SI-grunneiningar

ISQ Base Quantity	SI Base Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Electrical current	ampere (A)
Thermodynamic temperature	kelvin (K)
Amount of substance	mole (mol)
Luminous intensity	candela (cd)

Table 1.1 ISQ Base Quantities and Their SI Units

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Frá 2019 allar tengdar skammtafyrirbærum

Allar aðrar mælieiningar tengjast þessum, t.d.

lítri fyrir vökarúmmál
Pascal fyrir loftþrýsting
ohm fyrir rafviðnám
volt fyrir rafspennu.....

víddargreining

Base Quantity	Symbol for Dimension
Length	L
Mass	M
Time	T
Current	I
Thermodynamic temperature	Θ
Amount of substance	N
Luminous intensity	J

Table 1.3 Base Quantities and Their Dimensions

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En, einföldum okkur lífið aðeins:

Með heppilegri skölun jafna er hægt að notast aðeins við 3 grunnvíddir:

L, M, T

Víddagreining hjálpar okkur við að finna villur í reikningum

Dæmi um víddargreiningu

Hraði $[v] = \frac{L}{T}$

Hröðun $[a] = \frac{L}{T^2}$

Orka $[E] = M\frac{L^2}{T^2}$

$$s = s_0 + vt + \frac{1}{2} g t^2$$

$$L \qquad L \qquad \frac{L}{T}T \qquad \frac{L}{T^2}T^2$$

→ allt í samræmi!

Massaþéttleiki $[\rho] = \frac{M}{L^3}$

heppileg skölun, eða samantekt

Fínstrúktúrfastinn víddarlaus:

$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}, \qquad 1 = [\alpha] = \left[\frac{e^2}{\epsilon_0}\frac{1}{\hbar c}\right] = \left[\frac{e^2}{\epsilon_0}\right]\left(\frac{T}{ML^2}\right)\left(\frac{T}{L}\right)$$

$$\rightarrow \left[e^2/\epsilon_0\right] = \frac{ML^3}{T^2}$$

$$x = a\sin(\omega t) \rightarrow [a] = L, \quad [\omega t] = 1, \rightarrow [\omega] = \frac{1}{T}$$

$$E = E_0\cos(kx) \rightarrow [E_0] = M\frac{L^2}{T^2}, \quad [kx] = 1,$$

$$\rightarrow [k] = \frac{1}{L}$$

$$s = \int dt\, v \rightarrow [s] = T \cdot \frac{L}{T} = L$$

$$\frac{dx}{dt} = v \rightarrow [v] = \frac{L}{T}$$

5

Vigur- og skalarmælistærðir

Hraði (velocity) hefur lengd (speed, ferð) og stefnu, táknum sem vigur:

$$\vec{v}\,, \quad v = |\vec{v}|$$

Sama gildir um staðsetningu, hröðun, rafsvið, segulsvið, kraft,....

Massi, hleðsla, þéttleiki, vinna, afl og orka eru skalarstærðir með enga stefnu

Vigrar eru í sjálfu sér óháðir hnitakerfum, en tengum við þau seinna

Notaðir til að einfalda framsetningu án þess að drukkna í "bókhaldi" um hnit

6

Samsíða eða hornréttir ....

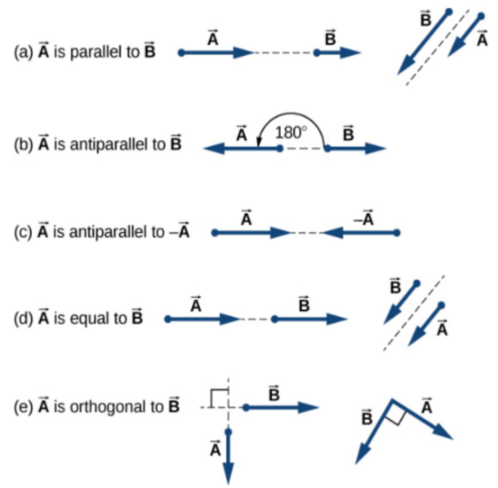

Figure 2.5 Various relations between two vectors $\vec{A}$ and $\vec{B}$. (a) $\vec{A} \neq \vec{B}$ because $A \neq B$. (b) $\vec{A} \neq \vec{B}$ because they are not parallel and $A \neq B$. (c) $\vec{A} \neq -\vec{A}$ because they have different directions (even though $|\vec{A}| = |-\vec{A}| = A$). (d) $\vec{A} = \vec{B}$ because they are parallel *and* have identical magnitudes $A = B$. (e) $\vec{A} \neq \vec{B}$ because they have different directions (are not parallel); here, their directions differ by 90°—meaning, they are orthogonal.

7

Samlagning og frádráttur vigra

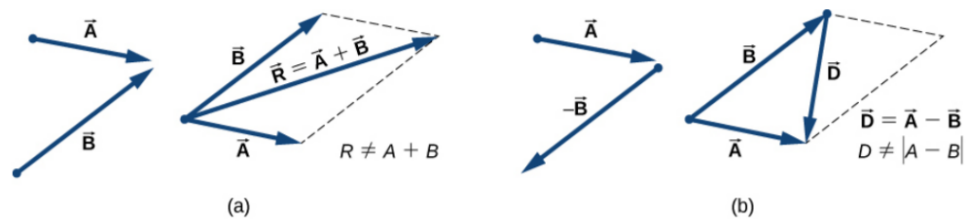

Figure 2.10 The parallelogram rule for the addition of two vectors. Make the parallel translation of each vector to a point where their

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Aflfræði Newtons, rafsegulfræði Maxwells og straumfræði hafa miklu einfaldari framsetningu en ella með vigrum...

