Hrif Zeemans

Ytra segulsvið hefur áhrif á hreyfingu rafeinda í vetnisatómi. Fyrir svið á tilraunastofu 0-20T eru áhrifin mest á hverfiþunga byggingu ástandanna. Í efni með virkan rafeindamassa miklu minni en m_e eru veruleg áhrif á alla brautarhreyfingu, líka r-þáttinn.

Línuleg áhrif $\bar{B}_{ext}$, (B^2-liðurinn sem meðal annars leiðir til Landau-stiga er sleppt hér)

$$H'_Z = -(\bar{\mu}_\ell + \bar{\mu}_s)\cdot \bar{B}_{ext}$$

$$\bar{\mu}_s = -\frac{e}{m}\bar{S} \qquad\qquad \bar{\mu}_\ell = -\frac{e}{2m}\bar{L}$$

"Áður voru áhrif innra sviðs (vegna hreyfing m.v. kjarnan)
sem valda braut-spuna víxlverkun.

Ef $B_{ext} \ll B_{int}$ $\rightarrow$ þá er Zeeman hrifin lítil leiðrétting ofan á fínuppbygginguna

$B_{ext} \sim B_{int}$ $\rightarrow$ 1. Stig leiðrétting á margföldum ástöndum í hlutrúmi

⋮

$B_{ext} \gg B_{int}$ $\rightarrow$ Zeeman hrifin eru grunnhrifin með fínbygginguna ofan á sem litla truflun

Veik Zeemanhrif

$B_{ext} \ll B_{int}$

Góðar skammtatölur

n, l, j, m_j en ekki m_l og m_s

Munum að orkurófið með fínuppbyggingunni var

$$E_{nj} = -R_y\left[1+\frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2}-\frac{3}{4}\right)\right]$$

óháð m_j, ástöndin eru margföld í m_j

Zeeman-truflunin mun eyða m_j-margfeldninni

$$E_z^1 = \langle nljm_j | H_z' | nljm_j \rangle$$

$$= \frac{e}{2m}\bar{B}_{ext}\cdot\langle \bar{L}+2\bar{S}\rangle$$

Hér er venja að skala orkuna á annan hátt, en

$$E_z^1 = \frac{\hbar}{2}\left(\frac{eB_{ext}}{m}\right)\frac{\bar{B}_{ext}}{B_{ext}}\cdot\frac{\langle \bar{L}+2\bar{S}\rangle}{\hbar}$$

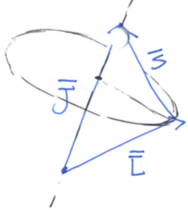
$$= \hbar\omega_c\cdot\hat{z}\cdot\frac{\langle \bar{L}+2\bar{S}\rangle}{\hbar}\cdot\frac{1}{2}$$

t.d.

Sígild tíðni hringhraðals

þurfum að reikna

$$\langle \bar{L} + 2\bar{S} \rangle$$



$$\bar{L} = \bar{J} - \bar{S}$$

$$\rightarrow \bar{L} + 2\bar{S} = \bar{J} + \bar{S}$$

Höfum engar góðar lýsingar á $\langle \bar{S} \rangle$, en

$$\bar{S}_{ave} = \frac{(\bar{S}\cdot\bar{J})}{J^2}\bar{J}$$

meðal ofanvarp $\bar{S}$ á $\bar{J}$

$\bar{S}\cdot\bar{J}$ fast líka frá $\bar{L} = \bar{J} - \bar{S}$

með

$$L^2 = J^2 + S^2 - 2\bar{J}\cdot\bar{S}$$

$$\rightarrow \bar{S}\cdot\bar{J} = \frac{1}{2}\left(J^2 + S^2 - L^2\right)$$

$$= \frac{\hbar^2}{2}\left\{ j(j+1) + s(s+1) - \ell(\ell+1) \right\}$$

og því

$$\langle \bar{L} + 2\bar{S} \rangle = \langle \bar{J} + \bar{S} \rangle$$

$$= \left\langle \left(1 + \frac{\bar{S}\cdot\bar{J}}{J^2} \right) \bar{J} \right\rangle$$

