a} - (\beta-1)^2 e^{-\kappa a}\right]$$

$$= 0$$

einföldum

$$\det M = -\beta^2 e^{-2\kappa a} + \beta^2 - 4\beta + 4 = 0 \qquad \beta = \frac{2m\alpha}{\hbar^2 \kappa}$$

$$\rightarrow e^{-2\kappa a} = \left(1 - \frac{2}{\beta}\right)^2 \qquad \beta = \frac{2m\alpha a}{\hbar^2 (\kappa a)}$$

$$e^{-\kappa a} = \pm\left(1 - \frac{2}{\beta}\right) \qquad = \frac{2ma^2 \left(\frac{\alpha}{a}\right)}{\hbar^2} \frac{1}{(\kappa a)}$$

$$= \left(\frac{\alpha}{E_1 a}\right) \frac{1}{(\kappa a)}$$

$$(*) \quad e^{-\kappa a} = \pm\left(1 - 2\left(\frac{E_1 a}{\alpha}\right)(\kappa a)\right)$$

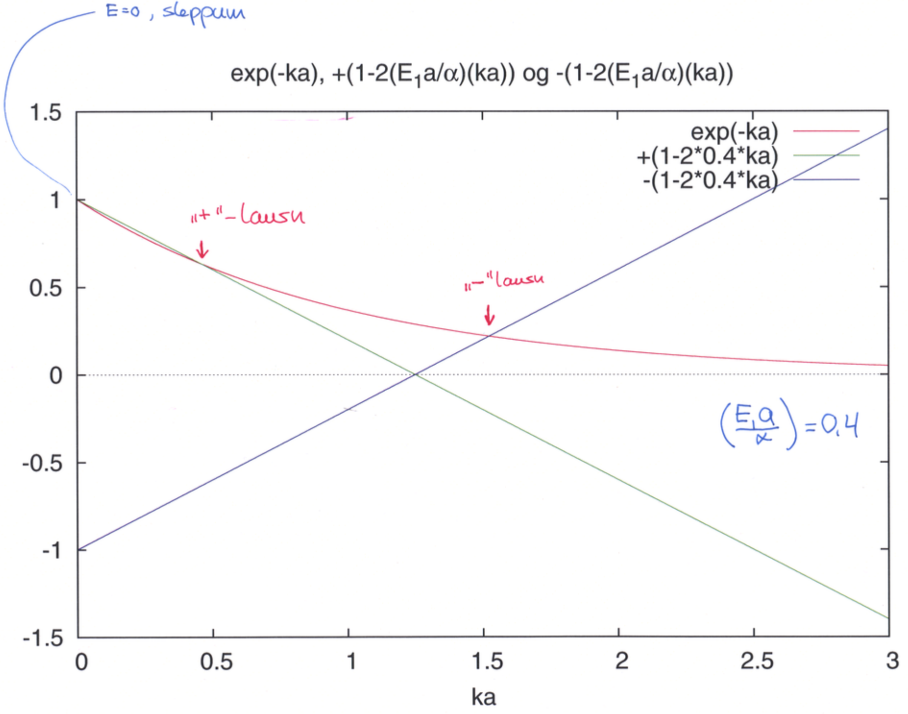
Könnum lausnir

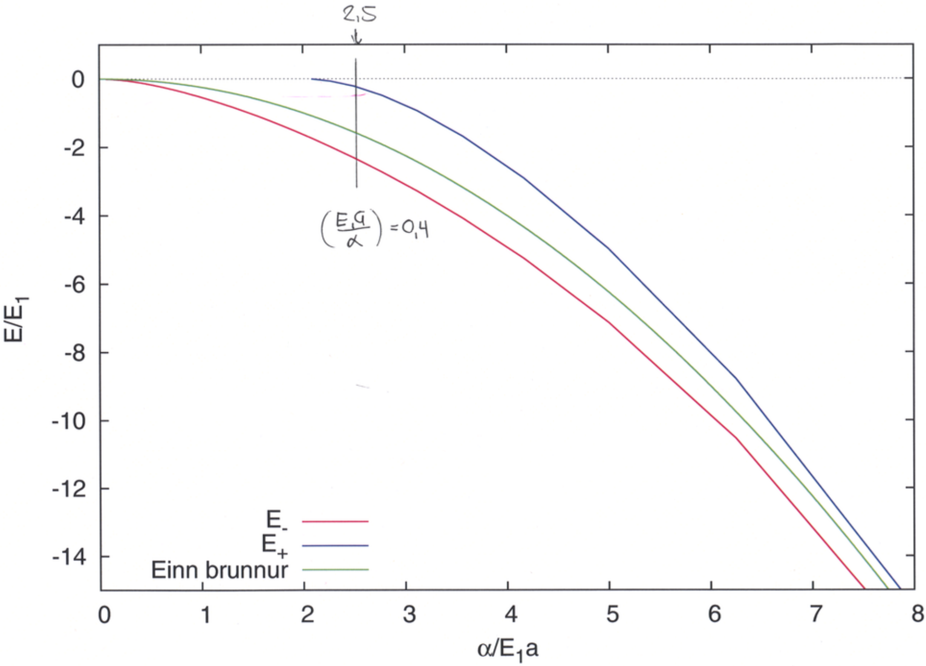
„$-$"-lausnin er til fyrir öll jákvæð gildi á $\left(\frac{E_1 a}{\alpha}\right)$

líðum á $e^{-\kappa a} \sim 1 - (\kappa a) + \frac{(\kappa a)^2}{2} + \cdots$

sýnir að „$+$" lausnin er til ef $2 \cdot \left(\frac{E_1 a}{\alpha}\right) < 1$

$$\rightarrow \frac{E_1 a}{\alpha} < \frac{1}{2} \quad \text{eða} \quad \boxed{\alpha > 2E_1 a}$$


E=0 , sleppum
