δ-brunnur

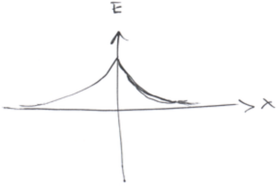
Höfum fundið eitt bundið ástand í δ-brunni

$$V(x) = -\alpha\,\delta(x)$$

$$\psi(x) = \sqrt{\kappa}\, e^{-\kappa|x|}$$

$$E = -\frac{m\alpha^2}{2\hbar^2}$$

$$\kappa = \frac{m\alpha}{\hbar^2}$$



Hvað með dreifi lausnir?

þegar $E > 0$

$$\left\{-\frac{\hbar^2}{2m}\partial_x^2 - \alpha\delta(x)\right\}\psi = E\psi$$

á svæði (I) þegar $x < 0$

$$d_x^2\psi = -\frac{2mE}{\hbar^2}\psi = -k^2\psi$$

með $k = \frac{\sqrt{2mE}}{\hbar} > 0,\ k \in \mathbb{R}$

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

Tökum báðar stöttu bylgjurnar til greina á svæði II $\quad x>0$

$$\psi(x) = Fe^{ikx} + Ge^{-ikx}$$

samfella í $x=0$

$$\psi^{I}(0) = \psi^{II}(0)$$

$$\to \boxed{A+B = F+G}$$

Fyrir afleiðurnar fæst

$$d_x\psi^{I}(0^-) = ik(A-B)$$

$$d_x\psi^{II}(0^+) = ik(F-G)$$

Áður höfðum við leitt út

$$\underbrace{d_x\psi(0^+) - d_x\psi(0^-)} = -\underbrace{\frac{2m\alpha}{\hbar^2}\psi(0)}$$

eða hér

$$\boxed{ik(F-G-A+B) = -\frac{2m\alpha}{\hbar^2}(A+B)}$$

<u>2 jöfnur og 4 óþekktir stuðlar</u>

Normun ekki möguleg

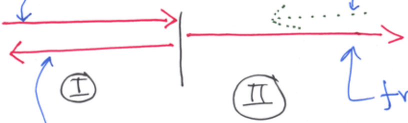
Hverju lýsir $\psi = \underbrace{Ae^{ikx}}_{\text{flöt bylgja frá vinstri}} + \underbrace{Be^{-ikx}}_{\text{flöt bylgja frá hægri}}$

Setjum þetta upp sem dreifingu (scattering) með jaðarskilyrðum

engin bylgja inn frá hægri

$\underline{G = 0}$

Bylgja inn frá vinstri



framferðar bylgja

endurkast frá árekstrarmætti

Við megum velja $A = 1$ ← ákvarðar magn straums inn

(eða höldum A t.þ.a. minna okkur á innstrauminn)

$$1 + B = F$$

$$ik\left(F - 1 + B\right) = -\frac{2m\alpha}{\hbar^2}(1 + B)$$

$$F - B = 1$$

$$ikF + B\left(ik + \frac{2m\alpha}{\hbar^2}\right) = \left(ik - \frac{2m\alpha}{\hbar^2}\right)$$

umritum þá sömu sem

$$F + B(1 - 2i\beta) = (1 + 2i\beta)$$

þ.s. $\beta = \dfrac{m\alpha}{\hbar^2 k}$

$$\boxed{\begin{aligned} F - B &= 1 \\ F + B(1-2i\beta) &= (1+2i\beta) \end{aligned}}$$

Sömu fyrir stærri verkefni er gott að sjá þetta sem

$$\begin{pmatrix} 1 & -1 \\ 1 & (1-2i\beta) \end{pmatrix}\begin{pmatrix} F \\ B \end{pmatrix} = \begin{pmatrix} 1 \\ (1+2i\beta) \end{pmatrix}$$

