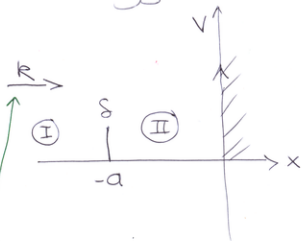


## Fjórði Stærktur

fína deifi ástönd wellisins

$V(x) = \alpha \delta(x+a)$  sem stöðsett  
er fyrir þann ástand þegar  
háan vegg í  $x=0$



Hugsum okkur umbylgju  
með bylgju vögu  $K$

Tvö stöðir þar sem við þurfum  
á fína  $\psi(x)$ , þar er ekkert  
móti  $\rightarrow$  notum fjálsalusa  
(fyrir fjálsaend)

$$\textcircled{\text{I}} \quad \psi_{\pm}(x) = e^{ikx} + B e^{-ikx}$$

$$\textcircled{\text{II}} \quad \psi_{\pm}(x) = C e^{ikx} + D e^{-ikx}$$

## Þöcr-Þilyrði

$$\psi(0) = 0$$

Umbylgja

$$\psi_{\text{II}}(-a) = \psi_{\text{I}}(-a)$$

bröt í afturu

$$\begin{aligned} & \psi'(-a^+) - \psi'(-a^-) \\ &= \frac{2m\alpha}{\hbar^2} \psi(-a) \end{aligned}$$

$$\psi(0) = 0 \rightarrow D = -C$$

Samfella i  $x = -a$ :

$$e^{-ika} + B e^{ika} = C e^{-ika} + D e^{ika} = C \{ e^{-ika} - e^{ika} \}$$
$$= -C 2i \sin(ka)$$

(A)

brott i  $x = a$ :

$$C i k e^{-ika} + C i k e^{ika} - i k e^{-ika} + i k e^{ika} B = \frac{2m \alpha x}{\hbar^2} \{ e^{-ika} + B e^{ika} \}$$
$$C 2 \cos(ka) + e^{ika} B - \beta B e^{ika} = e^{-ika} (1 + \beta) \quad , \quad \beta = \frac{2m \alpha x}{\hbar^2 i k}$$

$$2C \cos(ka) + B(1 - \beta) e^{ika} = (1 + \beta) e^{-ika}$$

(B)

Umformung (A)

$$Be^{ika} + C 2i \sin(ka) = -e^{-ika}$$

(A)

og (B)

$$B(1-\beta)e^{ika} + 2C \cos(ka) = (1+\beta)e^{-ika}$$

(B)

Twoer l nubegav j tur fyrir  pattku st r irnar B og C

(A) getur

$$C = -\frac{e^{-ika} + Be^{ika}}{2i \sin(ka)} \quad (**)$$

Notum i (B)

$$i \frac{(e^{-ika} + Be^{ika})}{\sin(ka)} \cos(ka) + B(1-\beta)e^{ika} = (1+\beta)e^{-ika}$$

Q.20

$$B \left[ i e^{ika} \cot(ka) + (1-\beta) e^{ika} \right] = e^{-ika} \left[ -i \cot(ka) + (1+\beta) \right]$$

$$\rightarrow B = \frac{e^{-2ika} \left[ -i \cot(ka) + (1+\beta) \right]}{\left[ i \cot(ka) + (1-\beta) \right]}$$

$$\rightarrow |B|^2 = B \cdot B^* = \frac{1 + |\beta|^2 + \cot^2(ka)}{1 - |\beta|^2 + \cot^2(ka)}$$

$$= \frac{(1 + \gamma^2) + \cot^2(ka)}{(1 + \gamma^2) + \cot^2(ka)} = 1$$

$$\text{ef } i\gamma = \beta$$

$$1 + |\beta|^2 = 1 + |i\gamma|^2 = 1 + \gamma^2$$

$$1 - |\beta|^2 = 1 - |i\gamma|^2 = 1 - \gamma^2$$

(4)

5  
Allar sinder sem steyma að málmu fara síkhvortúman  
þá þú aftur, en hér er margt fallegt! ~~Stöðum.~~

Bylgjufallid á (II) er

$$\psi(x) = A \sin(kx)$$

C fast með þú að selja B í jöfnun (\*\*)

$$C = \frac{i e^{-ika} \left[ 1 + \frac{\{(1+i\gamma) - i \cot(ka)\}}{\{(1-i\gamma) + i \cot(ka)\}} \right]}{2 \sin(ka)}$$

$$\gamma = -\frac{2m\alpha}{\hbar^2 k}$$

friðsind

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2ma^2} (ka)^2 = E_1 \cdot (ka)^2$$

$$\gamma = - \frac{2m\alpha}{\hbar^2 k} = - \frac{\alpha}{a} \frac{2ma^2}{\hbar^2 (ka)} = - \left( \frac{\alpha}{aE_1} \right) \frac{1}{ka}$$

