1D-hreintóna sveifill

$$E_n = \hbar\omega\left(n+\tfrac{1}{2}\right), \quad n = 0, 1, 2, \dots$$

$$\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}\, a}} H_n\left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}$$

$$a = \sqrt{\frac{\hbar}{m\omega}}$$ náttúrulegur lengdarskali fleygbogamættisins

Kerfið er truflað með $H' = \alpha\delta(x)$

Skoðum 1. stigs áhrifin á orkurófið.

Einföld orkustig

$$\rightarrow E_n' = E_n + \langle n|V|n\rangle$$ er nýja orkurófið fyrir truflaða kerfið

$$\langle n|V|n\rangle = \alpha \int_{-\infty}^{\infty} dx\, \Phi_n^*(x)\, \delta(x)\, \Phi_n(x) = \alpha\, |\Phi_n(0)|^2$$

$$= \alpha \frac{1}{2^n n! \sqrt{\pi}\, a} \left\{ H_n(0) \right\}^2$$

$$H_n(0) = \begin{cases} 0 & \text{fyrir oddatölu } n \\ \dfrac{(-1)^{n/2}\, n!}{(\frac{1}{2}n)!} & \text{fyrir jöfnu } n \end{cases}$$

eða

$$H_n(0) = \frac{2^n \sqrt{\pi}}{\Gamma\left(\frac{1-n}{2}\right)}$$

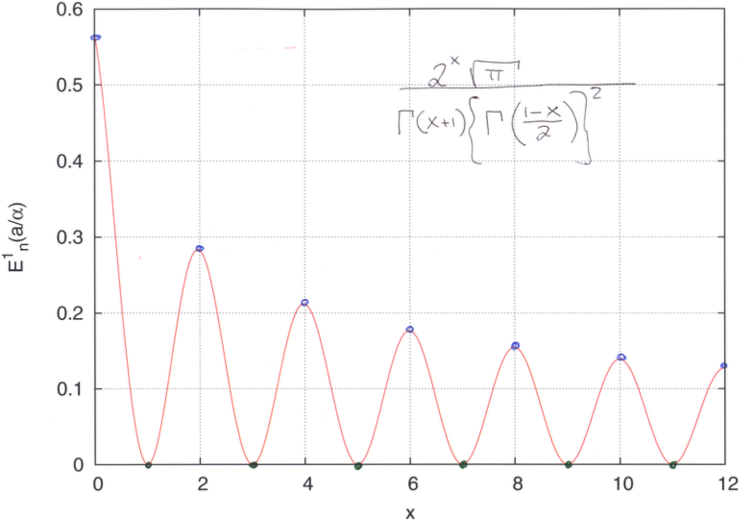
$$\langle n|V|n\rangle = \left(\frac{\alpha}{a}\right) \frac{2^n\sqrt{\pi}}{n!\left\{\Gamma\left(\frac{1-n}{2}\right)\right\}^2}$$

stærðfræðileg vídd
orku

$\Gamma(-n) \to \pm\infty$ and $\Gamma(0) \to \pm\infty$ fyrir $n \geq 0$

$\to$ ástönd með oddatölu n hreyfast ekki
and samhvert ástönd með núllstöð í 0

Ástönd með jöfnu n klofnast upp



$\frac{2^x \sqrt{\pi}}{\Gamma(x+1)\left\{\Gamma\left(\frac{1-x}{2}\right)\right\}^2}$
$E^1_n(a/\alpha)$
0
0.1
0.2
0.3
0.4
0.5
0.6
0
2
4
6
8
10
12
x

Tvær bósendir víxlverkast veikt með snertivíxlverkun

$$V(x_1 x_2) = \alpha\,\delta(x_1 - x_2) \qquad \alpha > 0 \text{ fráhrinding}$$

a) Finna orku grunn- og örvaðs ástands fyrir óvíxlverkandi bósendir

Einnar-eindar ástöndin eru

$$\psi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}\, a}}\, H_n\left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}$$

$$a = \sqrt{\frac{\hbar}{m\omega}}\,, \qquad E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

Grunnástandið er samhverft $\rightarrow$

$$\psi_g(x_1, x_2) = \psi_0(x_1)\psi_0(x_2) = \psi_g(x_2\, x_1)$$

Ókan er

$$E_g = 2 \cdot E_0 = \hbar\omega$$, engin víxlverkun

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fyrsta örvaða ástandið

samhverft fyrir bósóndir

$$\psi_e(x_1 x_2) = \frac{1}{\sqrt{2}}\left\{\psi_0(x_1)\psi_1(x_2) + \psi_1(x_1)\psi_0(x_2)\right\}$$

$$E_e = E_0 + E_1 = \frac{\hbar\omega}{2} + \frac{3\hbar\omega}{2} = 2\hbar\omega$$

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b) 1. stigs truflun fyrir E_g og E_e

$$E_g' = \langle g|V|g\rangle = \int_{-\infty}^{\infty} dx_1 dx_2\, \psi_g^*(x_1 x_2)\, V(x_1 x_2)\, \psi_g(x_1 x_2)$$

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$$= \alpha \int_{-\infty}^{\infty} dx_1 dx_2 \, \delta(x_1 - x_2) \, |\Phi_g(x_1, x_2)|^2 = \alpha \int_{-\infty}^{\infty} dx_1 \, |\Phi_g(x_1, x_1)|^2$$

$$= \frac{\alpha}{\pi a^2} \int_{-\infty}^{\infty} dx \, e^{-2\left(\frac{x}{a}\right)^2} = \left(\frac{\alpha}{a}\right) \frac{1}{\pi} \int_{-\infty}^{\infty} du \, e^{-2u^2}$$

$$= \left(\frac{\alpha}{a}\right) \frac{1}{\sqrt{2\pi}}$$

$$E_e^1 = \langle e | V | e \rangle = \int_{-\infty}^{\infty} dx_1 dx_2 \, \Phi_e^*(x_1, x_1) \, V(x_1, x_2) \Phi_e(x_1, x_2)$$

$$= \alpha \int_{-\infty}^{\infty} dx_1 \, |\Phi_e(x_1, x_1)|^2 = \alpha \frac{4}{2} \int_{-\infty}^{\infty} dx \, \left\{ \Phi_0(x) \Phi_1(x) \right\}^2$$

$$= 2\alpha \cdot \frac{1}{\sqrt{\pi}\,a} \frac{1}{2\sqrt{\pi}\,a} \int_{-\infty}^{\infty} dx \left(2\frac{x}{a}\right)^2 e^{-2\left(\frac{x}{a}\right)^2}$$

$$= \left(\frac{\alpha}{a}\right) \frac{4}{\pi} \int_{-\infty}^{\infty} du\, u^2 e^{-2u^2} = \left(\frac{\alpha}{a}\right) \frac{1}{\sqrt{2\pi}}$$

$$= \left(\frac{\alpha}{a}\right) \frac{1}{\sqrt{2\pi}} = E_g'$$

→ örvaða ástandið $|e\rangle$ hækkar jafnmikið og grunnástandið $|g\rangle$ vegna snerti fráhrindingar-innar