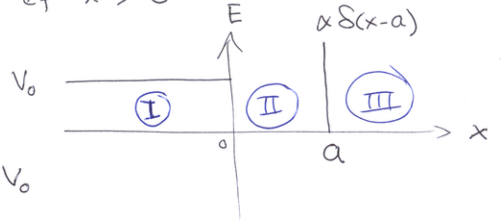
① ①

Mætti

$$V(x) = \begin{cases} V_0 & \text{ef } x < 0 \\ \alpha\delta(x-a) & \text{ef } x > 0 \end{cases}$$

$V_0 > 0$
$a > 0$



Gerum ráð fyrir að $\alpha > 0$

leitum dreifi-lausna með $E > V_0$
E-ið kemur inn frá vinstri

(I)

$$\psi_I(x) = e^{ikx} + Be^{-ikx}$$

$$k^2 = \frac{2m}{\hbar^2}(E - V_0)$$

(II)

$$\psi_{II}(x) = Ce^{iqx} + De^{-iqx}$$

$$q^2 = \frac{2m}{\hbar^2}E$$

(III)

$$\psi_{III}(x) = Fe^{iqx}$$

$$q^2 = \frac{2m}{\hbar^2}E$$

Á þessu svæði getur engin bylgja komið frá hægri

Lausn er samfelld í $x=0$

$$\psi_I(0) = \psi_{II}(0)$$

$$\rightarrow \quad 1 + B = C + D$$

Lausn er samfelld í $x=a$

$$\psi_{II}(a) = \psi_{III}(a)$$

$$Ce^{iqa} + De^{-iqa} = Fe^{iqa}$$

Afleiða lausnar er samfelld í $x=0$

$$\psi_I'(0) = \psi_{II}'(0)$$

$$ik - ikB = iqC - iqD$$

Afleiða lausnar er ósamfelld í $x=a$

$$\psi_{III}'(a^+) - \psi_{II}'(a^-) = \frac{2m\alpha}{\hbar^2}\psi_{III}(a)$$

eða

$$iqFe^{iqa} - \left\{iqCe^{iqa} - iqDe^{-iqa}\right\}$$

$$= \frac{2m\alpha}{\hbar^2}Fe^{iqa}$$

sözkonusu sonra yazılım

$$B - C - D = -1$$

$$-ikB - iqC + iqD = -ik$$

$$C + e^{-2iqa} D - F = 0$$

$$-iqC + iq e^{-2iqa} D + \left\{ iq - \frac{2m\alpha}{\hbar^2} \right\} F = 0$$

ya da

$$\begin{pmatrix} 1 & -1 & -1 & 0 \\ -ik & -iq & iq & 0 \\ 0 & 1 & e^{-2iqa} & -1 \\ 0 & -iq & iq e^{-2iqa} & \left[iq - \frac{2m\alpha}{\hbar^2} \right] \end{pmatrix} \begin{pmatrix} B \\ C \\ D \\ F \end{pmatrix} = \begin{pmatrix} -1 \\ -ik \\ 0 \\ 0 \end{pmatrix}$$

Setjum $s = e^{-2iqa}$ og $r = \left\{ iq - \frac{2m\alpha}{\hbar^2} \right\}$

$$\begin{pmatrix} 1 & -1 & -1 & 0 \\ -ik & -iq & iq & 0 \\ 0 & 1 & s & -1 \\ 0 & -iq & iqs & r \end{pmatrix} \begin{pmatrix} B \\ C \\ D \\ F \end{pmatrix} = \begin{pmatrix} -1 \\ -ik \\ 0 \\ 0 \end{pmatrix}$$

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lausn fundin í maxima er af ~~við~~ stilgreinum