8

Hægt að margfalda skalar og vigur

$$\boxed{\vec{F} = m\,\vec{a}}$$

"skölun á vigri", skalar og vigurstærðin þurfa ekki að hafa sömu vídd

Tvo vigra má innfalda

Scalar Product (Dot Product)

The **scalar product** $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is a number defined by the equation

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\varphi, \qquad (2.27)$$

where φ is the angle between the vectors (shown in Figure 2.27). The scalar product is also called the **dot product** because of the dot notation that indicates it.

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Innfeldi tveggja vigra býr til skalarstærð

9

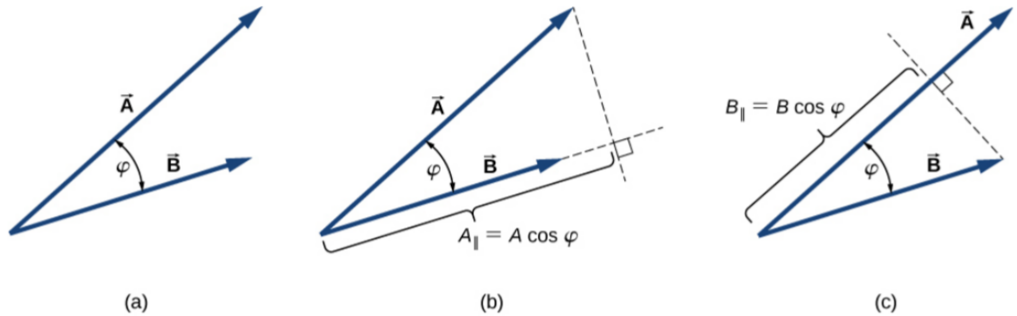


Figure 2.27 The scalar product of two vectors. (a) The angle between the two vectors. (b) The orthogonal projection $A_{||}$ of vector $\vec{A}$ onto the direction of vector $\vec{B}$. (c) The orthogonal projection $B_{||}$ of vector $\vec{B}$ onto the direction of vector $\vec{A}$.

Fyrir hornrétta vigra fæst

$$\bar{A}\cdot\bar{B} = 0$$

Víxlið

$$\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$$

Krossfeldi tveggja vigra

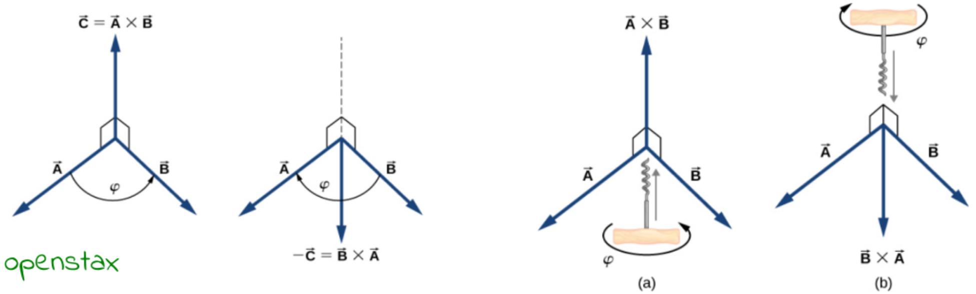
10

Vector Product (Cross Product)

The **vector product** of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\times\vec{B}$ and is often referred to as a **cross product**. The vector product is a vector that has its direction perpendicular to both vectors $\vec{A}$ and $\vec{B}$. In other words, vector $\vec{A}\times\vec{B}$ is perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$, as shown in Figure 2.29. The magnitude of the vector product is defined as

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\varphi, \qquad 2.35$$

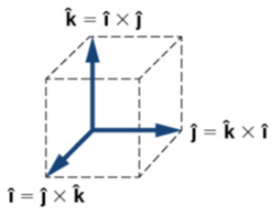
where angle φ, between the two vectors, is measured from vector $\vec{A}$ (first vector in the product) to vector $\vec{B}$ (second vector in the product), as indicated in Figure 2.29, and is between 0° and 180°.



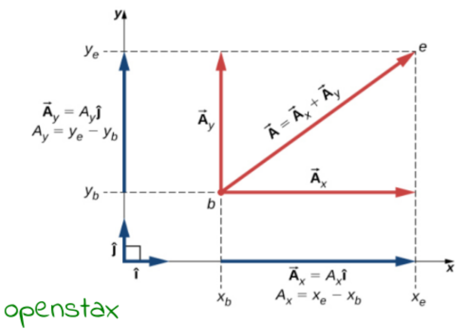
Krossfeldi tveggja vigra býr til þriðja vigurinn hornréttan í hina fyrri

11

Hnitakerfi, kartísk hnit



$$\begin{cases}\hat{\mathbf{i}}\times\hat{\mathbf{j}} = +\hat{\mathbf{k}},\\ \hat{\mathbf{j}}\times\hat{\mathbf{k}} = +\hat{\mathbf{i}},\\ \hat{\mathbf{k}}\times\hat{\mathbf{i}} = +\hat{\mathbf{j}}.\end{cases}$$



Vigur táknaður með hnitum eða eingarvigrum hnitakerfis

$$\bar{A} = A_x\hat{i} + A_y\hat{j}$$

$$\bar{A} = (A_x, A_y)$$

Lengd vigurs $\quad A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

12

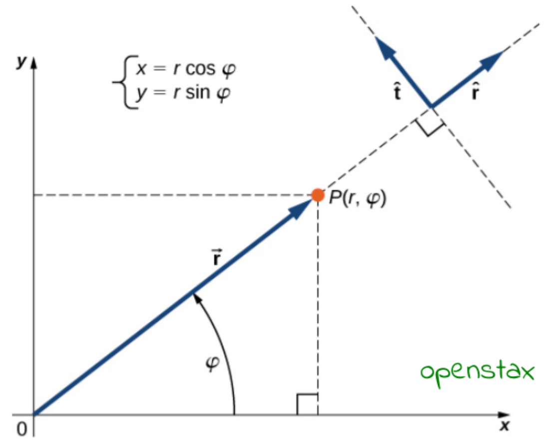
Pól- eða skauthnit

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}.$$

Tökum eftir að víddir r og φ eru ekki þær sömu

Einingarvigur fyrir hornstefnuna $\hat{t}$ eða $\hat{\varphi}$ er ekki með "fasta stefnu"

Lengd?



