9.4 Kerfinu er lýst með ①

$$i\hbar\, d_t \bar{C}(t) = \mathbb{H}'\bar{C}(t) \qquad \mathbb{H}' = \begin{pmatrix} H'_{aa} & H'_{ab}e^{-i\omega_0 t} \\ H'_{ba}e^{i\omega_0 t} & H'_{bb} \end{pmatrix}$$

eða með

$$\bar{C}(t) = \bar{C}(0) + \frac{1}{i\hbar}\int_0^t ds\, \mathbb{H}'(s)\bar{C}(s)$$

a)

Gerum ráð fyrir því að $C_a(0)=1$, $C_b(0)$, $\bar{C}(0) = \begin{pmatrix}1\\0\end{pmatrix}$

Í fyrirlestri var 1. stigs lausn leidd út:

$$\bar{C}^{(1)}(t) = \bar{C}(0) + \frac{1}{i\hbar}\int_0^t ds\, \mathbb{H}'(s)\bar{C}(0)$$

$$= \left[1 + \frac{1}{i\hbar}\int_0^t ds\, \mathbb{H}'(s)\right]\bar{C}(0)$$

$$\begin{pmatrix} C_a(t) \\ C_b(t) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{i\hbar}\int_0^t ds \begin{pmatrix} H'_{aa}(s) \\ H'_{ba}(s)\, e^{i\omega_0 s} \end{pmatrix}$$

άρα

$$C_a(t) = 1 + \frac{1}{i\hbar}\int_0^t ds\, H'_{aa}(s)$$

$$C_b(t) = \frac{1}{i\hbar}\int_0^t ds\, H'_{ba}(s)\, e^{i\omega_0 s}$$

$$|C_a|^2 = \left[1 - \frac{1}{i\hbar}\int_0^t ds\, H'_{aa}(s)\right]\left[1 + \frac{1}{i\hbar}\int_0^t dz\, H'_{aa}(z)\right]$$

$$= 1 + \left[\frac{1}{\hbar}\int_0^t ds\, H'_{aa}(s)\right]^2 = 1 + o\left((H')^2\right)$$

$$|C_b|^2 = o\left((H')^2\right)$$

$$\longrightarrow \quad |C_a|^2 + |C_b|^2 = 1 \quad + \; o\left((H')^2\right)$$

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b) skoðum

$$d_a(t) \equiv U_a(t)\, C_a(t)$$

$$d_b(t) \equiv U_b(t)\, C_b(t)$$

þar sem

$$U_a(t) = \exp\left\{-\frac{1}{i\hbar}\int_0^t ds\, H'_{aa}(s)\right\}$$

$$U_b(t) = \exp\left\{-\frac{1}{i\hbar}\int_0^t ds\, H'_{bb}(s)\right\}$$

tíma þróun vegna H'_{aa} og H'_{bb}, hér er henni haldið aðgreindri frá tímaþróun vegna H'_{ab} og H'_{ba}

$$\dot{d}_a(t) = \dot{U}_a(t)\, C_a(t) + U_a(t)\, \dot{C}_a(t)$$

$$= U_a(t)\left\{ -\frac{1}{i\hbar} H'_{aa}(t)\, C_a(t) + \dot{C}_a(t) \right]$$

og $\dot{C}_a(t) = \frac{1}{i\hbar}\left(H'_{aa}(t)\, C_a(t) + H'_{ab}(t)\, e^{-i\omega_0 t}\, C_b(t) \right)$

$$\rightarrow \dot{d}_a(t) = U_a(t)\left\{ H'_{ab}(t)\, e^{-i\omega_0 t}\, C_b(t) \right]\frac{1}{i\hbar}$$

$$= U_a(t)\, H'_{ab}(t)\, e^{-i\omega_0 t}\, U_b^*(t)\, d_b(t)\, \frac{1}{i\hbar}$$

$$= U_a(t)\, H'_{ab}(t)\, U_b^*(t)\, e^{-i\omega_0 t}\, d_b(t)\, \frac{1}{i\hbar}$$

$$= \exp\left\{ -\frac{1}{i\hbar}\int_0^t ds \left(H'_{aa}(s) - H'_{bb}(s) \right) \right\} e^{-i\omega_0 t}\, H'_{ab}\, d_b(t)\, \frac{1}{i\hbar} \quad (**)$$

$$\dot{d}_b(t) = \dot{U}_b(t) C_b(t) + U_b(t) \dot{C}_b(t)$$

$$= U_b(t) \left\{ -\frac{1}{i\hbar} H'_{bb}(t) C_b(t) + \dot{C}_b(t) \right\}$$

$$\dot{C}_b(t) = \frac{1}{i\hbar} \left(H'_{ba} e^{i\omega_0 t} C_a(t) + H'_{bb} C_b \right)$$

$$\rightarrow \dot{d}_a(t) = U_b(t) \left\{ H'_{ba} e^{i\omega_0 t} C_a(t) \right\} \frac{1}{i\hbar}$$

$$= U_b(t) H'_{ba} U_a^*(t) e^{i\omega_0 t} d_a(t) \frac{1}{i\hbar}$$

$$= \exp\left\{ -\frac{1}{i\hbar} \int_0^t ds \left(H'_{bb}(s) - H'_{aa}(s) \right) \right\} e^{-i\omega_0 t} H'_{ba} d_a(t) \frac{1}{i\hbar} \quad (*)$$

c) Lausn i.f. d_a og d_b

<u>Upphafsstilyrði</u> $C_a(0) = 1$, $C_b(0) = 0$

$U_a(0) = 1$ og $U_b(0) = 1 \rightarrow d_a(0) = 1$, $d_b(0) = 0$

líka núllta stigs lausn
(engin víxlverkun a og b)

<u>Fyrsta stigs nálgun</u>

nota í (**)

$$\rightarrow \dot{d}_a(t) = 0 \rightarrow d_a(t) = 1 \rightarrow U_a(t)\, C_a(t) = 1$$

$$\rightarrow C_a(t) = U_a^*(t) = \exp\left\{ \frac{1}{i\hbar} \int_0^t ds\, H'_{aa}(s) \right\}$$

Notum (*)

$$\dot{d}_b(t) = U_b(t)\, H'_{ba}\, U_a^*(t)\, e^{i\omega_0 t} \cdot 1 \cdot \frac{1}{i\hbar}$$

$$\to d_b(t) = \frac{1}{i\hbar} \int_0^t ds\, U_b(s)\, H'_{ba}\, U_a^*(s)\, e^{i\omega_0 s}$$

$$\to C_b(t) = \frac{1}{i\hbar} U_b^*(t) \int_0^t ds\, U_b(s)\, H'_{ba}\, U_a^*(s)\, e^{i\omega_0 s}$$

Þetta þarf að gera að 1. stig truflun í H'_{ab} $\qquad U_a \to 1$, $U_b \to 1$

$$\to C_b(t) \simeq \frac{1}{i\hbar} \int_0^t ds\, H'_{ba}(s)\, e^{i\omega_0 s}$$

til þess að fá réttan samanburð við lausn í bókinni