Lausnin er

$$F = \frac{1}{1 - i\beta}, \quad B = \frac{i\beta}{1 - i\beta}$$

Sitthvoru megin δ-mættisins er mættið jafnhátt $V = 0$

<u>Speglunarlíkur</u> (endurkast)

$$\boxed{R = \frac{|B|^2}{1} = \frac{\beta^2}{1 + \beta^2}}$$

innflæði

Framferðarlíkur

$$T = \frac{|F|^2}{1} = \frac{1}{1+\beta^2}$$

$$\beta^2 = \left(\frac{m\alpha}{\hbar k}\right)^2 , \quad k^2 = \frac{2mE}{\hbar^2}$$

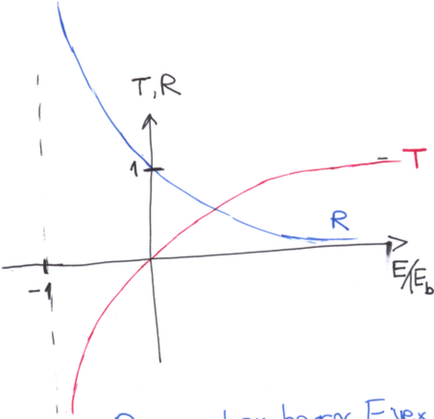
$$E_b = -\frac{m\alpha^2}{2\hbar^2}$$ Orka bundna ástandsins

Líkindini varðveitast

$$R+T = \frac{\beta^2}{1+\beta^2} + \frac{1}{1+\beta^2} = 1$$

$$R = \frac{1}{1+\frac{2\hbar^2 E}{m\alpha^2}} = \frac{1}{1+\frac{E}{|E_b|}}$$

$$T = \frac{1}{1+\frac{m\alpha^2}{2\hbar^2 E}} = \frac{1}{1+\frac{|E_b|}{E}}$$



R minkar þegar E vex

T vex —||—

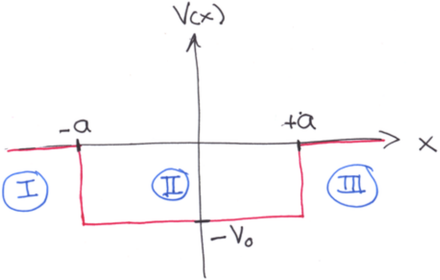
En við loyfdum ødeins $E > 0$

Føgur framlenging, tonging við dreififylki $\longrightarrow$ skaut í $E = -|E_b|$

Endanlegur brunnur

$$V(x) = \begin{cases} -V_0 & \text{þ.} \quad -a \leq x \leq a \\ 0 & \text{þ.} \quad |x| > a \end{cases}$$

$\boxed{\text{leitum fyrst að bundnum ástöndum með } E<0}$



Skiptum upp í þrjá hluta

I:

$$-\frac{\hbar^2}{2m} d_x^2 \psi = E\psi$$

$$d_x^2 \psi = \kappa^2 \psi$$

með $\kappa = \frac{\sqrt{-2mE}}{\hbar}$

tvær lausnir, sú sem dofnar og vex ekki fjarri brunninum er

$$\psi(x) = Be^{\kappa x}, \quad x < -a$$

í III er lausnin þá

$$\psi(x) = Fe^{-\kappa x} \quad x > a$$

þar á milli í II er

$$-\frac{\hbar^2}{2m} d_x^2 \psi - V_0 \psi = E\psi$$

eða

$$d_x^2 \psi = -l^2 \psi$$

með

$$l = \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

því $\quad E + V_0 > 0$

Lausnin er

$$\psi(x) = C\,\mathrm{Sin}(lx) + D\,\mathrm{Cos}(lx)$$

þ. $\quad -a < x < a$

Brunnurinn er samhverfur

Lausnirnar eru annaðhvort jafn- eða oddstæðar

$$\psi(-x) = \pm\psi(x)$$

Þá nægir að tryggja að ψ og ψ' séu samfelld öðru megin

jafnstæðu lausnirnar eru $\sim \mathrm{Cos}(lx)$

oddstæðu eru $\sim \mathrm{Sin}(lx)$

8

Skoðum jafnstæðu lausnirnar

(t.d. grunnástandið er jafnst.)