$$\Omega = q\left\{-ir(s+1) + k(s+1)\right\} + ikr(1-s) + q^2(s-1)$$

$$= q(s+1)(k-ir) + (s-1)(q^2-ikr)$$

$$B = -\frac{q(s+1)(-k-ir) - ikr(1-s) + q^2(s-1)}{\Omega}$$

$$= \frac{q(s+1)(k+ir) - (s-1)(q^2+ikr)}{\Omega}$$

$$C = \frac{2ks(q-ir)}{\Omega}$$

$$D = \frac{2k(ir+q)}{\Omega}$$

$$F = \frac{4kqs}{\Omega}$$

Ef $\underline{\alpha = 0}$, engin δ-toppar $\rightarrow$ $r = iq$

$$\rightarrow \Omega = q(s+1)(k+q) + (s-1)(q^2+kq)$$

$$= (s+1)(q^2+kq) + (s-1)(q^2+kq)$$

$$= \underline{2s(q^2+kq)}$$

og

$$B = \frac{q(s+1)(k-q) - (s-1)(q^2-kq)}{\Omega}$$

$$= \frac{2sq(k-q)}{2sq(q+k)} = -\frac{q-k}{q+k}$$

eins og í dømine í síðstu viku
þar sum $\alpha = 0$

Enn ef $\alpha = 0$

$$F = \frac{4kqs}{\Omega} = \frac{4kqs}{2s(q^2+kq)} = \frac{2k}{q+k}$$

eins og í dæminu í síðustu viku

Undirbúum grafík:

Miðum allt við orkuna $E_1 = \frac{\hbar^2}{2ma^2}$ ← var orka lægsta ástands í óendanlegum brunni

$$ka = \sqrt{\frac{2ma^2}{\hbar^2}(E-V_0)} = \sqrt{\frac{E}{E_1} - \frac{V_0}{E_1}} \qquad E > V_0$$

$$qa = \sqrt{\frac{E}{E_1}} \qquad ra = \left\{ i\,qa - \left(\frac{\alpha}{a}\right)\frac{1}{E_1} \right\}$$

↑ víddarlausar stærðir

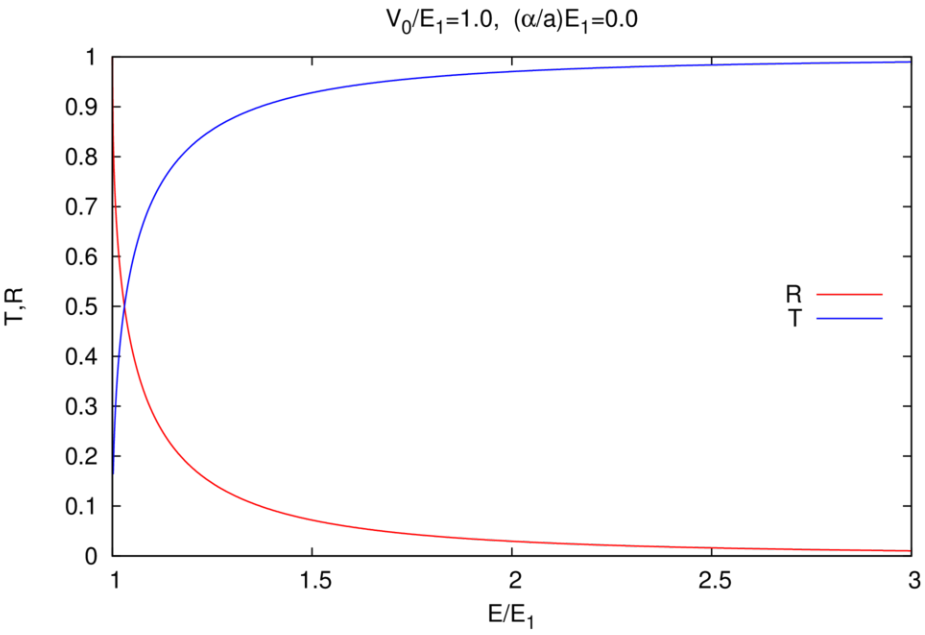
Styrkur δ-mættis í orku er $\frac{\alpha}{a}$, miðum hann líka við E.

