1 1

Eind í brunni með lengd a í ástandi lýst með

$$\Psi(x,0) = A\left\{\psi_3(x) - i\psi_5(x)\right\}$$

1 Finnið A. $\psi_n(x)$ eru stöðluð og hornrétt (raungild)

$$1 = |A|^2 \int_0^a dx \left\{\psi_3(x) - i\psi_5(x)\right\}^* \left\{\psi_3(x) - i\psi_5(x)\right\}$$

$$= |A|^2 \int_0^a dx \left\{\psi_3(x) + i\psi_5(x)\right\}\left\{\psi_3(x) - i\psi_5(x)\right\}$$

$$= |A|^2 \int_0^a dx \left\{ \overbrace{|\psi_3(x)|^2 + |\psi_5(x)|^2}^{\text{eru stöðluð}} \underbrace{- i\psi_3(x)\psi_5(x) + i\psi_5(x)\psi_3(x)}_{\text{eru hornrétt}} \right\} = 2|A|^2$$

$\rightarrow |A|^2 = \frac{1}{2}$, og við veljum, $A = \frac{1}{\sqrt{2}}$

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\left[\psi_3(x)e^{-i\omega_3 t} - i\psi_5(x)e^{-i\omega_5 t}\right]$$

þ.s. $\omega_n = \frac{E_n}{\hbar} = \frac{E_1}{\hbar}n^2 = \omega_1 n^2$

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(n\pi\frac{x}{a}\right)$$

2 Finna $\langle x \rangle$

$$\langle x \rangle = \int_0^a dx\, \Psi^*(x,t)\, x\, \Psi(x,t)$$

$$= \frac{1}{2}\int_0^a dx \left\{\psi_3(x)e^{i\omega_3 t} + i\psi_5(x)e^{i\omega_5 t}\right\} x \left\{\psi_3(x)e^{-i\omega_3 t} - i\psi_5(x)e^{-i\omega_5 t}\right\}$$

3

$$= \frac{1}{2}\int_0^a dx \left\{ \underbrace{|\psi_3(x)|^2 x} + \underbrace{|\psi_5(x)|^2 x} + i x \psi_5(x)\psi_3(x)\left[e^{i(\omega_5-\omega_3)t} - e^{-i(\omega_5-\omega_3)t}\right]\right\}$$

$$= \frac{a}{2} + \Delta(t)$$

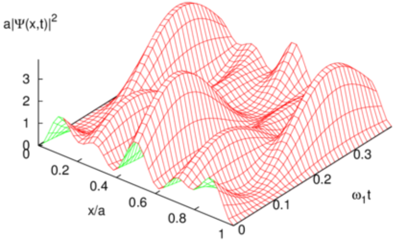
$$\Delta(t) = -\int_0^a dx \underbrace{\left\{x\psi_5(x)\psi_3(x)\right\}}_{\text{(tvískautsfylgjastak)}} \underbrace{\sin((\omega_5-\omega_3)t)}_{\text{óháð } x} = 0$$

$$\boxed{\langle x\rangle = \frac{a}{2}}$$

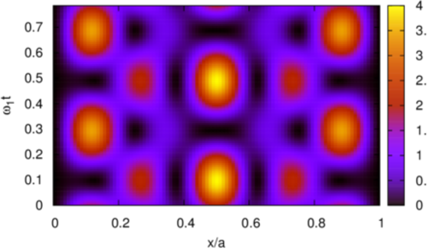
ekki háð tíma

(3) $$|\Phi(x,t)|^2 = \frac{1}{2}\left\{|\psi_3(x)|^2 + |\psi_5(x)|^2 - 2\psi_5(x)\psi_3(x)\sin[(\omega_5-\omega_3)t]\right\}$$

$$= \frac{1}{2}\left\{|\psi_3^2(x)| + |\psi_5(x)|^2 - 2\psi_5(x)\psi_3(x)\sin[16\omega_1 t]\right\}$$



$a|\Psi(x,t)|^2$
3
2
1
0
0
0.2
0.4
0.6
0.8
1
x/a
0
0.1
0.2
0.3
$\omega_1 t$



ω1t
x/a
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
3
3.5
4

(4) Ef orka $\Psi(x,t)$ er mæld?