$$\psi(x) = \begin{cases} F e^{-\kappa x} & x > a \\ D \cos(l x) & 0 < x < a \\ \psi(-x) & x < 0 \end{cases}$$

<u>Samfella ψ í $x = a$</u>

$$(1) \quad F e^{-\kappa a} = D \cos(la)$$

<u>Samfella ψ' í $x = a$</u>

$$(2) \quad -\kappa F e^{-\kappa a} = -l D \sin(la)$$

Saman gefa þær

$$\left((1)/(2) \right)$$

$$\kappa = l \tan(la)$$

Munum að $\kappa = \kappa(E)$ og $l = l(E)$ þ.a. við höfum hér óbeina jöfnu fyrir E

$$\kappa^2 = -\frac{2mE}{\hbar^2} \,, \quad l^2 = \frac{2m(E+V_0)}{\hbar^2}$$

$$\rightarrow \kappa^2 + l^2 = \frac{2mV_0}{\hbar^2} \left(= z_0^2/a^2 \right)$$

Veljum $z \equiv la$, $z_0 \equiv \frac{a}{\hbar}\sqrt{2mV_0}$

þ.a.

$$\tan(la) = \frac{\kappa}{l}$$

$$\left(\tan(z)\right)^2 = \frac{\kappa^2}{l^2}$$

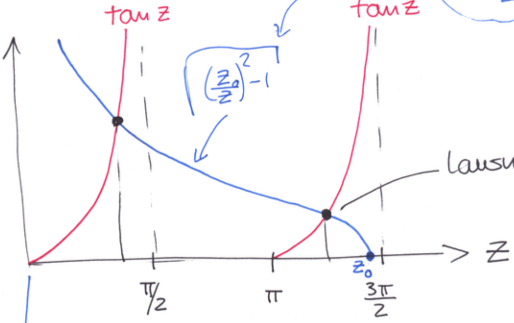
$$= \frac{\left(\frac{z_0}{a}\right)^2 - l^2}{l^2}$$

$$= \frac{z_0^2 - (la)^2}{(la)^2}$$

$$= \left(\frac{z_0}{z}\right)^2 - 1$$

$$\rightarrow \boxed{\tan z = \sqrt{\left(\frac{z_0}{z}\right)^2 - 1}}$$

<u>Óbein jafna</u> t.d. má skoða lausnir grafískt



$$z_0^2 = \frac{a^2\, 2mV_0}{\hbar^2}$$ þ.a. fjöldi lausna

$\rightarrow$ <u>fjöldi bundinna ástanda</u>

<u>fer eftir dýpt brunns V_0</u>

minnst eitt bundið ástand <u>(1D)</u>

Dreifilausnir fyrir endanlega brunninn

$E > 0$

Hugsum okkur inn-bylgju frá vinstri

I $\quad x < -a$

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

III $\quad x > a$

$$\psi(x) = Fe^{ikx}$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

II $\quad -a < x < a$

$$\psi(x) = C\sin(lx) + D\cos(lx)$$

$$l = \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

4 skilyrði:

$\psi(-a)$ samfellt

$$Ae^{-ika} + Be^{ika} = -C\sin(la) + D\cos(la)$$

$\psi'(-a)$ samfellt

$$ik\left(Ae^{-ika} - Be^{ika}\right) = l\left(C\cos(la) + D\sin(la)\right)$$

$\psi(a)$ samfellt

$$C\sin(la) + D\cos(la) = Fe^{ika}$$

$\psi'(a)$ samfellt

$$l\{C\cos(la) - D\sin(la)\} = ikFe^{ika}$$

Þar má taka saman sem

$$B = i\frac{\sin(2la)}{2kl}(l^2-k^2)F$$

$$F = \frac{e^{-2ika}A}{\cos(2la) - i\frac{(k^2+l^2)}{2kl}\sin(2la)}$$

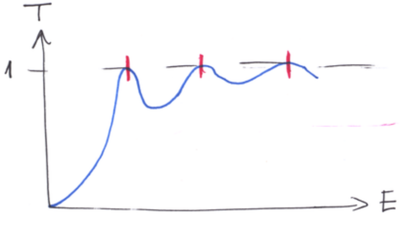
og

$$T = \frac{|F|^2}{|A|^2}$$

$$= \frac{1}{1+\frac{V_0^2}{4E(E+V_0)}\sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right)}$$