$$S = \exp\left\{-2iqa\right\}$$

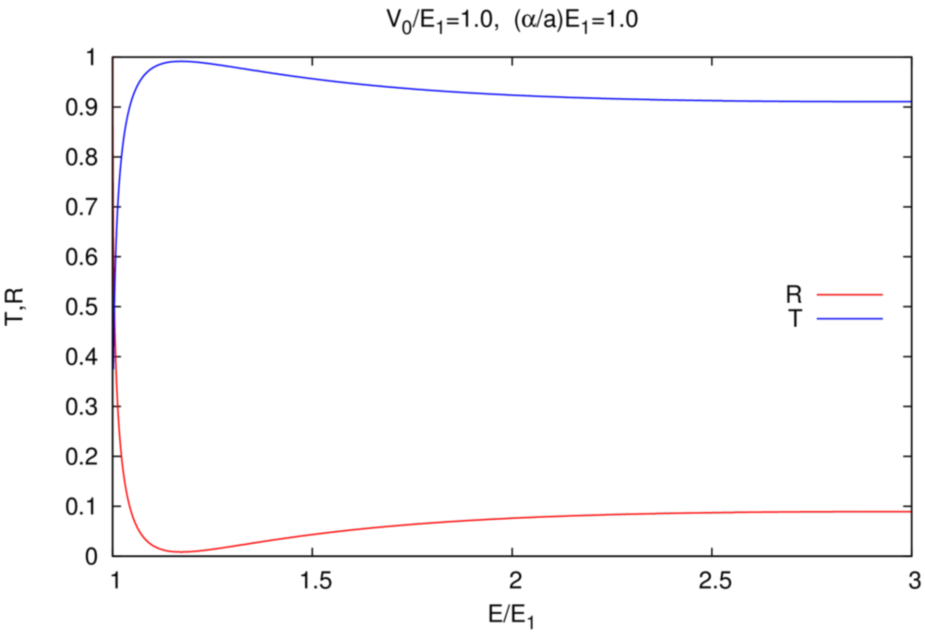
Mínnum úr dæminu í síðustu viku að

$$R = |B|^2$$

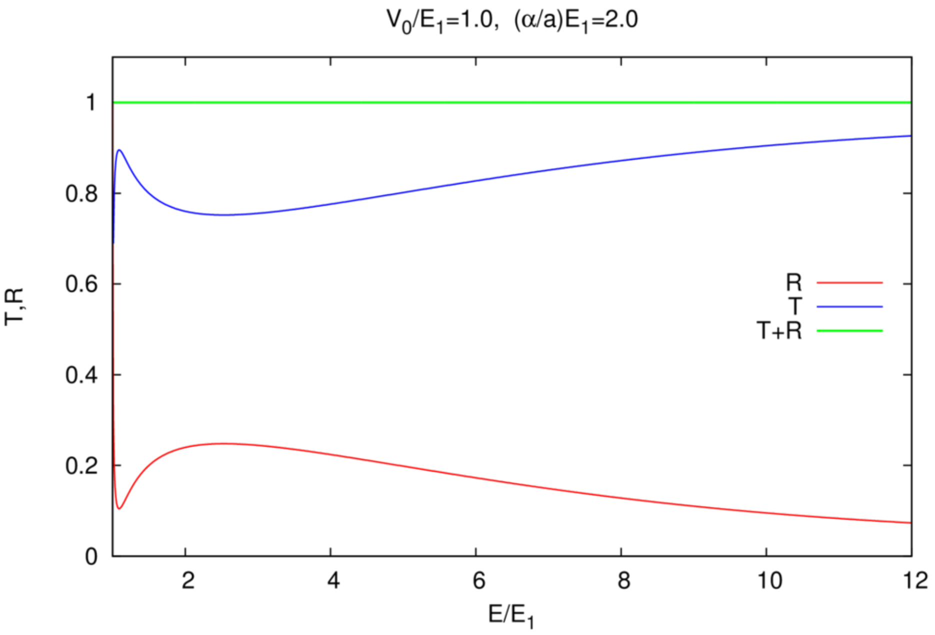
$$T = \frac{qa}{ka}|F|^2$$



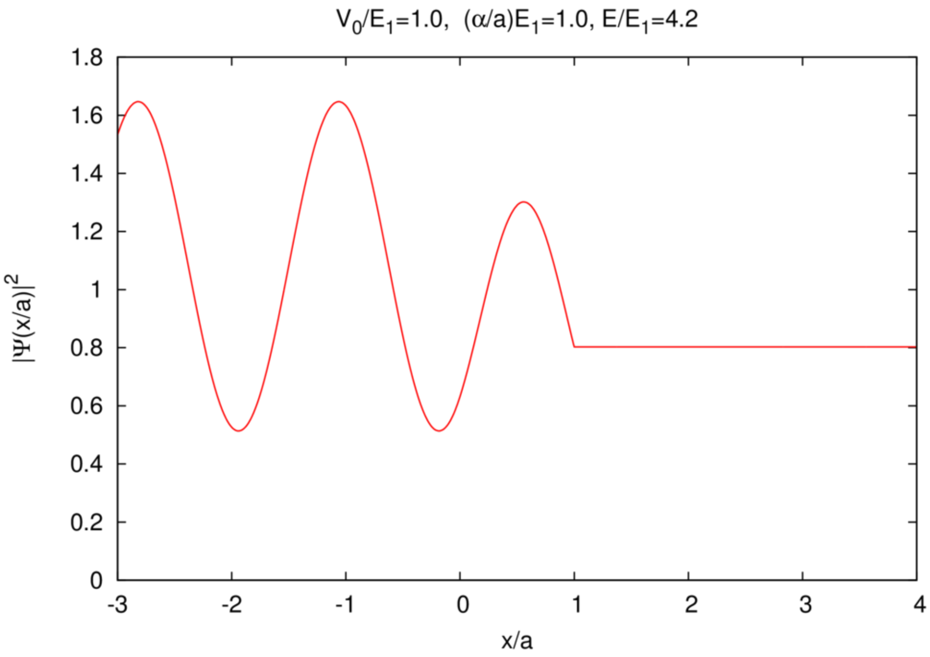