1

Einvíð hreyfilýsing, hliðrun (ekki skoðað hvað veldur hreyfingunni)

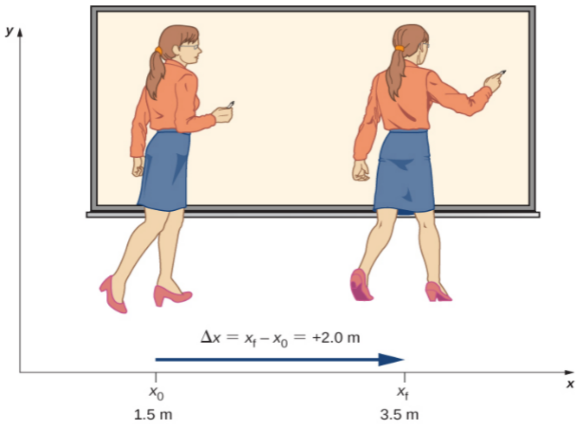


Figure 3.3 A professor paces left and right while lecturing. Her position relative to Earth is given by *x*. The +2.0-m displacement of the professor relative to Earth is represented by an arrow pointing to the right.

Displacement

Displacement Δx is the change in position of an object:

$$\Delta x = x_f - x_0, \qquad (3.1)$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

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2

Meðalhraði

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Average Velocity

If x_1 and x_2 are the positions of an object at times t_1 and t_2, respectively, then

$$\text{Average velocity} = \bar{v} = \frac{\text{Displacement between two points}}{\text{Elapsed time between two points}} \qquad (3.3)$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

Takið eftir að meðalhraðinn getur orðið neikvæður, stefna vigurs í 1-D fer eftir formerki eina hnits hans ...

3

Skoðum hreyfingu

A sketch of Jill's movements is shown in Figure 3.4.

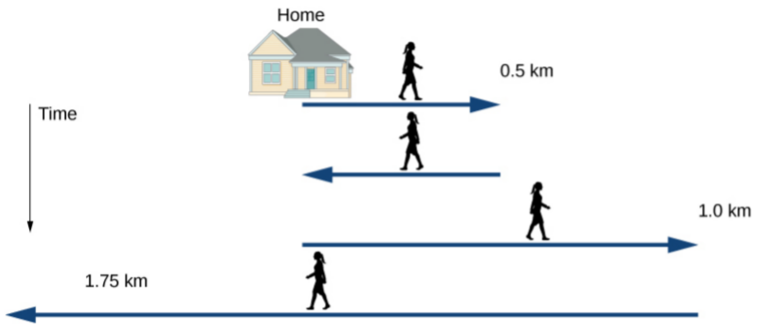

Figure 3.4 Timeline of Jill's movements.

Time t_i (min)	Position x_i (km)	Displacement Δx_i (km)
$t_0 = 0$	$x_0 = 0$	$\Delta x_0 = 0$
$t_1 = 9$	$x_1 = 0.5$	$\Delta x_1 = x_1 - x_0 = 0.5$
$t_2 = 18$	$x_2 = 0$	$\Delta x_2 = x_2 - x_1 = -0.5$
$t_3 = 33$	$x_3 = 1.0$	$\Delta x_3 = x_3 - x_2 = 1.0$
$t_4 = 58$	$x_4 = -0.75$	$\Delta x_4 = x_4 - x_3 = -1.75$

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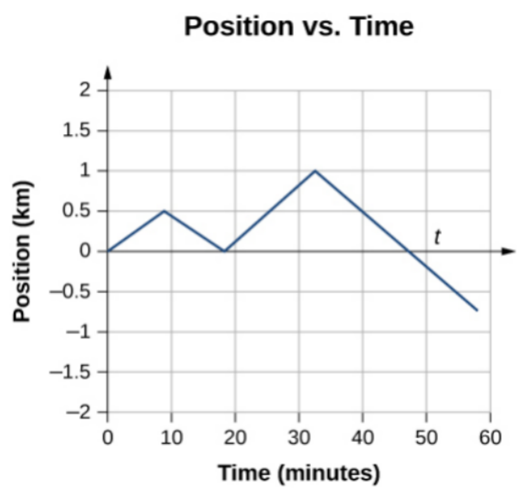
4

Myndræn framsetning, graf



Ekki gott dæmi til að skoða hröðun...

