(2.10) Hreintóna sveifill

1

a) finna $\psi_2(x)$

$$\psi_n(x) = \frac{1}{\sqrt{n!}} (a_+)^n \psi_0(x)$$

$$\psi_0(x) = \frac{1}{\sqrt{a\sqrt{\pi}}} e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}$$

$$a_+ = \sqrt{\frac{\hbar}{2\hbar m\omega}} \left\{ \mp \frac{ip}{\hbar} + \frac{m\omega x}{\hbar} \right\}$$

$$= \frac{a}{\sqrt{2}} \left\{ -\frac{ip}{\hbar} + \frac{x}{a^2} \right\}$$

ef $\quad a = \sqrt{\frac{\hbar}{m\omega}}$

$$a_+ = \frac{1}{\sqrt{2}} \left\{ -a\partial_x + \frac{x}{a} \right\}$$

$$\rightarrow \psi_2(x) = \frac{1}{\sqrt{2}\,2} \frac{1}{\sqrt{a\sqrt{\pi}}} \left(-a\partial_x + \frac{x}{a}\right)\left(-a\partial_x + \frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}$$

$$= \cdot \quad \cdot \quad \cdot \quad \cdot \; \cdot \, e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2} \left(\frac{x}{a} + \frac{x}{a}\right)$$

$$\Phi_2(x) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{a\sqrt{\pi}}}\left(-a\partial_x + \frac{x}{a}\right)\frac{x}{a}\, e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}$$

$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{a\sqrt{\pi}}}\left[\left(\frac{x}{a}\right)^2 - 1 + \left(\frac{x}{a}\right)^2\right] e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}$$

$$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{a\sqrt{\pi}}}\left\{2\left(\frac{x}{a}\right)^2 - 1\right\} e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}$$

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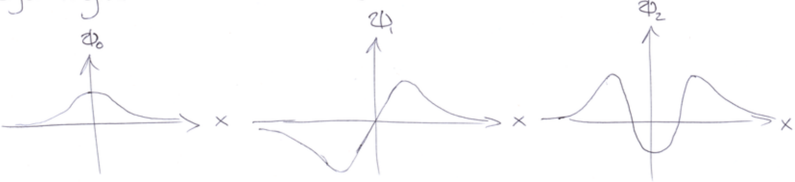
b) sjá mynd á bls. 8 í fyrirlestri 5



c)

$$\int_{-\infty}^{\infty} dx\, \psi_0(x)\, \psi_1(x) = 0$$ ← odd Staltfall

$$\int_{-\infty}^{\infty} dx\, \psi_2(x)\, \psi_3(x) = 0$$ ←

$$\int_{-\infty}^{\infty} dx\, \psi_0(x)\, \psi_2(x) = \frac{1}{\sqrt{2}}\,\frac{1}{a\sqrt{\pi}} \int_{-\infty}^{\infty} dx \left\{2\left(\frac{x}{a}\right)^2 - 1\right\} e^{-\left(\frac{x}{a}\right)^2}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} du \left\{2u^2 - 1\right\} e^{-u^2} = 0$$ samkvant Maxima

(2.11)

Reikna $\langle x \rangle, \langle p \rangle, \langle x^2 \rangle$ og $\langle p^2 \rangle$

fyrir ψ_0 og ψ_1 ~~með~~ heildum

$$\psi_0(x) = \frac{1}{\sqrt{a\sqrt{\pi}}} e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}, \quad \psi_1(x) = \frac{\sqrt{2}}{\sqrt{a\sqrt{\pi}}}\left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}$$

$n=0$

$\psi_0(x)$ er samhverft um $x=0$ $\longrightarrow$ $\langle x \rangle$ og $\langle p \rangle$ hverfa

$$\langle x^2 \rangle = \int dx\, \psi_0(x) x^2 \psi_0(x) = \frac{1}{a\sqrt{\pi}} \int dx\, x^2 e^{-\left(\frac{x}{a}\right)^2}$$

$$= \frac{a^3}{a\sqrt{\pi}} \int \frac{dx}{a} \left(\frac{x}{a}\right)^2 e^{-\left(\frac{x}{a}\right)^2} = \frac{a^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} du\, u^2 e^{-u^2} = \frac{a^2}{2}$$

$$\langle p^2 \rangle = \int dx\, \Phi_0(x)\, p^2\, \Phi_0(x) \quad = \frac{1}{a\sqrt{\pi}} \int dx\, e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2} \left[-\hbar^2 \partial_x^2\, e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}\right] \quad 5$$