V0/E1=1.0, (α/a)E1=0.0
T,R
E/E1
R
T
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1
1.5
2
2.5
3



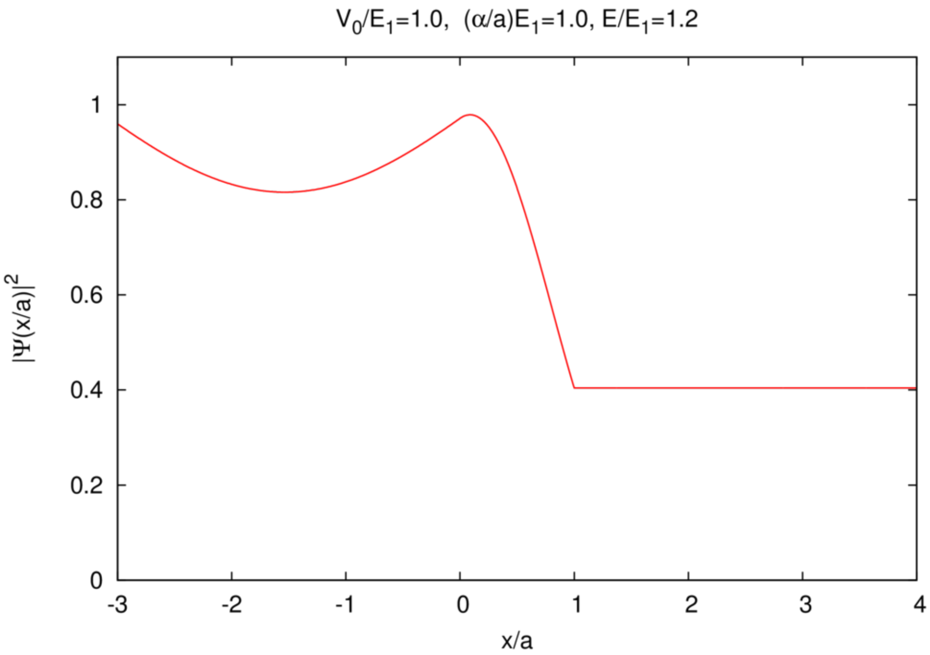
V0/E1=1.0, (α/a)E1=1.0
T,R
E/E1
R
T
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.5
2
2.5
3