exp(-ka), +(1-2(E1a/α)(ka)) og -(1-2(E1a/α)(ka))
exp(-ka)
+(1-2*0.4*ka)
-(1-2*0.4*ka)
"+"-løsn
"-"løsn
(E1a/α)=0.4
ka


2,5
$\left(\frac{E_1 a}{\alpha}\right) = 0,4$
0
-2
-4
-6
-8
-10
-12
-14
E/E$_1$
E_-
E_+
Einn brunnur
0
1
2
3
4
5
6
7
8
$\alpha/E_1 a$

Finnum bylgjuföllin

Höfum engar áhyggjur af normun. Útlit bylgjufalla sést vel án hennar. (Föllin eru normanleg hér).

Veljum því $G = 1$, þá eru jöfnurnar

$$\underline{Ae^{-ka} - Ce^{-ka} - De^{+ka} = 0}$$

$$\underline{C + D = 1}$$

$$Ce^{-ka} - De^{+ka} + A(\beta - 1)e^{-ka} = 0$$

$$\underline{-C + D = 1 - \beta}$$

fáum 3 jöfnur

Þar eru ekki allar óháðar

$$\begin{pmatrix} e^{-\kappa a} & -e^{-\kappa a} & -e^{\kappa a} \\ 0 & +1 & +1 \\ 0 & -1 & +1 \end{pmatrix} \cdot \begin{pmatrix} A \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1-\beta \end{pmatrix}$$

Reiknum þegar tvær lausnir eru til, t.d. þ. $\left(\frac{E_1 a}{\alpha}\right) = 0.4$

$$E_\pm = -E_1 \cdot (\kappa_\pm a)^2$$ tölulega lausn á (*) gefur

$(\kappa_- a) = 1.52266$ sjá mynd á bls 6

$(\kappa_+ a) = 0.464213$

$a\kappa_\pm$ eru núllstöðvar (*)