$$= -\frac{\hbar^2}{a^2\sqrt{\pi}} \int \frac{dx}{a}\, e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2} \left[a^2 \partial_x^2\, e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}\right]$$

$$= -\frac{\hbar^2}{a^2}\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} du\, e^{-\frac{u^2}{2}} \left\{\partial_u^2\, e^{-\frac{u^2}{2}}\right\}$$

$$= -\frac{\hbar^2}{a^2}\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} du\, e^{-u^2} \left\{u^2 - 1\right\} = -\frac{\hbar^2}{a^2}\frac{1}{\sqrt{\pi}}\left(-\frac{\sqrt{\pi}}{2}\right)$$

$$= \frac{\hbar^2}{2a^2}$$

$n=1$

$\psi_1(x)$ er andsamhverft $\to |\psi_1|^2$ er samhverft

$\to \langle x\rangle$ og $\langle p\rangle$ hverfa

$$\langle x^2\rangle = \frac{2}{a\sqrt{\pi}}\int dx \left(\frac{x}{a}\right)^2 x^2 e^{-\left(\frac{x}{a}\right)^2} \qquad \text{(sjá (2.62))}$$

$$= \frac{2a^2}{\sqrt{\pi}}\int_{-\infty}^{\infty} du\, u^4 e^{-u^2} = \frac{2a^2}{\sqrt{\pi}}\,\frac{3\sqrt{\pi}}{4} = \frac{3a^2}{2}$$

$$\langle p^2\rangle = \frac{2}{a\sqrt{\pi}}\int dx \left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}\left[-\hbar^2\partial_x^2 \left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}\right]$$

$$= \frac{2}{a^2\sqrt{\pi}}\int \frac{dx}{a} \left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}\left\{-\hbar^2 a^2\partial_x^2 \left(\frac{x}{a}\right) e^{-\frac{1}{2}\left(\frac{x}{a}\right)^2}\right\}$$

$$\langle p^2 \rangle = -\frac{2\hbar^2}{a^2\sqrt{\pi}} \int_{-\infty}^{\infty} du\, u e^{-u^2/2} \left\{ \partial_u^2\, u e^{-u^2/2} \right\}$$

$$= -\frac{2\hbar^2}{a^2\sqrt{\pi}} \int_{-\infty}^{\infty} du\, u^2 e^{-u^2} \left\{ u^2 - 3 \right\}$$

$$= \left(-\frac{2\hbar^2}{a^2\sqrt{\pi}}\right)\left(-\frac{3\sqrt{\pi}}{4}\right) = \frac{3\hbar^2}{2a^2}$$

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b) $\underline{n=0}$ $\quad \sigma_x = \frac{a}{\sqrt{2}}, \quad \sigma_p = \frac{\hbar}{\sqrt{2}\,a}$

$$\rightarrow \sigma_x \sigma_p = \frac{\hbar}{2}$$

<u>n=1</u>

$$\sigma_x = \sqrt{\frac{3}{2}}\,a\,, \qquad \sigma_p = \sqrt{\frac{3}{2}}\,\frac{\hbar}{a}$$

$$\rightarrow \quad \sigma_x \sigma_p = \frac{3}{2}\hbar$$

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c)

$$\langle T\rangle = \frac{\langle p^2\rangle}{2m}$$

$$\langle V\rangle = \frac{1}{2} m\omega^2 \langle x^2\rangle$$

<u>n=0</u>

$$\left.\begin{aligned} \langle T\rangle &= \frac{\hbar^2}{4ma^2} \\ \langle V\rangle &= \frac{1}{4} m\omega^2 a^2 \end{aligned}\right\} \quad \begin{aligned} \langle T\rangle + \langle V\rangle &= \frac{1}{4}\left\{\frac{\hbar^2}{ma^2} + m\omega^2 a^2\right\} \\ &= \frac{1}{4}\left\{\hbar\omega + \hbar\omega\right\} = \underline{\underline{\frac{\hbar\omega}{2}}} \end{aligned}$$

$\underline{n=1}$

$$\left.\begin{aligned} \langle T\rangle &= \frac{3\hbar^2}{4ma^2} \\ \langle V\rangle &= \frac{3}{4}m\omega^2a^2 \end{aligned}\right\} \longrightarrow \langle T\rangle + \langle V\rangle = \underline{\underline{\frac{3\hbar\omega}{2}}}$$

Í samræmi við að lægsta eigingildi hreintóna sveifils