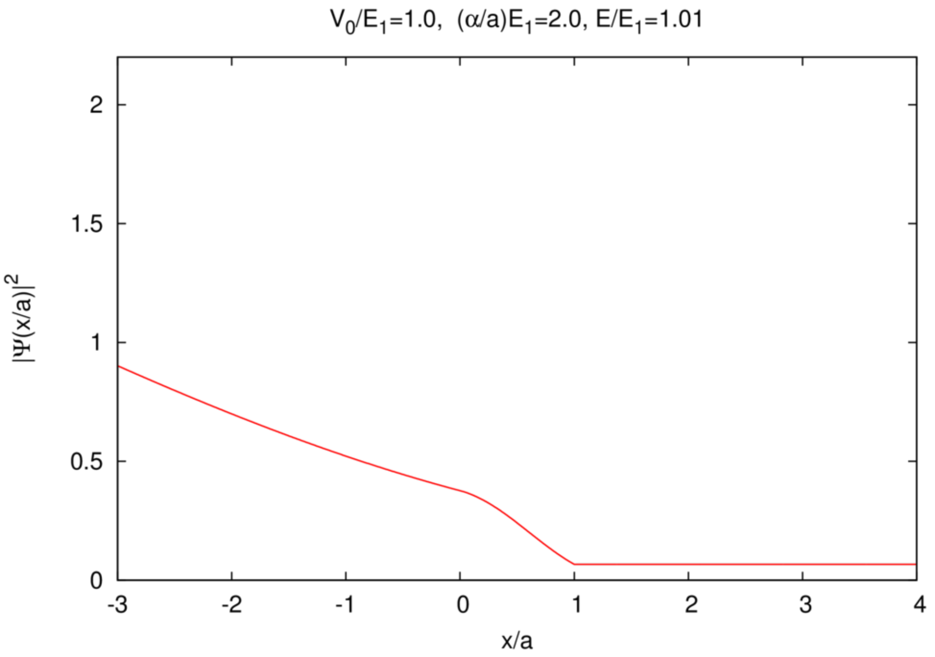
V0/E1=1.0, (α/a)E1=2.0
T,R
E/E1
1
0.8
0.6
0.4
0.2
0
2
4
6
8
10
12
R
T
T+R



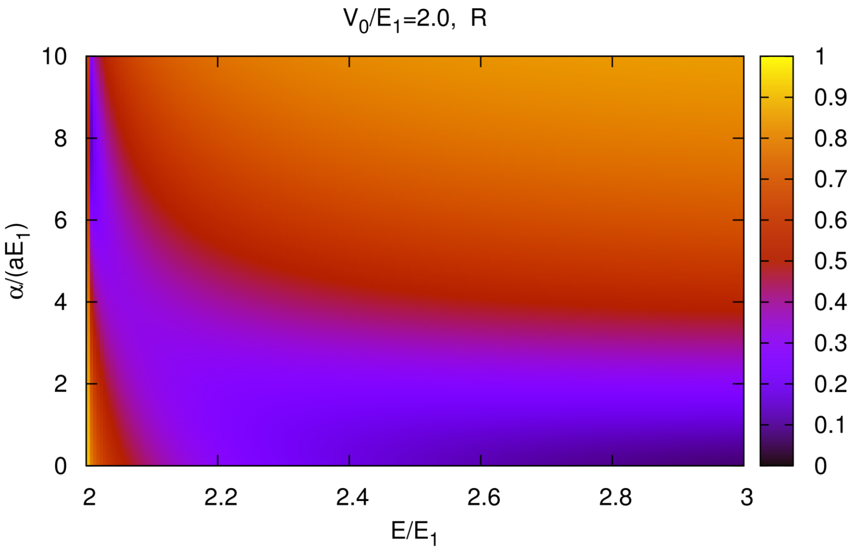
V0/E1=1.0, (α/a)E1=1.0, E/E1=4.2
|Ψ(x/a)|2
x/a
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-3
-2
-1
0
1
2
3
4



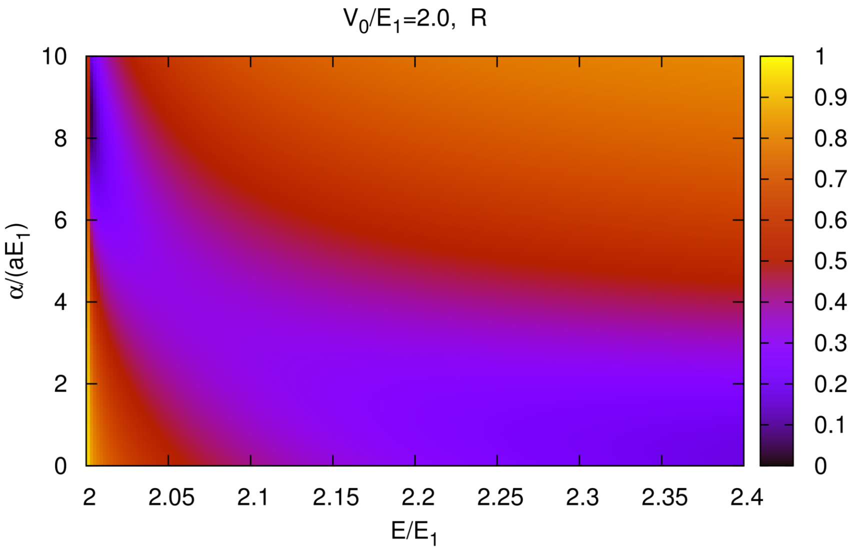
V0/E1=1.0, (α/a)E1=1.0, E/E1=1.2
|Ψ(x/a)|2
x/a
0
0.2
0.4
0.6
0.8
1
-3
-2
-1
0
1
2
3
4