Getum lesið meðalhraðann beint af grafinu

Viljum frekar notast við staðsetningu, og hraða og hröðun í hverjum tímapunkti

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5

Hraði á vissum tímapunkti

$$v(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} = \frac{dx(t)}{dt}.$$

Instantaneous Velocity

The instantaneous velocity of an object is the limit of the average velocity as the elapsed time approaches zero, or the derivative of x with respect to t:

$$v(t) = \frac{d}{dt}x(t). \qquad 3.4$$

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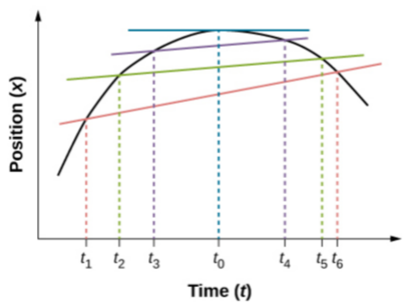
$v(t_0)$ = slope of tangent line



Hér sést hvernig rétt gildi fæst þegar tímabilið verður æ styttra

6

Dæmi (Ex. 3.4 í bók)

eining metrar

$$x(t) = (3t - 3t^2)\ \text{m}$$

$$v(t) = \frac{dx(t)}{dt} = (3 - 6t)\ \text{m/s}$$

einvíðir vigrar, staða og hraði

ferð (speed)

$$|v(t)| = |3 - 6t|\ \text{m/s}$$

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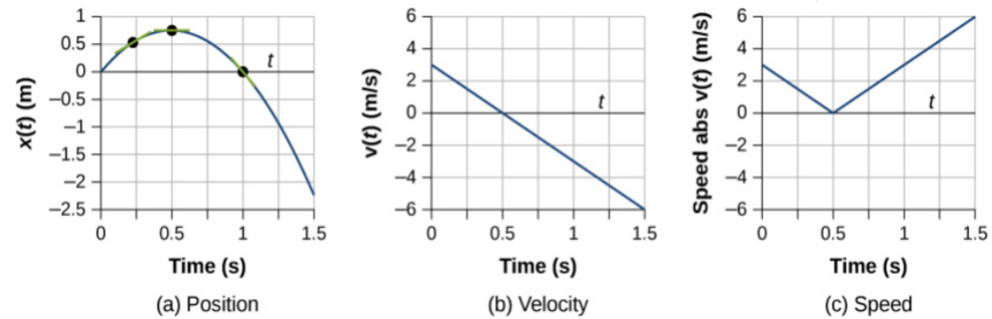

7

Meðalhröðun -- hröðun, (vigurstærðir)

Average Acceleration

Average acceleration is the rate at which velocity changes:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}, \qquad 3.8$$

where $\bar{a}$ is **average acceleration**, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

heppilegra að nota <a> fyrir meðaltal af stærðinni a í handskrift

Hröðun á tímapunkti

$$a(t) = \frac{d}{dt}v(t).$$

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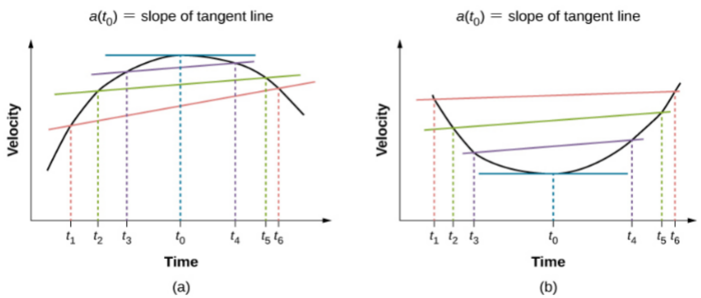

8

Dæmi (Ex. 3.6)

$$\bar{v}(t) = (20t - 5t^2)\ \text{m/s}$$

$$\bar{a}(t) = (20 - 10t)\ \text{m/s}^2$$

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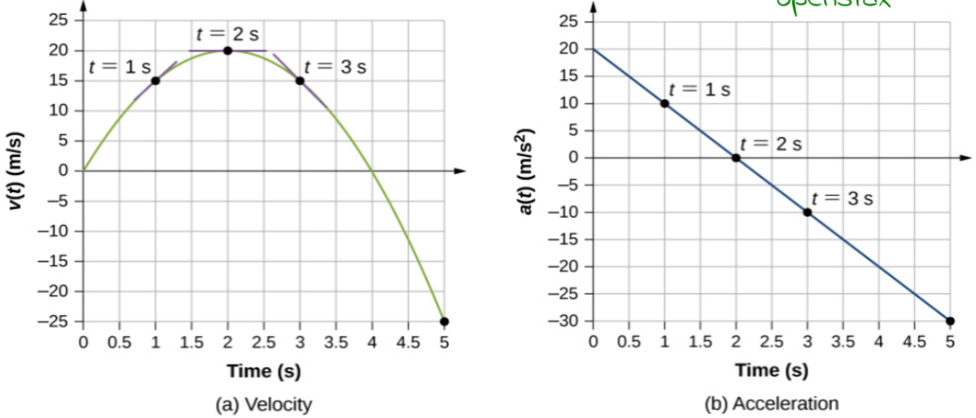

9

Einvíð hreyfing með fastri hröðun a

Almennt gildir $\frac{dv(t)}{dt} = a(t)$

Skoðum sértilfellið $\frac{dv(t)}{dt} = a$, a er fasti

$$\rightarrow \frac{dv(t)}{dt}dt = a\,dt \rightarrow dv(t) = a\,dt$$

$$\rightarrow \int_{v_0}^{v(t)} dv'(t) = a\int_{t_0}^{t} dt' \rightarrow v(t) - v_0 = a(t-t_0)$$

1 $$\rightarrow \boxed{v(t) = v_0 + a(t-t_0)}$$

10

Almennt gildir $\frac{dx(t)}{dt} = v(t) \rightarrow \frac{dx(t)}{dt}dt = v(t)dt$

Heildum $\int_{x_0}^{x(t)} dx'(t) = \int_{t_0}^{t} v(t')dt' = \int_{t_0}^{t} dt'\left[v_0 + a(t'-t_0)\right]$

$$\rightarrow x(t) - x_0 = v_0(t-t_0) + \frac{1}{2}a(t^2 - t_0^2) - at_0(t-t_0)$$

$$= v_0(t-t_0) + \frac{1}{2}a(t-t_0)^2$$

2 $$\rightarrow \boxed{x(t) = x_0 + v_0(t-t_0) + \frac{1}{2}a(t-t_0)^2}$$

11

Þorum við að taka saman þessar tvær jöfnur til að losna við t og finna samband staðsetningar og hraða?

