Eind er lýst með bylgjufallinu

$$\psi(x) = A x^2 \exp\left(-\left(\frac{x}{a}\right)^2\right)$$

1 Finna A

$$\int_{-\infty}^{\infty} dx\, |\psi(x)|^2 = |A|^2 \int_{-\infty}^{\infty} dx\, x^4 \exp\left\{-2\left(\frac{x}{a}\right)^2\right\}$$

$$= |A|^2 a^5 \int_{-\infty}^{\infty} \frac{dx}{a} \left(\frac{x}{a}\right)^4 \exp\left\{-2\left(\frac{x}{a}\right)^2\right\} = |A|^2 a^5 \int_{-\infty}^{\infty} du\, u^4 e^{-2u^2}$$

$$= |A|^2 a^5 \frac{3\sqrt{\pi}}{2^{9/2}} = 1 \quad \to \quad A = \frac{2^{9/4}}{\sqrt{3\sqrt{\pi}\, a^5}}$$

er lausn

2. Reikna $\langle x \rangle$ og $\langle x^2 \rangle$

$$\langle x \rangle = \int_{-\infty}^{\infty} dx \; x \; |\psi(x)|^2 = A^2 \int_{-\infty}^{\infty} dx \; x^5 \exp\left(-2\left(\frac{x}{a}\right)^2\right) = 0$$

því fallið undir heildinu er oddstætt.

$$\langle x^2 \rangle = A^2 \int_{-\infty}^{\infty} dx \; x^6 \exp\left\{-2\left(\frac{x}{a}\right)^2\right\} = A^2 a^7 \int_{-\infty}^{\infty} du \; u^6 e^{-2u^2}$$

$$= A^2 a^7 \frac{15\sqrt{\pi}}{2^{13/2}} = \frac{2^{9/2} a^7 \; 15\sqrt{\pi}}{3\sqrt{\pi}\, a^5 \; 2^{13/2}} = \frac{5}{4} a^2$$

3 Reikna $\langle p \rangle$ og $\langle p^2 \rangle$

$$p\psi(x) = -i\hbar\partial_x \psi(x) = -i\hbar A\,\partial_x\left\{x^2 \exp\left(-\left(\frac{x}{a}\right)^2\right)\right\}$$

$$= -i\hbar A\left\{-\frac{2}{a^2}\left(x^3 - a^2 x\right)e^{-\left(\frac{x}{a}\right)^2}\right\}$$

$$p^2\psi(x) = -\hbar^2\partial_x^2\psi(x) = -\hbar^2 A\partial_x^2\left\{x^2 e^{-\left(\frac{x}{a}\right)^2}\right\}$$

$$= -\hbar^2 A\left\{\frac{2}{a^4}\left(2x^4 - 5a^2x^2 + a^4\right)e^{-\left(\frac{x}{a}\right)^2}\right\}$$

$$\langle p \rangle = \int_{-\infty}^{\infty} dx\ \psi^*_{(x)}\, p\, \psi_{(x)} = +i\hbar|A|^2\frac{2}{a^2}\int_{-\infty}^{\infty} dx\,\underbrace{\left\{x^5 - a^2x^3\right\}e^{-2\left(\frac{x}{a}\right)^2}}_{} = 0$$

fallið er oddstøtt á $(-\infty, \infty)$

$$\langle p^2\rangle = \int_{-\infty}^{\infty} dx\, \Phi^*(x)\, p^2\, \Phi(x)$$

$$= -\hbar^2 |A|^2 \frac{2}{a^4} \int_{-\infty}^{\infty} dx \left\{ 2x^6 - 5a^2x^4 + a^4x^2 \right\} e^{-2\left(\frac{x}{a}\right)^2}$$

$$= -\hbar^2 |A|^2\, 2a^3 \int_{-\infty}^{\infty} d\left(\tfrac{x}{a}\right) \left\{ 2\left(\tfrac{x}{a}\right)^6 - 5\left(\tfrac{x}{a}\right)^4 + \left(\tfrac{x}{a}\right)^2 \right\} e^{-2\left(\frac{x}{a}\right)^2}$$

$$= -\hbar^2 |A|^2\, 2a^3 \int_{-\infty}^{\infty} du \left\{ 2u^6 - 5u^4 + u^2 \right\} e^{-2u^2}$$

$$= -\hbar^2 |A|^2\, 2a^3 \left\{ -\frac{7\sqrt{\pi}}{2^{11/2}} \right\} = +\hbar^2 2a^3 \left\{ \frac{2^{9/2}}{3\sqrt{\pi}\, a^5} \right\} \left\{ \frac{7\sqrt{\pi}}{2^{11/2}} \right\}$$

$$\langle p^2 \rangle = \frac{\hbar^2}{a^2} \cdot \frac{7}{3}$$

4 Reiknum Δx og Δp

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\langle x^2 \rangle} = \sqrt{\frac{5}{4}}\, a$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\langle p^2 \rangle} = \frac{\hbar}{a}\sqrt{\frac{7}{3}}$$

5 $$\Delta x \cdot \Delta p = \hbar \sqrt{\frac{5 \cdot 7}{4 \cdot 3}} = \hbar \sqrt{\frac{35}{12}} \sim \hbar \cdot 1{,}708$$

$$\geq \frac{\hbar}{2}$$

þannig að óvissulögmálið er uppfyllt fyrir þetta ástand! $\rightarrow$