$$= \underbrace{\left\{ 1 + \frac{j(j+1) - l(l+1) + \frac{3}{4}}{2j(j+1)} \right\}}_{\text{Landé g-stuðull}} \langle \bar{J} \rangle$$

$$\rightarrow E_Z^{1} = \mu_B \, g_J \, B_{ext} \quad m_J$$

þegar við setjum $\bar{B}_{ext} = B_{ext}\hat{z}$

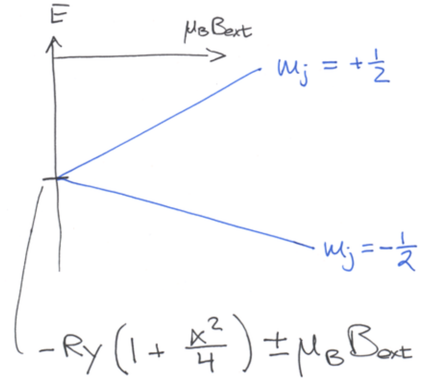
$$\mu_B = \frac{e\hbar}{2m} = 5.788 \cdot 10^{-5} \, eV/T$$

Bohr segulvægiseining

Grunnástandið $(n=1, l=0, j=\frac{1}{2})$

$$\rightarrow g_J = 2$$

Klofnar í tvö stig



Sterk Zeemanhvít

$$\underline{B_{ext} \gg B_{int}}$$

$$\bar{B}_{ext} = \hat{z}\, B_{ext}$$

Góðar skammtatølur

eru n, l, m_l og m_s

{ m_l og m_s eru miðaði við z-ått, ått $\bar{B}_{ext}$. m_j er partekki, ekki varðveitt vegna vagís á part }

$$H'_z = \frac{e}{2m} B_{ext}(L_z + 2S_z)$$

Án finbyggingar fast

$$E_{n m_l m_s} = -\frac{R_y}{n^2} + \mu_B B_{ext}\{m_l + 2m_s\}$$

Við finbyggingu

Sama E^1_r og áður

fyrir spunabrent fast

$$\langle \bar{S}\cdot\bar{L}\rangle = \underbrace{\langle S_x\rangle\langle L_x\rangle + \langle S_y\rangle\langle L_y\rangle}_{=0} + \langle S_z\rangle\langle L_z\rangle$$

$$= \hbar^2 m_l m_s$$

$$\rightarrow \boxed{E^1_{fs} = \frac{R_y}{n^3}\alpha^2\left\{\frac{3}{4n} - \underbrace{\left[\frac{l(l+1) - m_l m_s}{l(l+\frac{1}{2})(l+1)}\right]}_{=1 \text{ ef } l=0}\right\}}$$

Þegar Zeeman-truflunin
er að svipuðum styrk
og fs-truflunin þarf
að vinna með truflun

$$H' = H'_Z + H'_{fs}$$

á ótrufluða H-atóminu

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fyrir $n=2$ þurfum við að vinna
með 8 ástönd

$l = 0 \quad (\rightarrow j = 1/2)$

$l = 1 \quad \left(j = 1/2 \text{ og } \frac{3}{2} \right)$

$$
\begin{array}{ll}
 & \quad j\ m_j \qquad\qquad l\ m_l \quad s\ m_s \\
l=0 & \left\{ \begin{array}{l}
|1\rangle = |\tfrac{1}{2},\tfrac{1}{2}\rangle = |0,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle \\
|2\rangle = |\tfrac{1}{2},-\tfrac{1}{2}\rangle = |0,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle
\end{array} \right.
\end{array}
$$