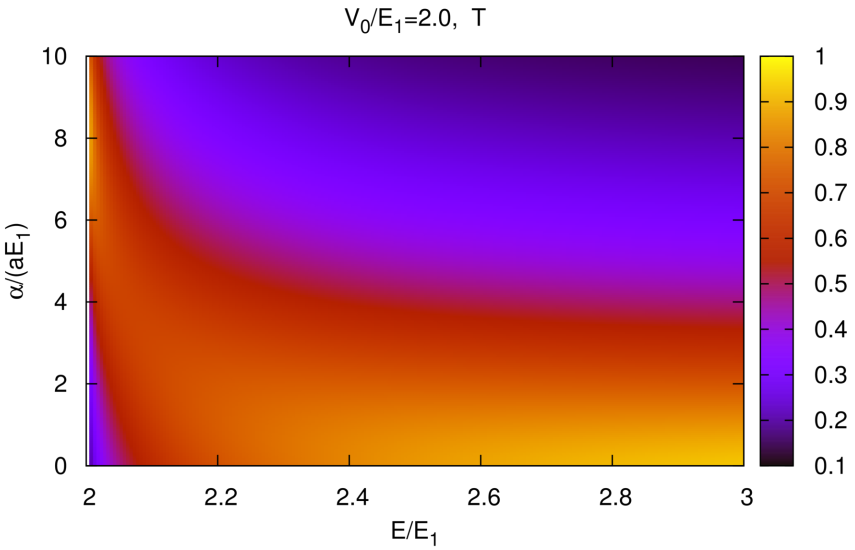
V0/E1=1.0, (α/a)E1=2.0, E/E1=1.01
|Ψ(x/a)|2
x/a
0
0.5
1
1.5
2
-3
-2
-1
0
1
2
3
4



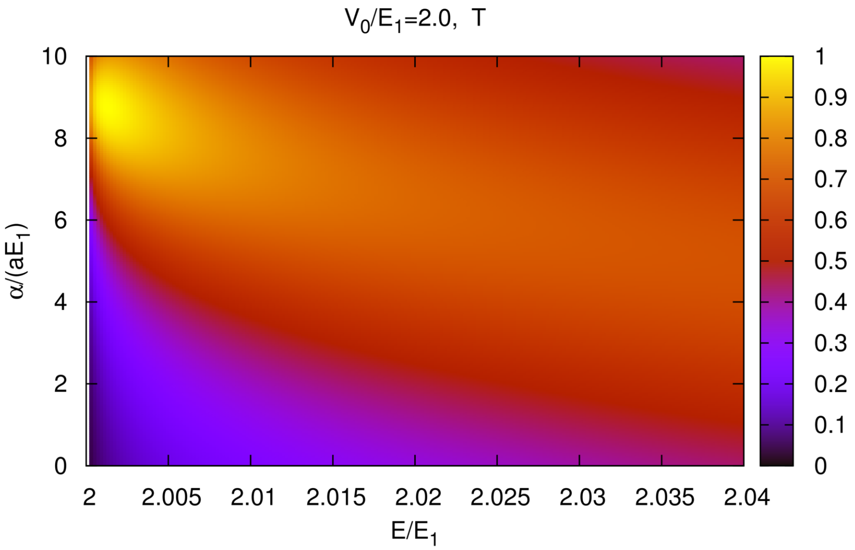
V0/E1=2.0, R
α/(aE1)
E/E1
0
2
4
6
8
10
2
2.2
2.4
2.6
2.8
3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



V0/E1=2.0, R
10
8
6
4
2
0
α/(aE1)
2
2.05
2.1
2.15
2.2
2.25
2.3
2.35
2.4
E/E1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0



V0/E1=2.0, T
α/(aE1)
E/E1
0
2
4
6
8
10
2
2.2
2.4
2.6
2.8
3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



V_0/E_1=2.0, T
$\alpha/(aE_1)$
0
2
4
6
8
10
2
2.005
2.01
2.015
2.02
2.025
2.03
2.035
2.04
E/E_1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1