4.20

①

Eind í mætti $V(\bar{r})$, sýna fram á að

$$d_t\langle \bar{L}\rangle = \langle \bar{N}\rangle$$

þ.s. $\bar{N} = \bar{r}\times\bar{F} = \bar{r}\times(-\bar{\nabla} V)$

Breyting á væntigildi hverfiþunga er vegna væntigildis vægis á kerfið

$$H = \frac{p^2}{2m} + V(\bar{r})$$

Höfum áður séð almenna jöfnu fyrir væntigildi

$$d_t\langle Q\rangle = \frac{i}{\hbar}\langle [H,Q]\rangle + \langle \partial_t Q\rangle$$

$\bar{L} = \bar{r} \times \bar{p}$ $\rightarrow$ í okkar Schrödingermynd $\partial_t L = 0$ ②

$$\rightarrow d_t \langle \bar{L} \rangle = \frac{i}{\hbar} \langle [H, \bar{L}] \rangle$$

Við þurfum því að reikna víxlin $[H, \bar{L}] = [\frac{p^2}{2m}, \bar{L}] + [V, \bar{L}]$

Skoðum

$$[p^2, L_z] = [p_x^2 + p_y^2 + p_z^2, L_z] = [p_x^2, L_z] + [p_y^2, L_z] + [p_z^2, L_z]$$

$$= p_x [p_x, L_z] + [p_x, L_z] p_x + p_y [p_y, L_z] + [p_y, L_z] p_y$$

$$+ p_z [p_z, L_z] + [p_z, L_z] p_z$$

þ.s. við notum $[AC, B] = A[C, B] + [A, B]C$

$$L_z = x p_y - y p_x$$

$$\rightarrow \; [p_x, L_z] = [p_x, x] p_y = -i\hbar p_y$$

$$[p_y, L_z] = -[p_y, y] p_x = i\hbar p_x$$

$$[p_z, L_z] = 0$$

$$\rightarrow \; [p_x^2, L_z] = -i\hbar p_x p_y - i\hbar p_y p_x$$

eins fæst

$$[p_y^2, L_z] = i\hbar p_x p_y + i\hbar p_y p_x$$

$$[p_x^2 + p_y^2, L_z] = 0$$

og því í heild $[p^2, L_z] = 0$

Samskonar fæst fyri hina parti $\bar{L}$ og tí

$$[p^2, \bar{L}] = 0$$

(hvør tilpungi frjálsvar eindar er hreifingarfasti)

Eftir Stendur

$$[V(\bar{r}), \bar{L}] = [V(\bar{r}), \bar{r} \times \bar{p}]$$

$$-i\hbar\,[V(\bar{r}), \bar{r} \times \bar{\nabla}]$$

ef við nýtum útsetingina í staðarrúminum við $\bar{p} = -i\hbar\bar{\nabla}$

Könnum

$$[V(\bar{r}), \bar{r} \times \bar{\nabla}]\, f(\bar{r}) = V(\bar{r}) \left\{ \bar{r} \times \bar{\nabla} f(\bar{r}) \right\} - \bar{r} \times \bar{\nabla} \left\{ V(\bar{r}) f(\bar{r}) \right\}$$

$$= \left\{ - f(\bar{r}) \right\} \bar{r} \times \bar{\nabla} V(\bar{r})$$

eda

$$-i\hbar\left[V(\bar{r}),\ \bar{r}\times\bar{\nabla}\right] = i\hbar\ \bar{r}\times\left\{\bar{\nabla}V(\bar{r})\right\}$$

en

$$d_t\langle\bar{L}\rangle = \frac{i}{\hbar}\left\langle\left[H,\bar{L}\right]\right\rangle = -\left\langle\bar{r}\times\left\{\bar{\nabla}V(\bar{r})\right\}\right\rangle$$

$$= \left\langle\bar{r}\times\left\{-\bar{\nabla}V(\bar{r})\right\}\right\rangle = \langle\bar{N}\rangle$$

of $\bar{N} = \bar{r}\times\bar{F} = \bar{r}\times\left\{-\bar{\nabla}V\right\}$

Einsleitur 3D hreintóna sveifill með orkuróf

$$E_{n\ell m} = E_0 \cdot \left(n + \frac{3}{2}\right)$$

er í ástandi

$$|\alpha\rangle = \left\{2|100\rangle + |211\rangle - |210\rangle + i|21{-1}\rangle\right\}$$

Gerum ráð fyrir því að $|n\ell m\rangle$ séu stöðluð, en við þurfum að staðla $|\alpha\rangle$

$$\langle\alpha|\alpha\rangle = \left\{4 + 1 + 1 + 1\right\} = 7$$

-> staðlað er

$$|\alpha\rangle = \frac{1}{\sqrt{7}}\left\{2|100\rangle + |211\rangle - |210\rangle + i|21{-1}\rangle\right\}$$

a) Vantigizdi H $\left(\text{notam} \quad H|nlm\rangle = E_{nlm}|nlm\rangle = E_n|nlm\rangle \right)$

$$\langle\alpha|H|\alpha\rangle = \frac{1}{7}\left\{4\cdot\frac{5}{2}E_0 + 1\cdot\frac{7}{2}E_0 + 1\cdot\frac{7}{2}E_0 + \frac{7}{2}E_0\right\}$$

$$= \frac{E_0}{7}\left\{\frac{20}{2} + \frac{21}{2}\right\} = E_0\cdot\frac{41}{14}$$

b) vantigizdi L^2 $\left(\text{notam} \quad L^2|nlm\rangle = \hbar^2 l(l+1)|nlm\rangle \right)$

$$\langle\alpha|L^2|\alpha\rangle = \frac{\hbar^2}{7}\left\{0 + 1\cdot 2 + 1\cdot 2 + 1\cdot 2\right\}$$

$$= \hbar^2\,\frac{6}{7}$$

c) valor médio de L_z $\quad$ (notem $L_z |nlm\rangle = \hbar m |nlm\rangle$)

$$\langle\alpha|L_z|\alpha\rangle = \frac{\hbar}{7}\left\{0 + 1 - 0 - 1\right\} = 0$$