1 $$\rightarrow (t-t_0) = \frac{v(t) - v_0}{a}$$

2 $$\rightarrow x(t) - x_0 = v_0(t-t_0) + \frac{a}{2}(t-t_0)^2$$

$$\rightarrow x(t) - x_0 = v_0\left(\frac{v(t)-v_0}{a}\right) + \frac{a}{2}\frac{(v(t)-v_0)^2}{a^2}$$

$$= \frac{v^2(t) - v_0^2}{2a}$$

3 $$\rightarrow \boxed{v^2(t) = v_0^2 + 2a(x(t) - x_0)}$$

12

Frjálst fall, (einvídd)

Hreyfijöfnurnar fyrir frjálsu falli fást því með að setja "a --> -g" þar sem þyngdarhröðunin er valin t.d. sem $g = 9.81\ m/s^2$

$$\boxed{\begin{aligned} a &= -g \\ v(t) &= v_0 - g(t-t_0) \\ y(t) &= y_0 + v_0(t-t_0) - \frac{g}{2}(t-t_0)^2 \\ v^2(t) &= v_0^2 - 2g(y-y_0) \end{aligned}}$$

Þær má einfalda með vali á upphafsgildum, en mikilvægt er að taka eftir og halda réttu bókhaldi um formerkin

þrí- og tvívíð hreyfing

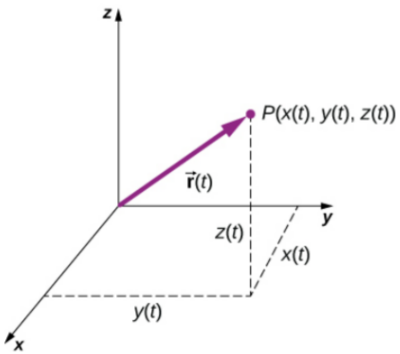

Figure 4.2 A three-dimensional coordinate system with a particle at position $P(x(t), y(t), z(t))$.

$$\bar{r} = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

$$= (x(t), y(t), z(t))$$

$$\vec{\mathbf{v}}(t) = \lim_{\Delta t \to 0} \frac{\vec{\mathbf{r}}(t+\Delta t) - \vec{\mathbf{r}}(t)}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}.$$

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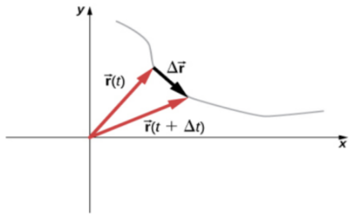
Staðsetning og hliðrun



$$v_x(t) = \frac{dx(t)}{dt}, \quad v_y(t) = \frac{dy(t)}{dt}, \quad v_z(t) = \frac{dz(t)}{dt}.$$

②

Independence of Motion

In the kinematic description of motion, we are able to treat the horizontal and vertical components of motion separately. In many cases, motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

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Við eigum eftir að læra um hreyfijöfnur, sem hægt er að leiða út frá lögmálum Newtons

$$\bar{F} = m\bar{a}$$

ef hreyfijöfnurnar fyrir hvert hnit "blanda" ekki hnitum þá er hægt að leysa hverja fyrir sig og líta svo á að þættir hreyfingarinnar séu óháðir

(Við notum kartísk hnit og eigum við einfalda krafta til að byrja með)

③

Dæmi (Ex. 4.6)

Upphaf $t=0$, $\bar{r}(0) = x_0\hat{i} + y_0\hat{j}$

$\bar{v}(0) = v_{0x}\hat{i} + v_{0y}\hat{j}$

$x_0 = 75\,m$

$y_0 = -50\,m$

$v_{0x} = 4{,}1\,m$

$v_{0y} = -1{,}1\,m$

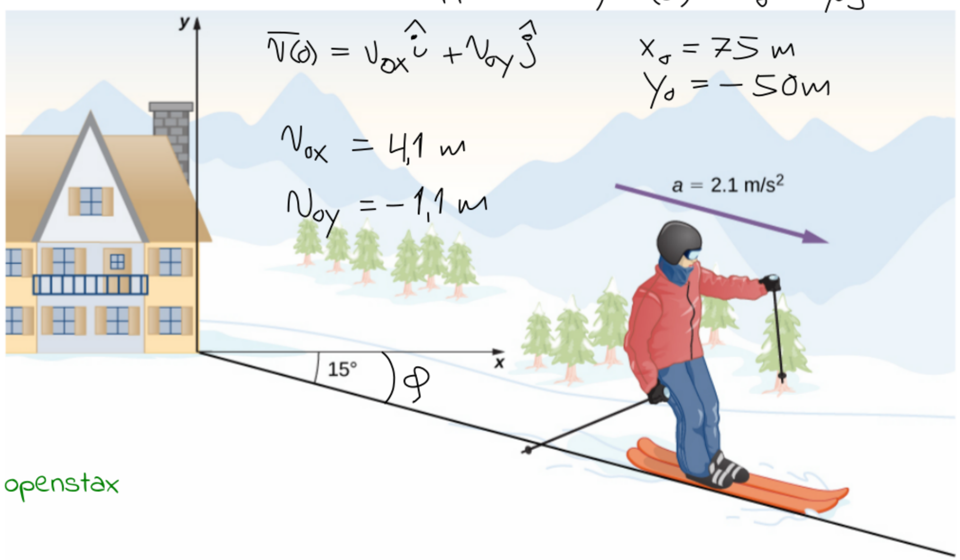

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Figure 4.10 A skier has an acceleration of $2.1\ m/s^2$ down a slope of $15°$. The origin of the coordinate system is at the ski lodge.

