(6.1)

Uendeleg djup brønn med eini eind

$$\psi_n^0(x) = \sqrt{\frac{2}{a}}\,\sin\left(n\pi\frac{x}{a}\right)$$

$$E_n^0 = E_1 \cdot n^2\,, \quad E_1 = \frac{\pi^2\hbar^2}{2ma^2}$$

Kor dei er forstyrra med $H' = \alpha\delta\left(x-\frac{a}{2}\right)$

a) Fyrste stigs forstyrring å orden

$$E_n = E_n^0 + \langle n|V|n\rangle$$

$$\langle n|V|n\rangle = 2\frac{\alpha}{a}\int_0^a dx\,\sin^2\left(n\pi\frac{x}{a}\right)\delta\left(x-\frac{a}{2}\right)$$

②

$$\langle n|V|n\rangle = 2\frac{\alpha}{a}\sin^2\left(\frac{n\pi}{2}\right) = \begin{cases} 0 & \text{ef } n = 2, 4, 6, \dots \\ \frac{2\alpha}{a} & \text{ef } n = 1, 3, 5, \dots \end{cases}$$

ástöndin með jöfn n eru með núllstöð í $x = \frac{a}{2}$, miðjum brunni, eru andsamhverf um þennan punkt.

b) Finna lægstu liðina í leiðréttingunni á grunnástandinu

$$|\psi_1^1\rangle = \sum_{m=2}^{\infty} \frac{\langle \psi_m | H' | \psi_1^0\rangle}{E_1^0 - E_m^0} |\psi_m^0\rangle$$

Lið m=1 er sleppt, þurfum enn að sjá hvaða m er í raun summað yfir þurfum að reikna

$$\langle \psi_m | H' | \psi_1^0 \rangle = 2\frac{\alpha}{a} \int_0^a dx \, \text{Sin}\left(\pi \frac{x}{a}\right) \delta\left(x - \frac{a}{2}\right) \text{Sin}\left(m\pi \frac{x}{a}\right)$$

$$= 2\frac{\alpha}{a} \, \text{Sin}\left(\frac{\pi}{2}\right) \text{Sin}\left(m\frac{\pi}{2}\right) = \begin{cases} 2\frac{\alpha}{a} \text{Sin}\left(\frac{m\pi}{2}\right) & \text{ef } m = 1,3,5,7 \\ 0 & \text{ef } m = 2,4,6,\ldots \end{cases}$$

Ennfremur er

$$E_1^0 - E_m^0 = E_1 - E_1 m^2 = E_1 (1 - m^2)$$

$$\rightarrow |\psi_1^1\rangle = \sum_{m=3,5,7\ldots}^{\infty} \left\{ \frac{2\left(\frac{\alpha}{a}\right)}{E_1} \right\} \frac{\text{Sin}\left(\frac{m\pi}{2}\right)}{1 - m^2}$$

Fyrstu 3 liðirnir eru

$$|\psi_1^1\rangle \simeq \frac{2\left(\frac{\alpha}{a}\right)}{E_1}\left\{\frac{-|\psi_3^0\rangle}{1-9}+\frac{|\psi_5^0\rangle}{1-25}+\frac{-|\psi_7^0\rangle}{1-49}\right\}$$

$$=\frac{2\left(\frac{\alpha}{a}\right)}{E_1}\left\{\frac{|\psi_3^0\rangle}{8}-\frac{|\psi_5^0\rangle}{24}+\frac{|\psi_7^0\rangle}{48}\right\}$$

$$\langle x|\psi_1^1\rangle \simeq \frac{2\left(\frac{\alpha}{a}\right)}{E_1}\sqrt{\frac{2}{a}}\left\{\frac{\sin\left(\frac{3\pi x}{a}\right)}{8}-\frac{\sin\left(\frac{5\pi x}{a}\right)}{24}+\frac{\sin\left(\frac{7\pi x}{a}\right)}{48}\right\}$$

Truflunin er spegilsamhverf um punktinn $x=\frac{a}{2}$ → aðeins bylgjuföll sem eru með sömu samhverfu eru liðuð saman í trufluðu bylgjufallið

(6.6) Tvö „góð" ótrufluð ástönd eru

$|\psi_a^0\rangle$ og $|\psi_b^0\rangle$ eru margföld

$$|\psi^0_\pm\rangle = \alpha_\pm |\psi^0_a\rangle + \beta_\pm |\psi^0_b\rangle \quad \Big| \quad \langle\psi^0_a|\psi^0_c\rangle = \delta_{ac} \text{ með } c = a, b$$

þar sem $\alpha_\pm$ og $\beta_\pm$ eru ákvörðuð með jöfnu (6.22)

a) Sýna að $\langle\psi^0_+|\psi^0_-\rangle = 0$

ákvörðunarjöfnurnar eru því

$$\left.\begin{aligned} \alpha_\pm W_{aa} + \beta_\pm W_{ab} &= \alpha_\pm E^1_\pm \\ \alpha_\pm W_{ba} + \beta_\pm W_{bb} &= \beta_\pm E^1_\pm \end{aligned}\right\} (*)$$

$$\begin{aligned} &\langle\psi^0_+|\psi^0_-\rangle \\ &= \left\{\alpha^*_+ \langle\psi^0_a| + \beta^*_+ \langle\psi^0_b|\right\} \\ &\quad \cdot \left\{\alpha_- |\psi^0_a\rangle + \beta_- |\psi^0_b\rangle\right\} \\ &= \alpha^*_+ \alpha_- + \beta^*_+ \beta_- \end{aligned}$$

