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$$\Psi(x,t) = A e^{-\lambda|x|} e^{-i\omega t}$$

$\lambda > 0$
$A > 0$
$\omega > 0$

a) Norma

$$|A|^2 \int_{-\infty}^{\infty} dx\, \Psi^*(x,t)\, \Psi(x,t) = 2|A|^2 \int_0^{\infty} dx\, e^{-2\lambda x}$$

$$= 2|A|^2 \left\{ \frac{e^{-2\lambda x}}{-2\lambda} \Big|_0^{\infty} \right\} = 2|A|^2 \left\{ 0 + \frac{1}{2\lambda} \right\}$$

$$= |A^2| \frac{1}{\lambda} = 1, \quad \rightarrow \quad \underline{A = \sqrt{\lambda}}$$

Vídd λ er L^{-1}, Vídd Ψ er $L^{-1/2}$ sem er í samræmi við $A = \sqrt{\lambda}$

b) Reikna $\langle x \rangle$ og $\langle x^2 \rangle$

$|\Psi|^2$ er jafn stott $\rightarrow$ $x|\Psi|^2$ er odd stott,

$x^2|\Psi|^2$ er jafn stott

$\rightarrow$ $\underline{\langle x \rangle = 0}$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} dx \, x^2 |\Psi|^2 = 2\int_{0}^{\infty} dx \, x^2 e^{-2\lambda x} \lambda$$

$$= 2\lambda \left\{ -\frac{(2\lambda^2 x^2 + 2\lambda x + 1)e^{-2\lambda x}}{4\lambda^3} \Big|_0^\infty \right\}$$

$$= 2\lambda \left\{ 0 + \frac{1}{4\lambda^3} \right\} = \frac{1}{2\lambda^2}, \quad \underline{\langle x^2 \rangle = \frac{1}{2\lambda^2}}$$

$$\Delta x = \sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle} = \sqrt{\frac{1}{2\lambda^2} - 0} = \frac{1}{\sqrt{2}\,\lambda}$$

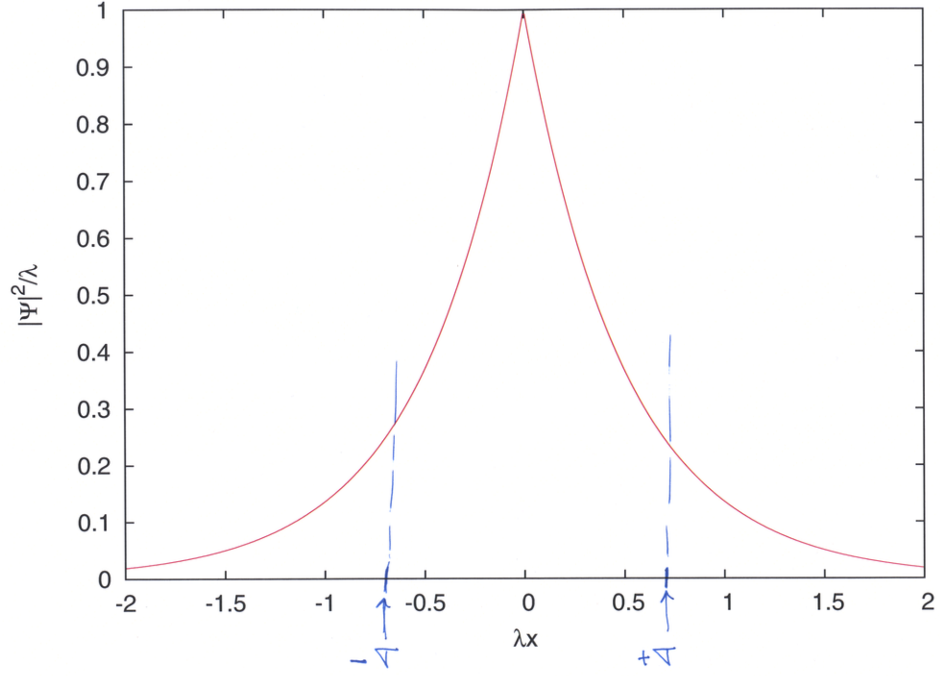
Teikna $|\Psi|^2$ og merkja punktana $(\langle x \rangle + \sigma_x)$ og $(\langle x \rangle - \sigma_x)$

$|\Psi|^2 = \lambda e^{-2\lambda|x|}$ pui er ~~[illegible]~~ ~~[illegible]~~

teikna vidderlausu Stordina

$$\frac{1}{\lambda}|\Psi|^2 = e^{-2\lambda|x|} = e^{-2|u|} \quad \text{v.s.} \quad u$$

$$u = \lambda x \qquad \sigma_x = \frac{1}{\sqrt{2}\,\lambda} \quad \to \quad \sigma_u = \frac{1}{\sqrt{2}}$$


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
$|\Psi|^2/\lambda$
λx
$-\Delta$
$+\Delta$

Líkindi þess að finna eindina utan $\pm\sigma$ (5)

$$2\int_{\sigma_x}^{\infty} dx\, |\Psi|^2 = 2\lambda\int_{\sigma_x}^{\infty} dx\, e^{-2\lambda x} = 2\int_{1/\sqrt{2}}^{\infty} du\, e^{-2u}$$

$$= 2\left\{ -\frac{e^{-2u}}{2}\Big|_{1/\sqrt{2}}^{\infty} \right\} = \exp\left\{-\frac{2}{\sqrt{2}}\right\} = e^{-\sqrt{2}}$$

$\sim 0{,}243$

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finna $d_t\langle p\rangle$

þar sem x er óháð t er

$$d_t\langle p\rangle = d_t\left[\int \Psi^*(-i\hbar\partial_x\Psi)\,dx\right]$$

$$= \int dx\left\{(\partial_t\Psi^*)(-i\hbar\partial_x\Psi) + \Psi^*(-i\hbar\partial_x\partial_t\Psi)\right\}$$

$$= \int dx\left\{\left(-\frac{H}{i\hbar}\Psi^*\right)(-i\hbar\partial_x\Psi) + \Psi^*\left(-i\hbar\partial_x\frac{H}{i\hbar}\Psi\right)\right\}$$

Munum að

$$H = -\frac{\hbar^2}{2m}\partial_x^2 + V(x)$$

$$d_t\langle p\rangle = \int dx\left\{-\frac{\hbar^2}{2m}\left(\partial_x^2\Psi^*\partial_x\Psi - \Psi^*\partial_x^3\Psi\right)\right\}$$

$$+\int dx\left\{V\Psi^*(\partial_x\Psi) - \Psi^*\left(\partial_x(V\Psi)\right)\right\}$$

$$= I_1 - \int dx\ \Psi^*(\partial_x V)\Psi = I_1 - \langle\partial_x V\rangle$$

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stadium

$$I_1 \sim \int dx\left\{\partial_x^2\Psi^*\partial_x\Psi - \Psi^*\partial_x^3\Psi\right\}$$

$$= \int dx\left\{-\partial_x\Psi^*\partial_x^2\Psi + \partial_x\Psi^*\partial_x^2\Psi\right\} + \left\{\partial_x\Psi^*\partial_x\Psi\Big|_{-\infty}^{\infty}\right\}$$

$$-\left\{\partial_x\Psi^*\partial_x\Psi\Big|_{-\infty}^{\infty}\right\} = 0$$