(4.30) (1)

Almenn stefna í kúluhnitum

$$\hat{r} = \sin\theta\cos\phi \cdot \hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}$$

Finnum S_r, fylkið fyrir spuna þáttinn í þessa stefnu

$$\bar{S}\cdot\hat{r} = S_r = S_x\hat{r}_x + S_y\hat{r}_y + S_z\hat{r}_z$$

$$= \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\sin\theta\cos\phi + \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\sin\theta\sin\phi$$

$$+ \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\cos\theta$$

$$= \frac{\hbar}{2}\begin{pmatrix}\cos\theta & \sin\theta(\cos\phi - i\sin\phi)\\ \sin\theta(\cos\phi + i\sin\phi) & -\cos\theta\end{pmatrix}$$

$$\to S_r = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ \sin\theta\, e^{i\phi} & -\cos\theta \end{pmatrix}$$

Reynum til gamans $\hat{r} = \hat{z}$ þegar $\theta = 0$, þá fæst

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

og samskonar má sannreyna fyrir x og y - pættina

Finnum eigingildi og vigra

Skoðum hvat maxima gefur

Eigingildi $-1 \cdot \frac{\hbar}{2}$ og $+1 \cdot \frac{\hbar}{2}$

með eiginvigra

$$\begin{pmatrix} 1 \\ -\left(\frac{1+\cos\theta}{\sin\theta}\right)e^{+i\phi} \end{pmatrix} \quad \text{og} \quad \begin{pmatrix} 1 \\ \left(\frac{1-\cos\theta}{\sin\theta}\right)e^{+i\phi} \end{pmatrix}$$

Notum $1+\cos\theta = 2\cos^2\frac{\theta}{2}$ og $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$1-\cos\theta = 2\sin^2\frac{\theta}{2}$$

þá eru vigrarnir

$$\begin{pmatrix} 1 \\ -\cot\frac{\theta}{2}\cdot e^{i\phi} \end{pmatrix} \quad \text{og} \quad \begin{pmatrix} 1 \\ \tan\frac{\theta}{2}\cdot e^{+i\phi} \end{pmatrix}$$

Normum, þá þurfum við

$$1 + \tan^2\frac{\theta}{2} = \sec^2\frac{\theta}{2} = \frac{1}{\cos^2\frac{\theta}{2}}$$

$$1 + \cot^2\frac{\theta}{2} = \csc^2\frac{\theta}{2} = \frac{1}{\sin^2\frac{\theta}{2}}$$

Og þá verða normuðu eigin vigrarnir

$$\underbrace{\begin{pmatrix} \sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2}\cdot e^{+i\phi} \end{pmatrix}}_{\substack{\text{samsvarandi} \\ -\frac{\hbar}{2}}} \quad \text{og} \quad \underbrace{\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\cdot e^{+i\phi} \end{pmatrix}}_{\substack{\text{samsvarandi} \\ +\frac{\hbar}{2}}}$$

5

Vigrarnir eru greinilega hornréttir, og þar sem normunin er frjáls m.t.t. fasa skiftum við

$$\begin{pmatrix} \sin\frac{\theta}{2}\cdot e^{-i\phi} \\ -\cos\frac{\theta}{2} \end{pmatrix} \quad \text{og} \quad \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\cdot e^{+i\phi} \end{pmatrix}$$

og sjáum fyrir $\hat{r} = \hat{z}$ þegar $\theta = 0$

$$\begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad \text{og} \quad \begin{pmatrix} +1 \\ 0 \end{pmatrix}$$

eins og við þekkjum

4.33 Rafeind föst í segulsviði

$$\overline{B} = B_0 \cos(\omega t)\,\hat{k}$$

a) finna H fyrir kerfið. Samkvæmt jöfnu (4.160)

$$H = -\gamma\, \overline{B} \cdot \overline{S} = -\gamma\, B_0 \cos(\omega t)\, S_z$$

$$= -\gamma\, B_0 \cos(\omega t)\, \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= -\frac{\gamma B_0 \hbar}{2} \cos(\omega t) \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$$