$$
l=1 \left\{ \begin{array}{l}
|3\rangle = |\tfrac{3}{2},\tfrac{3}{2}\rangle = |1,1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle \\
|4\rangle = |\tfrac{3}{2},-\tfrac{3}{2}\rangle = |1,-1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle \\
|5\rangle = |\tfrac{3}{2},\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}|1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}|1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle \\
|6\rangle = |\tfrac{1}{2},\tfrac{1}{2}\rangle = -\sqrt{\tfrac{1}{3}}|1,0\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}|1,1\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle \\
|7\rangle = |\tfrac{3}{2},-\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}|1,-1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}|1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle \\
|8\rangle = |\tfrac{1}{2},-\tfrac{1}{2}\rangle = -\sqrt{\tfrac{2}{3}}|1,-1\rangle|\tfrac{1}{2},\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}|1,0\rangle|\tfrac{1}{2},-\tfrac{1}{2}\rangle
\end{array} \right.
$$

Clebsch-Gordan stuðlar

Í þessu hlutrúmi eru fylkjastök H'_{fs} öll á hornalínuformi

Ef $\gamma = \left(\frac{\alpha}{8}\right)^2 R_y$

og $\beta = \mu_B B_{ext}$

þá fæst fyrir

$-W$

$$
\begin{pmatrix}
5\gamma-\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 5\gamma+\beta & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma-2\beta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma+2\beta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma-\frac{2}{3}\beta & \frac{\sqrt{2}}{3}\beta & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\sqrt{2}}{3}\beta & 5\gamma-\frac{1}{3}\beta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \gamma+\frac{2}{3}\beta & \frac{\sqrt{2}}{3}\beta \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{3}\beta & 5\gamma+\frac{\beta}{3}
\end{pmatrix}
$$

Lausnir verða

$\epsilon_1 = E_2 - 5\gamma + \beta$

$\epsilon_2 = E_2 - 5\gamma - \beta$

$\epsilon_3 = E_2 - \gamma + 2\beta$

$\epsilon_4 = E_2 - \gamma - 2\beta$

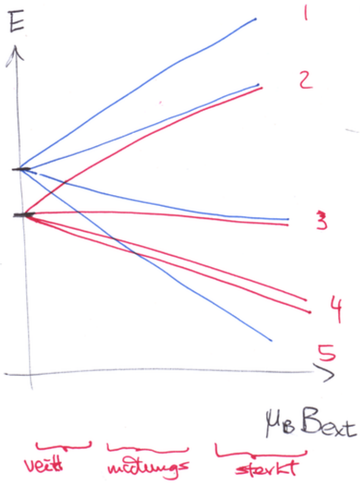
$\epsilon_5 = E_2 - 3\gamma + \beta/2 + \sqrt{4\gamma^2 + \frac{2\gamma\beta}{3} + \frac{\beta^2}{4}}$

$\epsilon_6 = E_2 - 3\gamma + \beta/2 - \sqrt{4\gamma^2 + \frac{2\gamma\beta}{3} + \frac{\beta^2}{4}}$

$\epsilon_7 = E_2 - 3\gamma - \beta/2 + \sqrt{4\gamma^2 - \frac{2\gamma\beta}{3} + \frac{\beta^2}{4}}$

$\epsilon_8 = E_2 - 3\gamma - \beta/2 - \sqrt{4\gamma^2 - \frac{2\gamma\beta}{3} + \frac{\beta^2}{4}}$

Þegar lausnin fyrir lágt og miðlungs B_{ext} er skoðuð koma í ljós 2 4-klofin ástönd



sterkt svið

$$E_{n m_l m_s} = -\frac{R_y}{n^2} + \mu_B B_{ext}\,(m_l + 2m_s)$$

$n = 2\ ,\quad l = 0, 1$

$$\left.\begin{aligned} m_l &= -1, 0, +1 \\ 2m_s &= -1, +1 \end{aligned}\right\} \; m_l + 2m_s = \begin{cases} -2 \\ -1 \\ 0 \\ +1 \\ +2 \end{cases}$$

