1 Væntigildi p^4 og x^4 fyrir n-ástand H.O.

Byrjum á p^4

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}\left\{\mp ip + m\omega x\right\}$$

$$\rightarrow \quad a_+ - a_- = \frac{1}{\sqrt{2\hbar m\omega}}\left\{-ip - ip\right\} = -\sqrt{\frac{2}{\hbar m\omega}}\, ip$$

$$= -\frac{ip}{\hbar}\sqrt{\frac{2\hbar}{m\omega}} = -\frac{i}{\hbar}\sqrt{2}\, a\, p$$

því $a = \sqrt{\frac{\hbar}{m\omega}}$

$$\rightarrow \quad p = \frac{i\hbar}{a}\frac{1}{\sqrt{2}}(a_+ - a_-)$$

$$p^4 = \frac{\hbar^4}{a^4}\frac{1}{4}(a_+ - a_-)^4$$

$$\langle p^4 \rangle = \int dx\, \psi_n^*(x)\, p^4\, \psi_n(x)$$

Munum að ψ_n mynda hornrættan grunn

$$\rightarrow \qquad \int dx\, \psi_n^*(x)\psi_m(x) = \delta_{n,m}$$

Einu liðirnir sem ekki hverva i $\langle p^4 \rangle$ eru liðir með javnan fjølda hakkunar og lækkunarvirkja

$$\langle p^4 \rangle = \frac{\hbar^4}{a^4 4}\int dx\, \psi^*(x)\Big\{a_+a_+a_-a_- + a_-a_-a_+a_+ + a_+a_-a_+a_- + a_-a_+a_-a_- + a_+a_-a_-a_+ + a_-a_+a_+a_-\Big\}\psi_n(x)$$

notum $a_-\psi_n = \sqrt{n}\,\psi_{n-1}$ og $a_+\psi_n = \sqrt{n+1}\,\psi_{n+1}$

$$\langle p^4\rangle = \frac{\hbar^4}{4a^4}\int dx\,\psi_n^*(x)\Big\{ n(n-1) + (n+1)(n+2) + n^2 + (n+1)^2 + n(n+1) + (n+1)n \Big\}\psi_n(x)$$

$$= \frac{\hbar^4}{4a^4}\Big\{6n^2 + 6n + 3\Big\} = \frac{\hbar^4}{a^4}\Big\{\frac{3}{2}n^2 + \frac{3}{2}n + \frac{3}{4}\Big\}$$

Gætum notað sömu aðferð fyrir x^4, en ég reyni aðra aðferð

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!\,\sqrt{\pi}\,a}}\, H_n\left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}$$

Vitum

$$\left.\begin{aligned} H_0 &= 1 \\ H_1 &= 2x \\ H_2 &= 4x^2 - 2 \\ H_3 &= 8x^3 - 12x \\ H_4 &= 16x^4 - 48x^2 + 12 \end{aligned}\right\} \longrightarrow x^4 = \left\{\frac{1}{16} H_4(x) + \frac{3}{4} H_2(x) + \frac{3}{4} H_0(x)\right\}$$

þuí er

$$\langle x^4 \rangle = \int_{-\infty}^{\infty} dx\, \psi_n^*(x)\, x^4\, \psi_n(x)$$

$$= \frac{a^4}{2^n n! \sqrt{\pi}} \int_{-\infty}^{\infty} du\, H_n(u) H_n(u) e^{-u^2} \left[\frac{1}{16} H_4(u) + \frac{3}{4} H_2(u) + \frac{3}{4} H_0(u)\right]$$

Notum GR- 7.375.2

$$\int_{-\infty}^{\infty} e^{-x^2} H_k(x) H_m(x) H_n(x) dx = \frac{2^{\frac{m+n+k}{2}} \sqrt{\pi}\, k!\, m!\, n!}{(s-k)!\,(s-m)!\,(s-n)!}$$

$2s = m+n+k$ ($k+m+n$ er jöfn tala)

$m=n$, k er 0, 2, 4 hjá okkur

$$\langle x^4 \rangle = \frac{a^4}{2^n n! \sqrt{\pi}} \frac{2^n \sqrt{\pi}\, n!\, n!}{1} \left\{ \frac{2^2 \cdot 4!}{16 \cdot (n-2)!\,(2)!\,(2)!} \right.$$

$$\left. + \frac{2 \cdot 2! \cdot 3}{4 \cdot (n-1)!\,(1)!\,(1)!} + \frac{3}{4 \cdot (n)!\, 0!\, 0!} \right\}$$

$$\langle x^4 \rangle = a^4 \left\{ \frac{3}{2} n\cdot(n-1) + 3n + \frac{3}{4} \right\}$$

$$= a^4 \left\{ \frac{3}{2} n^2 + \frac{3}{2} n + \frac{3}{4} \right\}$$

svipað $\langle p^4 \rangle$? . . . . . .

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2 $[A,B] = AB - BA$

því er líka $[A,B] = -[B,A]$

$$[A,BC] = ABC - BCA$$

$$\left.\begin{aligned} B[A,C] &= BAC - BCA \\ [A,B]C &= ABC - BAC \end{aligned}\right\} \to \begin{aligned} &B[A,C] + [A,B]C \\ &= [A,BC] \end{aligned}$$

2 fine

$$[x, H] \quad , \quad H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$$

$$\rightarrow [x, H] = \frac{1}{2m} [x, p^2] \qquad \{ \text{poi } [x, x] = 0 \}$$

$$= \frac{1}{2m} \{ p[x, p] + [x, p] p \} = \frac{i\hbar \cdot 2p}{2m}$$

$$= \frac{i\hbar p}{m} \qquad \left([x, p] = i\hbar \right)$$

$$[p, H] = \frac{1}{2} m\omega^2 [p, x^2] = \frac{1}{2} m\omega^2 \{ x [p, x] + [p, x] x \}$$

$$= \frac{1}{2} m\omega^2 \{ - x \cdot 2 i\hbar \} = - m\omega^2 i\hbar x$$

3

$$[a_-, H] = [a_-, \hbar\omega(a_+a_- + \tfrac{1}{2})] = \hbar\omega\,[a_-, a_+a_-]$$

$$= \hbar\omega\left\{ a_+[a_-, a_-] + [a_-, a_+]\,a_- \right\}$$

$$= \hbar\omega\,[a_-, a_+]\,a_- = \hbar\omega\, a_-$$

$$[a_+, H] = \hbar\omega\,[a_+, a_+a_-]$$

$$= \hbar\omega\; a_+\,[a_+, a_-] = -\hbar\omega\, a_+$$