2.4 Óendanlegur brunnur, Reikna $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$ og σ_x og σ_p

Ástand $|n\rangle$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)$$

$$\langle x \rangle = \int_0^a dx\, \psi_n^*(x)\, x\, \psi_n(x) = \frac{2}{a} \int_0^a dx\, x \sin^2\left(\frac{n\pi}{a} x\right)$$

$$= \frac{a}{4n^2\pi^2} \left\{ 1 - 2n\pi \sin(2n\pi) - \cos(2\pi n) + 2n^2\pi^2 \right\}$$

$$= \frac{a}{4n^2\pi^2} \left\{ 1 - 1 + 2n^2\pi^2 \right\} = \frac{a}{2}$$

ψ_n er alltaf jafnstætt eða oddstætt svo niðurstaðan kemur ekki á óvart

(2)

$$\langle x^2\rangle = \int_0^a dx\, \Psi_n^*(x)\, x\, \Psi_n(x) = \frac{2}{a}\int_0^a dx\, x^2 \sin^2\left(\frac{n\pi}{a}x\right)$$

$$= \frac{2}{a}\left\{\frac{-6a^3 n\pi + 4a^3 n^3\pi^3}{24\, n^3\pi^3}\right\} = \frac{-6a^2 n\pi + 4a^2 n^3\pi^3}{12\, n^3\pi^3}$$

$$= a^2\left\{\frac{1}{3} - \frac{1}{2\pi^2 n^2}\right\}$$

$$\langle p\rangle = \int_0^a dx\, \Psi_n^*\, p\, \Psi_n = -i\hbar\int_0^a dx\, \Psi_n^*(x)\left(\frac{\partial}{\partial x}\Psi_n(x)\right)$$

$$= m\frac{d\langle x\rangle}{dt} = 0$$

notum (1.31) úr bók

$$\langle p^2 \rangle = -\hbar^2 \int_0^a dx\, \Phi_n(x) \left[\frac{\partial^2}{\partial x^2} \Phi_n(x) \right]$$

$$= +\hbar^2 \frac{n^2\pi^2}{a^2} \int_0^a dx\, |\Phi_n(x)|^2 = \frac{\hbar^2 n^2 \pi^2}{a^2}$$

p.s. vi utnyttjar att $\Phi_n''(x) = -\frac{n^2\pi^2}{a^2}\Phi_n(x)$

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = a\sqrt{\left\{\frac{1}{3} - \frac{1}{2(\pi n)^2}\right\} - \frac{1}{4}}$$

$$= a\sqrt{\frac{1}{12} - \frac{1}{2(\pi n)^2}} = \frac{a}{2}\sqrt{\frac{1}{3} - \frac{2}{(\pi n)^2}}$$

$$\sigma_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \frac{\hbar n\pi}{a}$$

$$\rightarrow \sigma_x \sigma_p = \frac{\hbar}{2}\left[n\pi \sqrt{\frac{1}{3} - \frac{2}{(\pi n)^2}} \right]$$

$$\sigma_x \sigma_p \geq \frac{\hbar}{2} \quad \text{ef} \quad n\pi \sqrt{\frac{1}{3} - \frac{2}{(\pi n)^2}} \geq 1$$

$$\rightarrow \text{ef} \sqrt{\frac{n^2\pi^2}{3} - 2} \geq 1$$

sem er alltaf rétt fyrir $n = 1, 2, \dots$

Lægst er óvissan fyrir $n = 1$

2.8

Eind með massa m hefur jafnar líkur á að finnast vinstra megin í óendanlegum brunni klukkan $t=0$

a) Finna $\Phi(x,0)$

$$\Phi(x,0) = \begin{cases} A & \text{ef } 0 < x < \frac{a}{2} \\ 0 & \text{annars} \end{cases}$$

$$1 = \int_0^a dx \, |\Phi(x,0)|^2 = |A|^2 \int_0^{a/2} dx = |A|^2 \frac{a}{2}$$

$\rightarrow A = \sqrt{\frac{2}{a}}$ er möguleg lausn

b) Með hverjum líkum fæst $1 \cdot \frac{\pi^2\hbar^2}{2ma^2} = E_0 \cdot 1$ í orkumælingu

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Orkurófið er eigingildi H. Eiginástönd H eru

$$\psi_n(x) = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi}{a}x\right)$$

Þurfum táknum $\Psi(x,0)$ í ~~þessum~~ grunni

$$\Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x)$$

mælingin setur kerfið í eiginástönd H. Vogi ψ_1, c_1 er þá líkindavísirinn

$$c_n = \int_0^a dx\, \Psi(x,0)\, \psi_n^*(x) = \sqrt{\frac{2}{a}} \int_0^a \sin\left(\frac{n\pi}{a}x\right) \Psi(x,0)\, dx$$

$$C_n = \frac{2}{a}\int_0^{a/2} dx \,\sin\left(\frac{n\pi}{a}x\right) = 2\int_0^{1/2} du \,\sin(n\pi u)$$

$$= \frac{2}{n\pi}\left\{1 - \cos\left(\frac{n\pi}{2}\right)\right\}$$

$\rightarrow C_1 = \frac{2}{\pi}$ $\rightarrow$ liternor em $|C_1|^2 = \frac{4}{\pi^2}$

$\simeq 0{,}4$