$$= \frac{1}{1+\frac{V_0^2}{4E(E+V_0)}\sin^2\left(\pi\sqrt{\frac{E+V_0}{\hbar\omega_1}}\right)}$$

$$\hbar\omega_1 = \hbar\left(\frac{\pi^2\hbar}{2m(2a)^2}\right)$$



Þetta er skilyrði þess að öll bylgjan varpist frá $-a$ aftur inn á brunnsvæðið

T=1 herma í hvert sinn sem

$$\sqrt{\frac{E+V_0}{\hbar\omega_1}} = n\,, \quad n = 1, 2, 3, \ldots$$

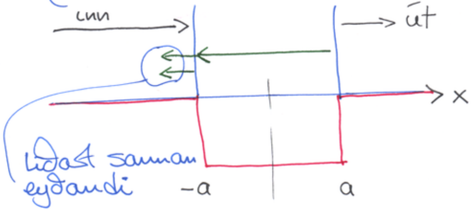
Hermuástönd

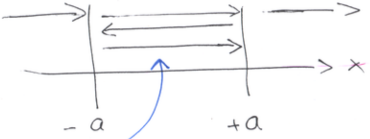
finnast í tilraunum

$$E + V_0 = \hbar\omega_1 n^2$$

þegar $E = \hbar\omega_1 n^2 - V_0$

sama skilyrði og fyrir óendanlegan brunn





Í henni er því hægt að sjá meiri líkur þess að ögn finnist á brunnsvæðinu

Hennar ástöndin fá endanlega meðalævi þess vegna er toppurinn með endanlega breidd.

þegar svæðið milli −a og +a er skoðað betur sést að lausnin jafngildir því að allir mögulegar fyrir "n"– summan endurkast sé teknir með í reikninginn

Dæmi um hermu í dreifingu

hermutoppurinn

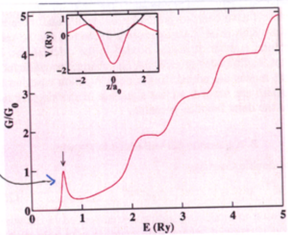
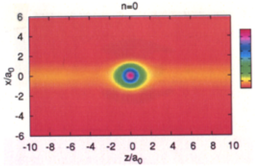


FIG. 9. (Color online). Conductance of a parabolic wall quantum wire in units of $G_0=2e^2/h$ as a function of energy in the presence of a double Gaussian scattering potential. The inset shows the scattering potential, whose parameters are $V_1=2.02$ Ry, $V_2=3.71$ Ry, $\alpha_1=0.29\,a_0^{-2}$ and $\alpha_2=0.96\,a_0^{-2}$, in the cross section $x=0$. The parabolic confinement potential ($\hbar\omega=1.01$ Ry) is also shown. The total number of modes is $N=9$. The arrow points at the value of the energy at which the probability density in Fig. 10 is calculated.



Líkindin fyrir hermuna

Dvalartími eindar innan brunns

FIG. 10. (Color online). The probability density of the scattering states ψ_{nE}^{+} in the parabolic quantum wire in the presence of the double Gaussian scattering potential of Fig. 9. The total energy of the incident particle is $E=0.64$ Ry, coinciding with a resonance in the conductance (cf. the arrow in Fig. 9). The incoming wave is in mode $n=0$.

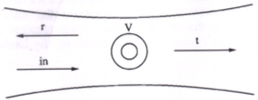


FIG. 1. Schematic view of the system. An incoming wave is partly transmitted and partly reflected by the finite range scattering potential V.

fleygboga skammtavír