Notum (*) t.p.a. unnita $\beta_\pm$ i $\alpha_\pm$

$$\beta_\pm W_{ab} = \alpha_\pm E'_\pm - \alpha_\pm W_{aa} \quad \rightarrow \quad \beta_\pm = \frac{\alpha_\pm (E'_\pm - W_{aa})}{W_{ab}}$$

$$\rightarrow \beta_+^* \beta_- = \frac{\alpha_+^* \alpha_-}{W_{ab}^* W_{ab}} (E'_+ - W_{aa}^*)(E'_- - W_{aa}), \quad W_{aa}^* = W_{aa}$$

þá fæst

$$\langle \psi_+^0 | \psi_-^0 \rangle = \frac{\alpha_+^* \alpha_-}{|W_{ab}|^2} \left[|W_{ab}|^2 + (E'_+ - W_{aa})(E'_- - W_{aa}) \right] \qquad (**)$$

$$= \frac{\alpha_+^* \alpha_-}{|W_{ab}|^2} \left[E'_+ E'_- - W_{aa}(E'_+ + E'_-) + |W_{ab}|^2 + W_{ab}^2 \right]$$

en samkvæmt (6.27)

$$E'_+ + E'_- = \frac{W_{aa} + W_{bb}}{2} \cdot 2$$

og $$E_+^1 E_-^1 = \frac{1}{4}\left\{ (W_{aa}+W_{bb})^2 - (W_{aa}-W_{bb})^2 - 4|W_{ab}|^2 \right\}$$

$$\rightarrow \langle \Phi_+^0 | \Phi_-^0 \rangle = \frac{\alpha_+^* \alpha_-}{|W_{ab}|^2} \left[\frac{1}{4}(W_{aa}+W_{bb})^2 - \frac{1}{4}(W_{aa}-W_{bb})^2 - \cancel{|W_{ab}|^2} \right.$$

$$\left. - W_{aa}^2 - W_{aa}W_{bb} + \cancel{|W_{ab}|^2} + W_{aa}^2 \right]$$

$$= \frac{\alpha_+^* \alpha_-}{|W_{ab}|^2} \left[\cancel{W_{aa}W_{bb}} - \cancel{W_{aa}^2} - \cancel{W_{aa}W_{bb}} + \cancel{W_{aa}^2} \right]$$

$$= 0$$

b) Sýna að $\langle \psi_+^0 | H' | \psi_-^0 \rangle = 0$

$$\langle \psi_+^0 | H' | \psi_-^0 \rangle = \left\{ \alpha_+^* \langle \psi_a^0 | + \beta_+^* \langle \psi_b^0 | \right\} H' \left\{ \alpha_- | \psi_a^0 \rangle + \beta_- | \psi_b^0 \rangle \right\}$$

$$= \alpha_+^* \alpha_- \langle \psi_a^0 | H' | \psi_a^0 \rangle + \alpha_+^* \beta_- \langle \psi_a^0 | H' | \psi_b^0 \rangle$$

$$+ \beta_+^* \alpha_- \langle \psi_b^0 | H' | \psi_a^0 \rangle + \beta_+^* \beta_- \langle \psi_b^0 | H' | \psi_b^0 \rangle$$

$$= \alpha_+^* \alpha_- W_{aa} + \alpha_+^* \beta_- W_{ab} + \beta_+^* \alpha_- W_{ba} + \beta_+^* \beta_- W_{bb}$$

$$= \alpha_+^* \alpha_- \left[W_{aa} + (E_-^1 - W_{aa}) + \frac{(E_+^1 - W_{aa}) W_{ba}}{W_{ab}^*} + \frac{(E_+^1 - W_{aa})}{W_{ab}^*} \frac{(E_-^1 - W_{aa})}{W_{ab}} W_{bb} \right]$$

$$= \alpha_+^* \alpha_- \left[W_{aa} + (E_-' - W_{aa}) + (E_+' - W_{aa}) + \frac{(E_+' - W_{aa})(E_-' - W_{aa})}{|W_{ab}|^2} W_{bb} \right]$$

från (**) såg att $(E_+' - W_{aa})(E_-' - W_{aa}) = -|W_{ab}|^2$

$$\Rightarrow \langle \psi_+^0 | H' | \psi_-^0 \rangle = \alpha_+^* \alpha_- \left[W_{aa} + (E_-' - W_{aa}) + (E_+' - W_{aa}) - W_{bb} \right]$$

$$= \alpha_+^* \alpha_- \left[E_-' + E_+' - W_{aa} - W_{bb} \right] = 0$$

$\Uparrow$

samtidigt (6.27)

c) Σύγκα σε

$$\langle \psi^0_\pm | H' | \psi^0_\pm \rangle = E^1_\pm$$

$$\langle \psi^0_\pm | H' | \psi^0_\pm \rangle = \left\{ \alpha^*_\pm \langle \psi^0_a | + \beta^*_\pm \langle \psi^0_b | \right\} H' \left\{ \alpha_\pm | \psi^0_a \rangle + \beta_\pm | \psi^0_b \rangle \right\}$$

$$= |\alpha_\pm|^2 W_{aa} + \alpha^*_\pm \beta_\pm W_{ab} + \beta^*_\pm \alpha_\pm W_{ba} + |\beta_\pm|^2 W_{bb}$$

$$= |\alpha_\pm|^2 \left\{ W_{aa} + \frac{(E^1_\pm - W_{aa})}{W_{ab}} W_{ab} \right\} + |\beta_\pm|^2 \left\{ W_{bb} + \frac{(E^1_\pm - W_{bb})}{W_{ba}} W_{ba} \right\}$$

$$= |\alpha_\pm|^2 \left\{ E^1_\pm \right\} + |\beta_\pm|^2 \left\{ E^1_\pm \right\} = \underline{E^1_\pm}$$

$$= \left\{ |\alpha_\pm|^2 + |\beta_\pm|^2 \right\} E^1_\pm = 1 \cdot E^1_\pm$$