Viðfarlausar  
stærðir

$$(ka)^2 = \frac{E}{E_1} \rightarrow ka = \sqrt{\left( \frac{E}{E_1} \right)}$$

$$\rightarrow \frac{\alpha}{aE_1} = -(ka)\gamma$$

Notum sem stíka í gröfum

Við höfum að  $|B|^2 = 1$  sem táknað er allar eindir (7)  
koma einhverutíman til baka

$\lim_{r \rightarrow 0} |C|^2 = 1$  þú þegar  $r \rightarrow 0$  eða  $x \rightarrow 0$

er engin S-toppur og þá flæða öll litindin  
alveg að veggnum

$|C|^2$  hefur sérstöðupunkta, eins og gröfin Sjva  
og jafnan fyrir  $c$  bendir til. Það eru  
kernur, þá festist eindir einhverntíma  
milli S-toppis og veggis

↑ eindir eyðir löngum tíma far  $\rightarrow$  mestu  
litindin á þeir að fuma kanna þar,

kæf bylgjuvægrir ka

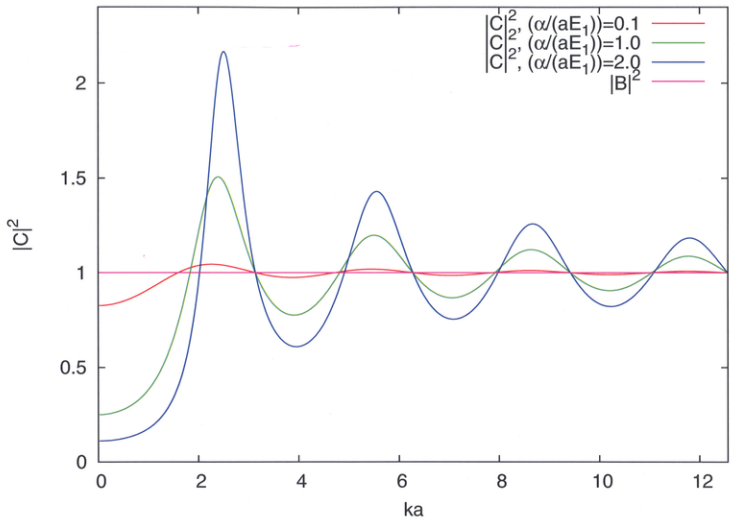
(8)

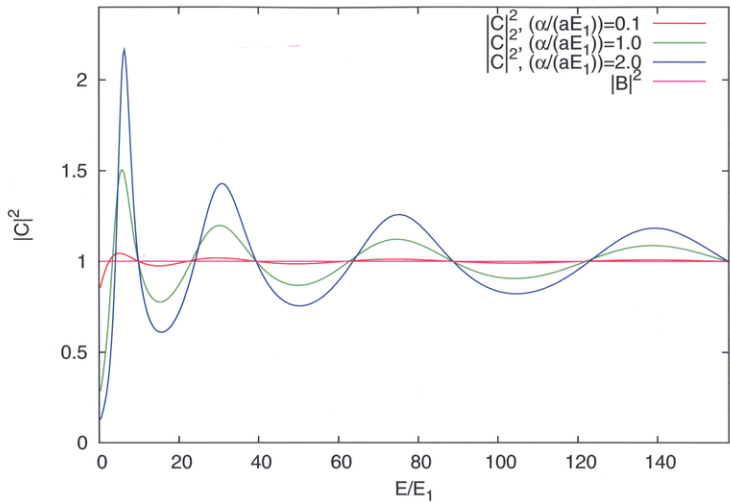
Hér toppur  $\alpha \rightarrow \infty$  títil líkindi að eind fannist milli topps og veggis, nema fyrir hermu ástand

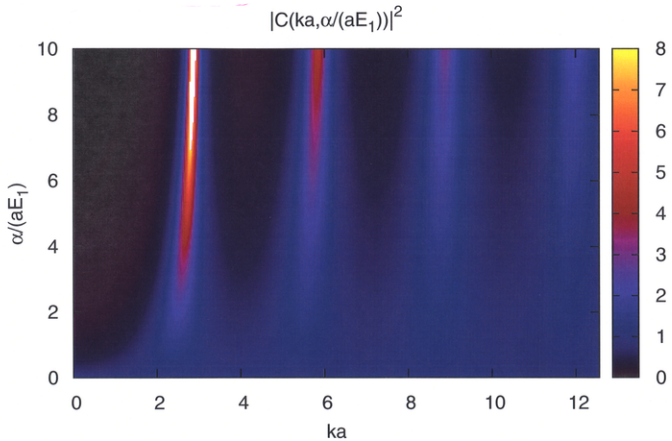
Í hermu passar bylgjulengd einder milli topps og veggis (hálf sáa heilt margfeldi hálfvægr.)