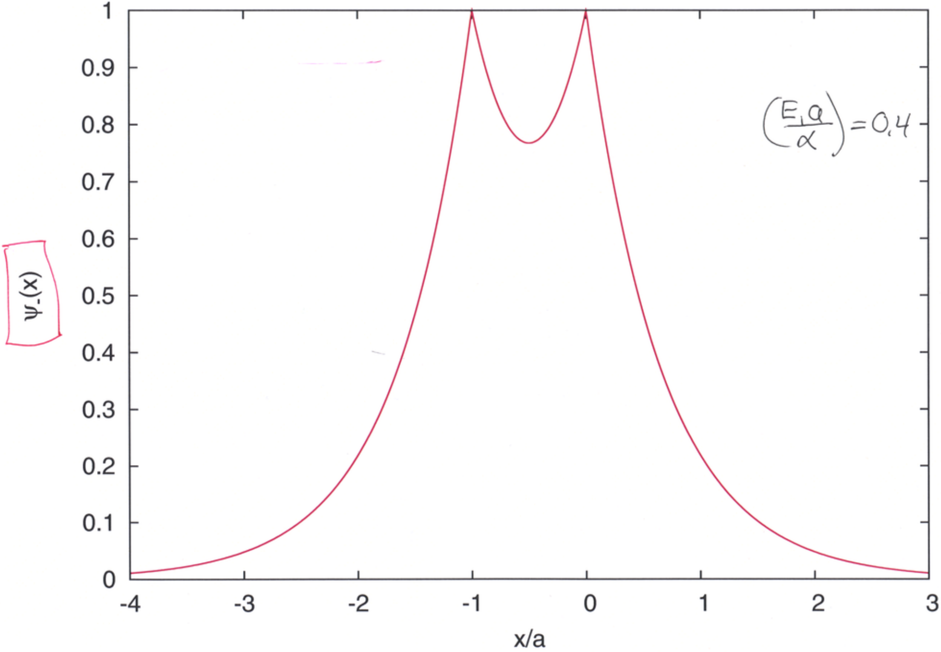
$$\beta_\pm = \left(\frac{\alpha}{E_1 a}\right) \frac{1}{(\kappa_\pm a)}$$

"–"-lausu

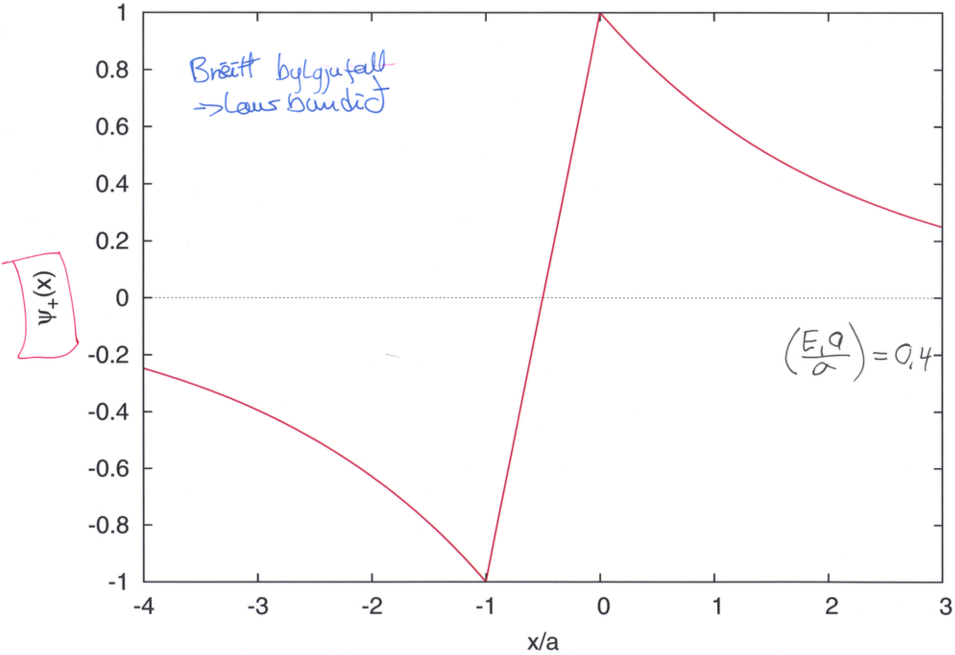
$$\left.\begin{aligned} e^{-\kappa a_-} &= 0.21813 \\ e^{\kappa a_-} &= 4.5844 \\ \beta_- &= 1.6419 \end{aligned}\right\} \longrightarrow \begin{aligned} A &= 4.5843 \\ C &= 0{,}82094 \\ D &= 0{,}17906 \end{aligned}$$