Finna staðsetningu og hraða sem fall af tíma

④

Seinna lærum við að þyngdarkrafturinn sé lóðréttur, þannig að lárétti þáttur hröðunarinnar gæti aðeins komið frá vindi eða einhverjum hreyfli sem skíðakonan hefði, en látum það vera.

$$a_x = a\cos(-\varphi) = a\cos\varphi, \quad a = |\bar{a}|$$

$$a_y = a\sin(-\varphi) = -a\sin\varphi, \quad \varphi = 15°$$

$$\rightarrow \bar{a} = a(\cos\varphi, -\sin\varphi)$$

$$\boxed{\begin{aligned} v_x(t) &= v_{x0} + a\cos\varphi \cdot t \\ v_y(t) &= v_{y0} - a\sin\varphi \cdot t \end{aligned}}, \quad t_0 = 0$$

5

$$x(t) = x_0 + v_{0x} t + \frac{a\cos\varphi}{2} t^2$$

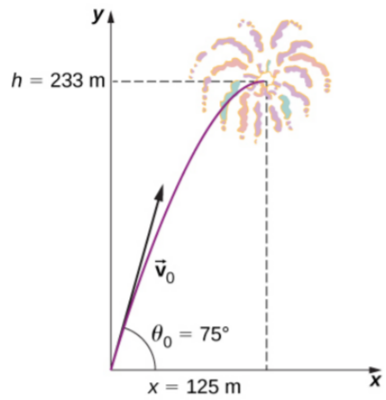
$$y(t) = y_0 + v_{0y} t - \frac{a\sin\varphi}{2} t^2$$

Síðan get ég sett inn tölurnar sem voru gefnar í upphafi til að svara enn nákvæmar, en ég setti ekki inn tölur fyrir hröðunina og hornið í upphafi því á þessum ham get ég spurt spurninga um hvað gerist þegar horninu er breytt og álíka spurningum.

Því getur verið best að bíða með að setja inn tölur til að halda meiri upplýsingum í jöfnunum. Eins er þægilegt að þurfa ekki að burðast um með einingar. Hér að lokum er auðvelt að sannreyna að allar víddir eru í lagi.

6

Dæmi (Ex. 4.7)



Einungis þyngdarhröðun
Springur í hæsta punkti

$v_0 = 70$ m/s

Finna Δt til sprengingar
Finna h og Δx
Finna fjarlægð s frá upphafspunkti og horn

Setjum $t_0 = 0$, $x_0 = 0$, $y_0 = 0$

$a_x = 0$, $a_y = -g$, $g = 9{,}81\ m/s^2$

7

Hápunktur hreyfingarinnar næst þegar $v_y(t) = 0$

$$v_y^2(t) = v_{y0}^2 - 2gy$$

$$0 = v_{y0}^2 - 2gy \rightarrow y = \frac{v_{y0}^2}{2g}$$

og $v_{y0} = v_0 \sin\theta_0$ $\rightarrow$ $\boxed{y = \frac{v_0^2 \sin^2\theta_0}{2g}}$

Þekkjum allar stærðir hér

$$y = \frac{(70)^2\, m^2/s^2 \sin^2\left(\frac{75\pi}{180}\right)}{2\cdot 9{,}81\ m/s^2} \approx 233\ m$$

8

Δt?

Höfum $v_y(t) = v_{y0} - gt$, Efst $v_y(t) = 0$

$$\rightarrow 0 = v_{y0} - gt \rightarrow \Delta t = t = \frac{v_{y0}}{g}$$

$$\rightarrow \Delta t = \frac{v_0 \sin\theta_0}{g} = \frac{70\cdot \sin\left(\frac{75\pi}{180}\right) s}{9{,}81} = 6{,}9\ s$$

Δx?

$$x(t) = v_{0x} t = v_0 \cos\theta_0\, \Delta t$$

$$= v_0 \cos\theta_0 \frac{v_0 \sin\theta_0}{g} = \frac{v_0^2}{g} \cos\theta_0 \sin\theta_0$$

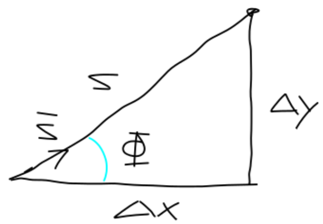
$$\simeq 125\ m$$

Heildarhliðrun að sprengingu

$$\vec{s} = \Delta x\,\hat{i} + \Delta y\,\hat{j} = \left(125\,\hat{i} + 233\,\hat{j}\right)\,\mathrm{m}$$

$$\rightarrow \quad |\vec{s}| = \sqrt{\Delta x^2 + \Delta y^2} \simeq 264\,\mathrm{m}$$