Hermu ástand færast til  $z$  ortu þegar  $\alpha$  vex og nælgast ræfist fyrir áendabegam þrunk þegar  $\alpha \rightarrow \infty$









```

#!/usr/bin/gnuplot -persist
#
#      GNU P L O T
#      Version 4.6 patchlevel 0   last modified 2012-03-04
#      Build System: Linux x86_64
#
#      Copyright (C) 1986-1993, 1998, 2004, 2007-2012
#      Thomas Williams, Colin Kelley and many others
#
#      gnuplot home:      http://www.gnuplot.info
#      faq, bugs, etc:   type "help FAQ"
#      immediate help:   type "help" (plot window: hit 'h')
#
#      Type 'load "all.dem"' to display a large number of examples.
#      They are located at /usr/share/doc/packages/gnuplot/demo/*
#
set terminal postscript landscape enhanced defaultplex \
leveldefault color colortext \
solid dashlength 1.0 linewidth 1.0 butt noclip \
nobackground \
paletteparam 2009,0.003 \
"Helvetica" 18 fontscale 1.0
set output 'C2-ka.ps'

set xlabel "ka"
set ylabel ""
set x2label "offset character 0, 0, 0 font "" textcolor lt -1 norotate
set xrange [*:*] norverse nowriteback
set x2range [*:*] norverse nowriteback
set ylabel "C|-2"
set y2label "offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270"
set y2range [*:*] norverse nowriteback
set y2range [*:*] norverse nowriteback
set zlabel ""
set zlabel "offset character 0, 0, 0 font "" textcolor lt -1 norotate
set zrange [*:*] norverse nowriteback
set cblabel ""
set cblabel "offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270"
set zero 1e-08
set locale "en_US.UTF-8"
set key right top

# ----- FOL1
I=(0.1.0) # pverinling 1
# ymkka
# ymkama
plot(x) = I/tan(x)
gm(x) = (1.0+I*x)
C(x,y) = (I*exp(-I*x))/(2*sin(x))*(1+gm(y)-Icot(x))/(gm(y)+Icot(x))
# -----
plot [0:4.0*pi][0:2:4] abs(C(x,-0.1/x))**2 w l title '|C|-2, ((/Symbol a)/(/SE 1))=0.1' lw 2,\
abs(C(x,-1.0/x))**2 w l title '|C|-2, ((/Symbol a)/(/SE 1))=1.0' lw 2,\
abs(C(x,-2.0/x))**2 w l title '|C|-2, ((/Symbol a)/(/SE 1))=2.0' lw 2,\
1.0 w l title '|B|-2' lt 4 lw 2
EOF

```

```

#!/usr/bin/gnuplot -persist
#
#      G N U P L O T
#      Version 4.6 patchlevel 0   last modified 2012-03-04
#      Build System: Linux x86_64
#
#      Copyright (C) 1986-1993, 1998, 2004, 2007-2012
#      Thomas Williams, Colin Kelley and many others
#
#      gnuplot home:   http://www.gnuplot.info
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#      immediate help: type "help" (plot window: hit 'h')
#
#      Type 'load "all.dem"' to display a large number of examples.
#      They are located at /usr/share/doc/packages/gnuplot/demo/*
#
set terminal postscript landscape enhanced defaultplex \
    leveldefault color colortext \
    solid dashlength 1.0 linewidth 1.0 butt noclip \
    nobackground \
    paletteparam 2000,0.003 \
    "Helvetica" 18 fontscale 1.0
set output 'C2-E.ps'

set xlabel "E/E_1"
set ylabel offset character 0, 0, 0 font "" textcolor lt -1 norotate
set x2label offset character 0, 0, 0 font "" textcolor lt -1 norotate
set xrange [ * : * ] norverse nowriteback
set x2range [ * : * ] norverse nowriteback
set ylabel "[C|^2"
set y2label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set y2range [ * : * ] norverse nowriteback
set yrange [ * : * ] norverse nowriteback
set y2range [ * : * ] norverse nowriteback
set xlabel ""
set ylabel offset character 0, 0, 0 font "" textcolor lt -1 norotate
set zlabel offset character 0, 0, 0 font "" textcolor lt -1 norotate
set zrange [ * : * ] norverse nowriteback
set clabel offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c2label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c3label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c4label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c5label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c6label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c7label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c8label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c9label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c10label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c11label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c12label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c13label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c14label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c15label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c16label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c17label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c18label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c19label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c20label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c21label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c22label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c23label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c24label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c25label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c26label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c27label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c28label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c29label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c30label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c31label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c32label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c33label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c34label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c35label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c36label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c37label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c38label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c39label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c40label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c41label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c42label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c43label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c44label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c45label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c46label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c47label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c48label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c49label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c50label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c51label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c52label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c53label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c54label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c55label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c56label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c57label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c58label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c59label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c60label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c61label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c62label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c63label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c64label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c65label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c66label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c67label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c68label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c69label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c70label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c71label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c72label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c73label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c74label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c75label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c76label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c77label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c78label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c79label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c80label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c81label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c82label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c83label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c84label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c85label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c86label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c87label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c88label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c89label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c90label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c91label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c92label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c93label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c94label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c95label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c96label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c97label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c98label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c99label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set c100label offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270
set locale "en_US.UTF-8"
set key right top
I=(0:0:1.0) # pverringing 1
# xkxk
# gamma
# gamma
icot(x) = 1/tan(x)
gpl(x) = (1.0+I*x)
gm(x) = (1.0-I*x)
C(x,y) = (I*exp(-I*x)/(2*sin(x)))+(1+(gpl(y)-icot(x))/(gm(y)+icot(x)))
#
#
plot [0:16.0*pi**2][0:2:4] abs(C(sqrt(x)), -0.1/sqrt(x))**2 w l title '|C|^2, ((Symbol a)/(ae_1))=0.1' lw 2, \
    abs(C(sqrt(x)), -1.0/sqrt(x))**2 w l title '|C|^2, ((Symbol a)/(ae_1))=1.0' lw 2, \
    abs(C(sqrt(x)), -2.0/sqrt(x))**2 w l title '|C|^2, ((Symbol a)/(ae_1))=2.0' lw 2, \
    1.0 w l title '|B|^2' lt 4 lw 2
EOF