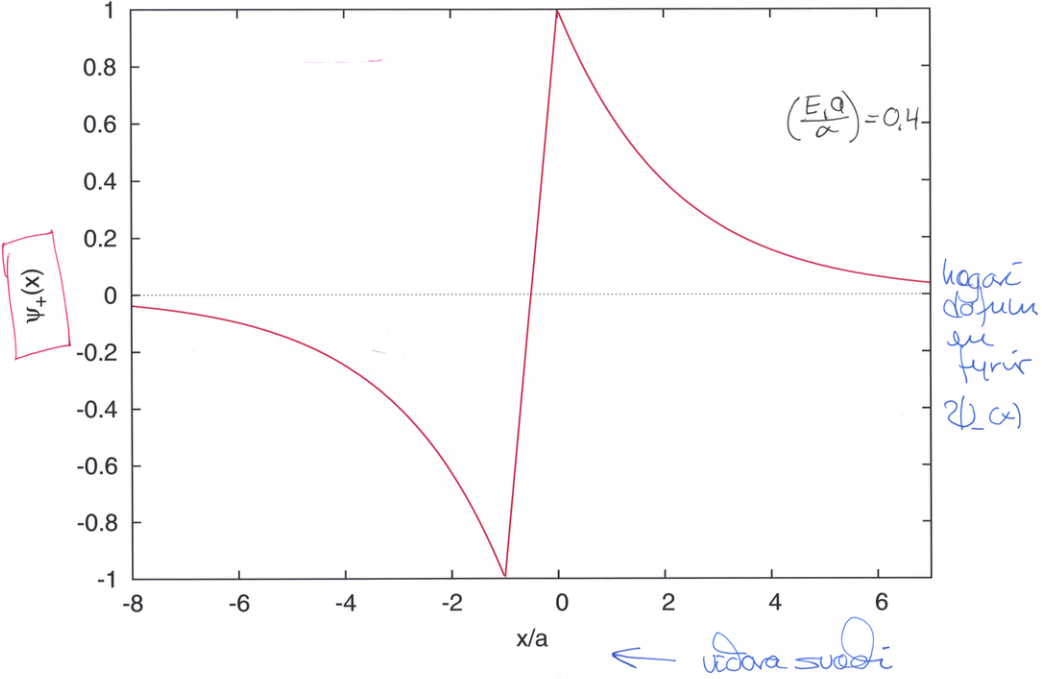
"+"-lausu

$$\left.\begin{aligned} e^{-\kappa_+ a} &= 0{,}62863 \\ e^{\kappa_+ a} &= 1.5908 \\ \beta_+ &= 5.3855 \end{aligned}\right\} \longrightarrow \begin{aligned} A &= -1{,}59079 \\ C &= 2.69275 \\ D &= -1{,}69275 \end{aligned}$$



ψ_(x)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
x/a
(E_1 a / α) = 0.4


Brett bylgjufall
-> laus bundið
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-4
-3
-2
-1
0
1
2
3
x/a
ψ+(x)
(E₁a/α) = 0,4



1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-8
-6
-4
-2
0
2
4
6
x/a
$\psi_+(x)$
$\left(\frac{E_1 a}{\alpha}\right) = 0.4$
hægri dofnun en fyrir $\psi_-(x)$
víðara svæði