Stefna að sprengistað, Φ



$$\tan\Phi = \frac{\Delta y}{\Delta x}$$

$$\rightarrow \quad \arctan\left(\frac{\Delta y}{\Delta x}\right) = \Phi$$

$$= \arctan\left(\frac{233}{125}\right) = 1{,}078\,\mathrm{rad}$$

$$= 1{,}078 \cdot \frac{180}{\pi} = 61{,}8^\circ$$

Kastbrautin, flugtími og seilni

Án loftviðnáms er brautin samhverf um hápunkt, við vorum búin að finna tímann frá upphafi þegar t = o að hápunkti, tvöfaldur þessi tími er þá flugtíminn

$$\boxed{T_{Total} = \frac{2(v_0 \sin\theta_0)}{g}}$$

Við vorum með

① $x = (v_0\cos\theta_0)t$, $\quad x_0 = 0$, $t_0 = 0$, $y_0 = 0$

og

② $y = (v_0\sin\theta_0)t - \frac{g}{2}t^2$

getum við losnað við t og fengið brautar jöfnu: $\overbrace{y = ax + bx^2}^{\text{Fleygbogi}}$?

reynum

① → $t = \dfrac{x}{v_0\cos\theta_0}$

→ ② → $y = \dfrac{v_0\sin\theta_0}{v_0\cos\theta_0}x - \dfrac{g}{2}\left(\dfrac{x}{v_0\cos\theta_0}\right)^2$

$$\rightarrow \quad \boxed{y = \tan\theta_0 \cdot x - \frac{g}{2(v_0\cos\theta_0)^2}x^2}$$

Fleygbogi $\quad y = ax + bx^2$

með fasta $\quad a = \tan\theta_0 \quad$ og $\quad b = -\dfrac{g}{2(v_0\cos\theta_0)^2}$

Seilni

$$y = \left[\tan\theta_0 - \frac{g\,x}{2(v_0\cos\theta_0)^2}\right]x$$

nú gildir að y = o í upphafi og í lokin

Lausnin í upphafi er x = o, en jafnan hefur aðra núllstöð

$$y = 0 \quad \text{þegar} \quad \tan\theta_0 - \frac{gx}{2(v_0\cos\theta_0)^2} = 0$$

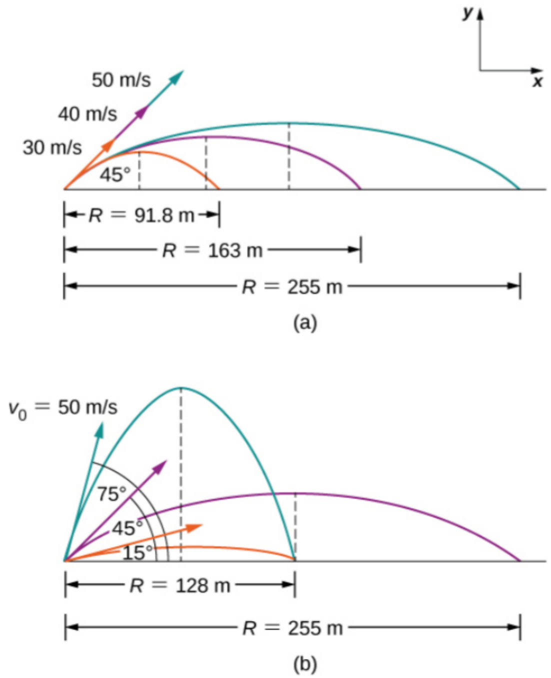
$$\rightarrow \quad x = \tan\theta_0 \frac{2(v_0\cos\theta_0)^2}{g} = \frac{2v_0^2}{g}\sin\theta_0\cos\theta_0$$

því er seilnin R

$$\boxed{R = \frac{v_0^2}{g}\sin(2\theta_0)}, \qquad 2\sin\theta_0\cos\theta_0 = \sin(2\theta_0)$$