tímaháða spuna ástandinu (kerfisins) er lýst með

$$\chi(t) = \begin{pmatrix} \alpha(t) \\ \beta(t) \end{pmatrix} \quad \text{og} \quad \boxed{i\hbar\,\partial_t \chi(t) = H(t)\,\chi(t)}$$

er hreyfijafna χ

b) Klukkan $t=0$ er upphafsástandið

$$\chi(0) = \chi_+^{(x)}$$

spinn uppástandið fyrir x-ásinum

Samkvæmt (4.151) er

$$\chi_+^{(x)} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Jøfnurnar fyrir $\alpha(t)$ og $\beta(t)$ eru ekki tengdar og tí er loysnarferðin einfóld

finnum $\chi(t)$ fyrir $t \geq 0$

Hreyfijøfnuna má skriva

$$i\hbar \begin{pmatrix} \dot{\alpha}(t) \\ \dot{\beta}(t) \end{pmatrix} = -\frac{\gamma B_0 \hbar}{2} \cos(\omega t) \begin{pmatrix} \alpha(t) \\ -\beta(t) \end{pmatrix}$$

Skoðum fyrri jöfnuna

$$\dot{\alpha}(t) = i\,\frac{\gamma B_0}{2}\cos(\omega t)\cdot\alpha(t)$$

1. stigs jafna

$$\rightarrow \alpha(t) = C\cdot\exp\left\{i\,\frac{\gamma B_0}{2\omega}\sin(\omega t)\right\}$$

reyna með innsetningu

upphafsskilyrði $\alpha(0) = C = \frac{1}{\sqrt{2}}$

Jafnan fyrir $\beta(t)$ er nokkurn eins, nema hvað er annað

$$\rightarrow \quad \beta(t) = \frac{1}{\sqrt{2}}\exp\left\{-i\,\frac{\gamma B_0}{2\omega}\sin(\omega t)\right\}$$

og því

$$\chi(t) = \begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\exp\left\{i\,\frac{\gamma B_0}{2\omega}\sin(\omega t)\right\}\\ \exp\left\{-i\,\frac{\gamma B_0}{2\omega}\sin(\omega t)\right\}\end{pmatrix}$$

c) Hver eru líkindin að mæla S_x og fá gildi $-\frac{\hbar}{2}$?

þ.e. hve stór $\chi_-^{(x)}$ þátturinn í $\chi(t)$?

Notum innfeldið

$$\left\{\chi_-^{(x)}\right\}^* \chi(t) = \frac{1}{\sqrt{2}}(1,-1)\cdot\frac{1}{\sqrt{2}}\begin{pmatrix} \exp\left\{i\frac{\gamma B_0}{2\omega}\sin(\omega t)\right\} \\ \exp\left\{-i\frac{\gamma B_0}{2\omega}\sin(\omega t)\right\}\end{pmatrix}$$

$$= \frac{1}{2}\left\{\exp\{i\cdots\} - \exp\{-i\cdots\}\right\}$$

$$= i\sin\left\{\frac{\gamma B_0}{2\omega}\sin(\omega t)\right\}$$

líkindin $\mathcal{P}_-^{(x)}(t) = \left|\left\{\chi_-^{(x)}\right\}^*\chi(t)\right|^2 = \sin^2\left\{\frac{\gamma B_0}{2\omega}\sin(\omega t)\right\}$

d) Hvert er lágmarkssviðið til þess að snúa S_x alveg?

Algersnúningur verður þegar $P_-^{(x)}(t) = 1$
því við byrjuðum með spuna upp $\chi(0) = \chi_+^{(x)}$

-> hvenær verður $\sin^2\left\{\frac{\gamma B_0}{2\omega}\sin(\omega t)\right\} = 1$

eða $\frac{\gamma B_0}{2\omega}\sin(\omega t) = \frac{\pi}{2}$

$\sin(\omega t)$ tekur gildi á $[-1, 1]$

-> $\frac{\gamma B_0}{2\omega} = \frac{\pi}{2}$ -> $B_0 = \frac{\pi\omega}{\gamma}$

annars tekst ekki að snúa spunanum!