```

```
#!/usr/bin/gnuplot -persist
#
# G N U P L O T
# Version 4.6 patchlevel 0      last modified 2012-03-04
# Build System: Linux x86_64
#
# Copyright (C) 1986-1993, 1998, 2004, 2007-2012
# Thomas Williams, Colin Kelley and many others
#
# gnuplot home:      http://www.gnuplot.info
# faq, bugs, etc:   type "help FAQ"
# immediate help:   type "help" (plot window: hit 'h')
#
# Type 'load "all.dem"' to display a large number of examples.
# They are located at /usr/share/doc/packages/gnuplot/demo/*
#
set terminal postscript landscape enhanced defaultplex \
leveldefault color colortext \
solid dashlength 1.0 linewidth 1.0 butt noclip \
nobackground \
paletteparam 2000,0:003 \
"Helvetica" 18 fontscale 1.0
set output "C2-ka-3D.ps"

set xlabel "ka"
set ylabel "offset character 0, 0, 0 font "" textcolor lt -1 norotate"
set x2label "offset character 0, 0, 0 font "" textcolor lt -1 norotate"
set xrange [ * : * ] norverse nowriteback
set x2range [ * : * ] norverse nowriteback
set ylabel "(Symbol a)/(a_e 1)"
set y2label "offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270"
set y2range [ * : * ] norverse nowriteback
set y2label "offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270"
set y2range [ * : * ] norverse nowriteback
set xlabel ""
set ylabel "offset character 0, 0, 0 font "" textcolor lt -1 norotate"
set zrange [ * : * ] norverse nowriteback
set cblabel ""
set cblabel "offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270"
set c2label "offset character 0, 0, 0 font "" textcolor lt -1 rotate by -270"
set cchange [ * : * ] norverse nowriteback
set zero 1e-08
set tmargin -1
set bmargin -1
set rmargin -1
set locale "en_US.UTF-8"
set pm3d explicit at s
set pm3d scansautomatic
set pm3d interpolate 1,1 flush begin nofrangles nohidden3d corners2color mean
set palette positive nops_allcf maxcolors 0 gamma 1.5 color model RGB
set palette rgbformulae 7, 5, 15
set colorbox default
set colorbox vertical origin screen 0.9, 0.2, 0 size screen 0.05, 0.6, 0 front default
set style boxplot candles range 1.50 outliers pt 7 separation 1 labels auto unsorted
set loadpath
set fontpath
set psdir
set fit noerrorvariables
set fit I=(0,0,1,0) # pverrindng I
#----- F011 -----
# x=ka
# y=gamma
# lct(x) = I/tan(x)
# gp(x) = (1.0+I*x)
```

```
gm(x) = (1.0-I*x)
C(x,y) = [I*exp(-I*x)/(2*sin(x))]*(1+(gp(y)-I*cot(x))/(gm(y)+I*cot(x)))
#.....
unset key
set title '[C(ka,{Symbol} a)/(ae_1)]|^2'
set pm3d map
set samples 400
set isosamples 400
plot [0:4.0*pi][0:10][0:8.0] abs(C(x,-y/x))^2 w pm3d
# EOF
```