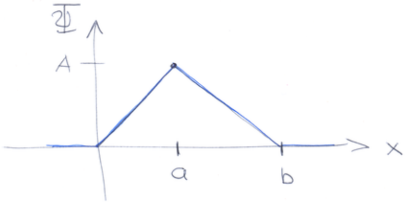
1,4 Klukkan $t = 0$ er bylgjufall einder

$$\Phi(x,0) = \begin{cases} A\dfrac{x}{a} & \text{ef } 0 \le x \le a \\[2ex] A\dfrac{(b-x)}{(b-a)} & \text{ef } a \le x \le b \\[2ex] 0 & \text{annars} \end{cases}$$

A, a, b eru fastar (jákvæðir)

Teiknum upp til að sjá útlitið



Hér væri æskilegra að gera graf af $\frac{\Phi}{A}$ v.s. $\frac{x}{a}$ og velja gildi á b/a.

a) Stuðla Φ

Likindi ~~þess~~ að finna eindina á öllu svæðinu eru

$$\int_{-\infty}^{\infty} dx \left| \Phi(x,0) \right|^2 = 1$$

$$= \int_0^a dx \left| \Phi(x,0) \right|^2 + \int_a^b dx \left| \Phi(x,0) \right|$$

$$= |A|^2 \int_0^a dx \left(\frac{x}{a}\right)^2 + |A|^2 \int_a^b dx \left(\frac{b-x}{b-a}\right)^2$$

$$|A|^2 \left\{ a \int_0^a \frac{dx}{a} \left(\frac{x}{a}\right)^2 + \frac{a^3}{(b-a)^2} \int_a^b \frac{dx}{a} \left(\frac{b}{a} - \frac{x}{a}\right)^2 \right\}$$

$$= |A|^2 a \left\{ \int_0^1 du\, u^2 + \frac{a^2}{(b-a)^2} \int_1^{b/a} du \left(\frac{b}{a} - u\right)^2 \right\}$$

$$= |A|^2 a \left\{ \frac{1}{3} + \frac{a^2}{(b-a)^2} \frac{1}{3} \left(\left(\frac{b}{a}\right)^3 - 3\left(\frac{b}{a}\right)^2 + 3\left(\frac{b}{a}\right) - 1 \right) \right\}$$

$$= |A|^2 a \frac{1}{3} \left\{ 1 + \frac{a^2}{(b-a)^2} \left(\left(\frac{b}{a}\right) - 1 \right)^3 \right\} = \frac{a}{3} |A|^2 \left\{ 1 + \frac{(b-a)}{a} \right\}$$

$$= \frac{a}{3} |A|^2 \frac{b}{a} = \frac{b}{3} |A|^2 = 1$$

$\rightarrow \quad A = \sqrt{\frac{3}{b}}$ er lausn

A hefur viddina $\frac{1}{\sqrt{L}}$ eins og normunin krefst

$$\int \underbrace{dx\, |\psi|^2}_{\text{viddarlaust}} = 1$$

b) sjá 1. blað.

c) Hvar er líklegast að finna eindina klukkan $t = 0$?

Í hápunkti $|\Psi|^2$ í $x = a$

d) Hver eru líkurnar fyrir því að finna eindina f. $x \leq a$

$$P(x \leq a) = \int_0^a dx \, |\Psi(x,0)|^2 = |A|^2 a \int_0^1 du \, u^2$$

$$= |A|^2 \frac{a}{3} = \frac{3}{b} \frac{a}{3} = \frac{a}{b}$$ heiltala, án víddar

Þegar $a \to b \longrightarrow P(x \leq a) \to 1$, enda er allt bylgjuf. á vinstra megin við a þegar $b \to 2a$, þá $a \to \frac{b}{2}$

$\to P(x \leq a) = \frac{1}{2}$ enda er bylgjufallið þá samhverft um $x = a$

e) Væntigildi $\langle x \rangle$?