fimmföld klofnun

Ofurfíngerð

Róteindin og rafeindin eru með segulvægi

$$\bar{\mu}_e = -\frac{e}{m_e}\bar{s}_e$$

$$\bar{\mu}_p = \frac{g_p e}{2m_p}\bar{s}_p$$

$$g_p \approx 5.59$$

Róteindin er samsett eind

Segulvægi veldur líka segulsviði

$$\bar{B} = \frac{\mu_0}{4\pi r^3}\left\{3(\bar{\mu}\cdot\hat{r})\hat{r} - \bar{\mu}\right\} + \underbrace{\frac{2\mu_0}{3}\bar{\mu}\,\delta^3(\bar{r})}$$

markgildi fyrir punkteind

Segulvægi róteindar framkallar segulsvið sem hefur áhrif á segulvægi rafeindar (ekki spuna-brautar víxlverkun)

Segulvíxlverkun milli tveggja einda með segulvægi

beint spunavíxlverkun

$$H'_{hf} = \frac{\mu_0 g_p e^2}{8\pi m_e m_p}\left\{\frac{3(\bar{S}_p\cdot\hat{r})(\bar{S}_e\cdot\hat{r}) - \bar{S}_p\cdot\bar{S}_e}{r^3}\right\} + \frac{\mu_0 g_p e^2}{3 m_p m_e}\bar{S}_p\cdot\bar{S}_e\,\delta^3(\bar{r})$$

$$\rightarrow E^1_{hf} = \frac{\mu_0 g_p e^2}{8\pi m_p m_e}\left\langle\frac{3(\bar{S}_p\cdot\hat{r})(\bar{S}_e\cdot\hat{r}) - \bar{S}_p\cdot\bar{S}_e}{r^3}\right\rangle$$

$$+ \frac{\mu_0 g_p e^2}{3 m_p m_e}\langle\bar{S}_p\cdot\bar{S}_e\rangle\,|\psi(0)|^2$$

þessi liður hverfur vegna kúlusamhverfu

Skoðum grunnástand vetnis

$$|\psi_{100}(0)|^2 = \frac{1}{(\pi a^3)}\quad,\quad l=0$$

$$\rightarrow E'_{hf} = \frac{\mu_0 g_p e^2}{3\pi m_p m_e a^3} \langle \bar{S}_p \cdot S_e \rangle$$

spunarnir tengjast saman,
víxlverka → heildarspuni
varðveitist $\bar{S} = \bar{S}_e + \bar{S}_p$

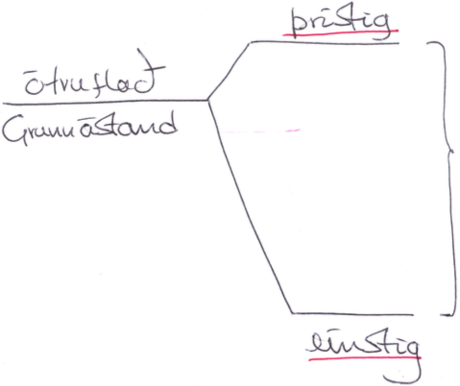
$$\bar{S}_p \cdot \bar{S}_e = \frac{1}{2}\left(S^2 - S_e^2 - S_p^2\right)$$

Báðar fermíeindir → $S_e^2 = S_p^2 = \frac{3}{4}\hbar^2$

einstig : $s = 0$ $\quad S^2 = 0$

þrístig : $s = 1$ $\quad S^2 = 2\hbar^2$

$$\hookrightarrow E'_{hf} = \frac{4 g_p \hbar^4}{3 m_p m_e^2 c^2 a^4} \begin{cases} +\frac{1}{4} & \underline{\text{þrístig}} \\ -\frac{3}{4} & \underline{\text{einstig}} \end{cases}$$



$$\Delta E = \frac{4 g_p \hbar^4}{3 m_p m_e^2 c^2 a^4} \simeq 5.88 \cdot 10^{-6}\,\mathrm{eV}$$

$$\nu = \frac{\Delta E}{h} \approx 1420\ \mathrm{MHz}$$

Tíðni ljóseindar milli orkustiganna í 1. stigs truflun

$\frac{c}{\nu} = 21\ \mathrm{cm}$ örbylgja