$\Psi(x,t)$ er ekki eiginástand H, en það er sett saman úr tveimur eigin ástöndum með sama vogi

því fæst E_3 með líkum $\frac{1}{2}$

og E_5 — " —

Eftir mælingu er eindin annaðhvort í ástandi 3 eða 5

(5)

$$\langle H \rangle = \int_0^a dx\, \Psi^*(x,t)\, H\, \Psi(x,t)$$

$$= \frac{1}{2}\int_0^a dx \left[\psi_3(x)e^{i\omega_3 t} + i\psi_5(x)e^{i\omega_5 t}\right]\left\{E_3\psi_3(x)e^{-i\omega_3 t} - iE_5\psi_5(x)e^{-i\omega_5 t}\right\}$$

5

$$= \frac{1}{2}\int_0^a dx \left\{ E_3 |\psi_3(x)|^2 + E_5 |\psi_5(x)|^2 \right\} = \frac{E_3 + E_5}{2}$$

eins og við mátti búast af tölunum á undan.

2 Eind í óendanlegum brunni lýst með

$$\Psi(x) = A \cdot x \cdot (a-x) \cdot \left(x - \frac{a}{2}\right)$$

1 Normum

$$1 = \int_0^a dx \, |A|^2 \, x^2 (a-x)^2 \cdot \left(x - \frac{a}{2}\right)^2 = |A|^2 \cdot \frac{a^7}{840}$$

Veljum því $\quad A = \sqrt{\dfrac{840}{a^7}}$

Hér er ágætt að athuga hvernig bylgjufallið hefur rétta vídd

2 Hvað gefur orkumæling?

Hvernig er hægt að liða Ψ í grunnföllum (eiginföllum) Hamiltonvirkjans, orkuvirkjans?

$$\Phi(x) = \sum_{n=1}^{\infty} c_n \phi_n(x)$$

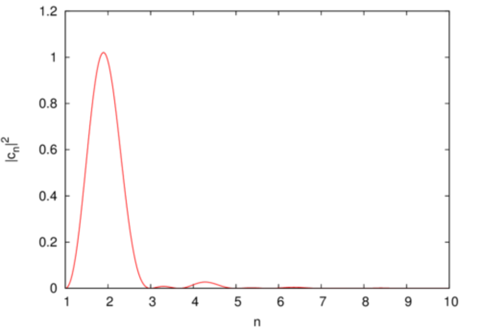
$$c_n = \int_0^a dx\, f(x)\, \phi_n^*(x) = \sqrt{\frac{1680}{a^8}} \int_0^a dx\, x(a-x)(x-\tfrac{a}{2}) \sin(n\pi \tfrac{x}{a})$$

$$= \sqrt{1680} \int_0^1 du\, u\cdot(1-u)\cdot(u-\tfrac{1}{2})\cdot \sin(n\pi u)$$

$$= \sqrt{1680}\left\{ -\frac{(n^2\pi^2-12)\sin(\pi n) + 6\pi n \cos(\pi n)}{2\pi^4 n^4} - \frac{3}{n^3\pi^3}\right\}$$

Mæling gefur $E_n = E_1 \cdot n^2$ með líkum $|c_n|^2$

$c_1 = 0$, sjá mynd



1.2
1
0.8
0.6
0.4
0.2
0
$|c_n|^2$
1
2
3
4
5
6
7
8
9
10
n

3 Vautigi di H

$$\langle H \rangle = \int_0^a dx\, \Phi^*(x)\, H\, \Phi(x) = \sum_{n=1}^{\infty} |C_n|^2 E_n$$

$$= E_1 \sum_{n=1}^{\infty} |C_n|^2 n^2 \qquad = E_1\, 1680 \sum_{n=1}^{\infty} \left[\frac{3\left[(-1)^n + 1\right]}{\pi^3 n^3} \right]^2 n^2$$

$$= E_1 \quad 1680 \cdot \frac{9}{\pi^6} \sum_{n=1}^{\infty} \left[\frac{(-1)^n + 1}{n^3} \right]^2 n^2 \simeq E_1 \cdot 4 + \delta$$

$$\simeq E_2 + \delta$$