$$\langle x \rangle = \int_{-\infty}^{\infty} dx \cdot x \, |\Psi(x,0)|^2$$

$$= \int_0^a dx \; x \, |\Psi(x,0)|^2 + \int_a^b dx \cdot x |\Psi(x,0)|^2$$

$$= |A|^2 a^2 \left\{ \int_0^1 du \, u^3 + \frac{a^2}{(b-a)^2} \int_1^{b/a} du \; u \left(u - \frac{b}{a}\right)^2 \right\}$$

$$= |A|^2 a^2 \left\{ \frac{b^2 + 2ab}{12a^2} \right\} = \frac{3}{b} \, \frac{b^2 + 2ab}{12}$$

$$= \frac{b + 2a}{4}$$

med rätta vidd

1.14 $P_{ab}(t)$ eru líkur ~~þess~~ að finna eind á bilinu (a,b) á tíma t

a) Sýna að

$$d_t P_{ab} = J(a,t) - J(b,t)$$

þar sem

$$J(x,t) = \frac{i\hbar}{2m}\left\{\Psi\,\partial_x\Psi^* - \Psi^*\partial_x\Psi\right\}$$

Við vitum að

$$P_{ab}(t) = \int_a^b dx\,|\Psi(x,t)|^2$$

$$d_t P_{ab}(t) = d_t\left\{\int_a^b dx\,|\Psi(x,t)|^2\right\} = \int_a^b dx\, d_t\left[|\Psi(x,t)|^2\right]$$

x is here independent of t

$$\rightarrow d_t P_{ab}(t) = \int_a^b dx\, \partial_t |\Phi(x,t)|^2 = \int_a^b dx \left\{ (\partial_t \Phi^*)\Phi + \Phi^* (\partial_t \Phi) \right\}$$

Notum jöfnu Schrödingers

$$i\hbar \partial_t \Phi = H\Phi$$

$$-i\hbar \partial_t \Phi^* = H\Phi^*$$

$$d_t P_{ab}(t) = \int_a^b dx \left\{ \left(\frac{-1}{i\hbar} H\Phi^*\right)\Phi + \Phi^*\left(\frac{1}{i\hbar} H\Phi\right) \right\}$$

Ef nú

$$H = \frac{p^2}{2m} + V \quad \longrightarrow \quad -\frac{\hbar^2}{2m}\partial_x^2 + V$$

þá fæst

$$d_t P_{ab}(t) = \int_a^b dx \left\{ \frac{\hbar}{2mi}\left(\partial_x^2 \Phi^*\right)\Phi - \frac{\hbar}{2mi}\Phi^*\left(\partial_x^2 \Phi\right) \right\}$$

{krossliðirnir styttast út þ. afleiðan er tekin}

$$d_t P_{ab}(t) = \int_a^b dx\, \partial_x \left\{ \frac{\hbar}{2mi}\left[\left(\partial_x \Phi^*\right)\Phi - \Phi^*\left(\partial_x \Phi\right) \right] \right\}$$

$$= \int_a^b dx\, \partial_x \underbrace{\left\{ \frac{i\hbar}{2m}\left[\Phi^*\left(\partial_x \Phi\right) - \left(\partial_x \Phi^*\right)\Phi \right] \right\}}$$

$$= -\int_a^b dx\, \partial_x J(x,t) = -J(b,t) + J(a,t)$$

Breytingin á líkindunum innan bilsins eru aðeins vegna strauma inn eða út úr bilinu á jöðrum þess. Líkindin eru víddarlaus $\longrightarrow$ straumur, $d_t P_{ab}$ hefur vídd T^{-1}

b) $$\Psi(x,t) = A \exp\left\{-a\left(\frac{mx^2}{\hbar} + it\right)\right\}$$

Hér er $\Psi(x,t) = \phi(x)\, e^{-ait}$ þ.s. $\phi(x) \in \mathbb{R}$

$$\longrightarrow \quad J(x,t) = 0$$