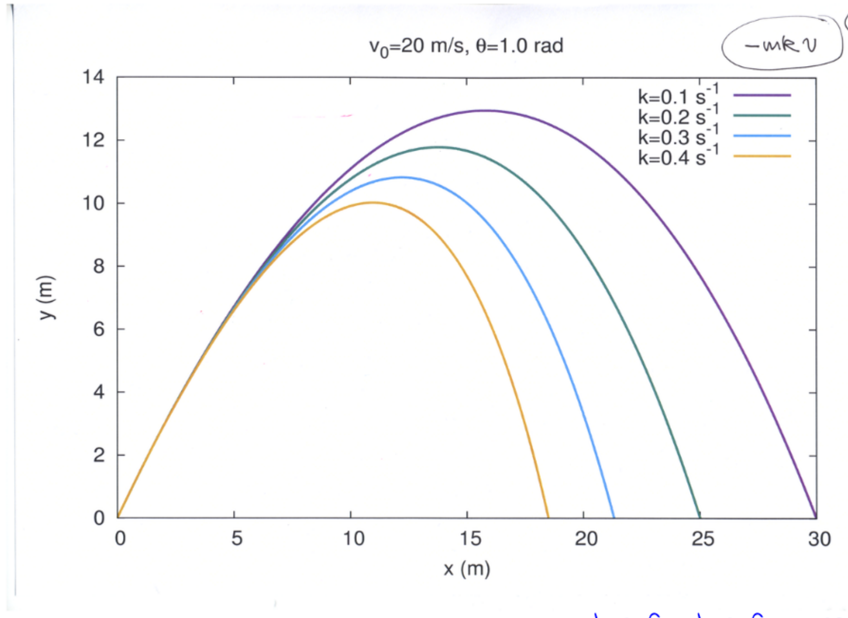
Þýðing niðurstaðna

án loftviðnáms



Með loftviðnámi í réttu hlutfalli við ferð

$-mkv$



samhverfan hverfur með loftviðnámi

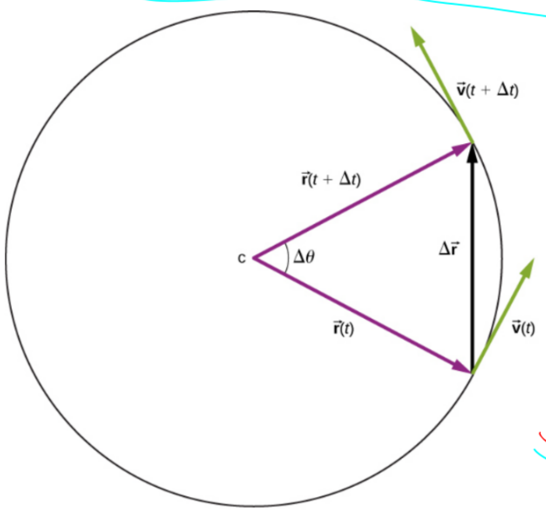
1

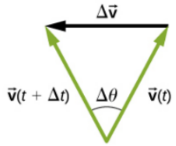
Stöðug hringhreyfing

$$|\vec v(t)| = |\vec v(t')|$$
$$|\vec r(t)| = |\vec r(t')|$$

fyrir öll t og t'

$$|\vec r(t)| = |\vec r(t+\Delta t)|$$
$$|\vec v(t)| = |\vec v(t+\Delta t)|$$





Jafnarma einslaga þríhyrningar

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \longrightarrow \Delta v = \left(\frac{v}{r}\right)\Delta r$$

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2

Radialhröðun -- hröðun útþáttar

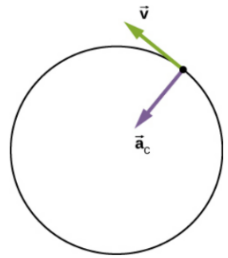
$$a = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(\frac{v}{r}\frac{\Delta r}{\Delta t}\right)$$

$$= \frac{v}{r}\lim_{\Delta t \to 0}\left(\frac{\Delta r}{\Delta t}\right) = \frac{v}{r}v = \frac{v^2}{r}$$

fyrir jafna hringhreyfingu verður að vera föst miðsóknarhröðun

$$\boxed{a_c = \frac{v^2}{r}}$$

að miðju hringbrautar



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3

Dæmi um stærð miðsóknarhröðunar

Object	Centripetal Acceleration (m/s^2 or factors of *g*)
Earth around the Sun	5.93×10^{-3}
Moon around the Earth	2.73×10^{-3}
Satellite in geosynchronous orbit	0.233
Outer edge of a CD when playing	5.78
Jet in a barrel roll	(2–3 *g*)
Roller coaster	(5 *g*)
Electron orbiting a proton in a simple Bohr model of the atom	9.0×10^{22}

Table 4.1 Typical Centripetal Accelerations

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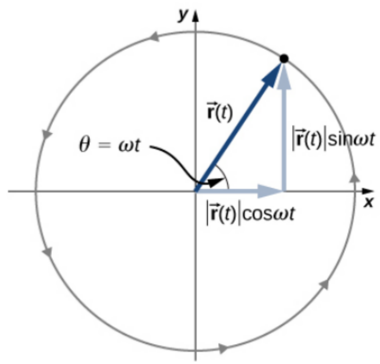
4

Lýsing jafnrar brautarhreyfingar

Hér væri hægt að nota pólhnit, en ....
Byrjum með kartísk hnit

Ef $A = |\vec r(t)|$ $\longrightarrow$

$$\boxed{\vec r(t) = A\cos(\omega t)\hat i + A\sin(\omega t)\hat j}$$



$\theta = \omega t$, ω horntíðni

$T = \frac{2\pi}{\omega}$, Lota

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5

Í kartískum hnitum fæst

$$\vec{\mathbf{v}}(t) = \frac{d\vec{\mathbf{r}}(t)}{dt} = -A\omega \sin \omega t\hat{\mathbf{i}} + A\omega \cos \omega t\hat{\mathbf{j}}.$$

og

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$$\vec{\mathbf{a}}(t) = \frac{d\vec{\mathbf{v}}(t)}{dt} = -A\omega^2 \cos \omega t\hat{\mathbf{i}} - A\omega^2 \sin \omega t\hat{\mathbf{j}}.$$

Því sést líka að

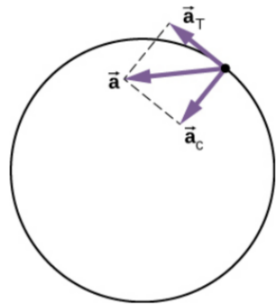
$$\boxed{\bar{a}(t) = -\omega^2 \bar{r}(t)}$$

Í pólhnitum er erfiðara að finna afleiðurnar því einingarvigrarnir eru líka háðir tíma. Svo er ekki í kartískum hnitum

6

Ójöfn hringhreyfing

Til viðbótar við miðsóknarhröðunina birtist snertilhröðun



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$$\boxed{a_T = \frac{d}{dt}|\bar{v}(t)|}$$

og heildarhröðunin verður

$$\boxed{\bar{a} = \bar{a}_c + \bar{a}_T}$$

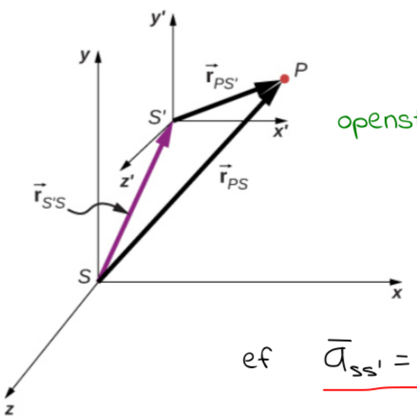
Þar sem miðsóknarhröðunina má reikna á sama hátt og áður

7

Afstæður hraði

T.d. hraði flugvélar miðað við jörð eða loft

Tvö viðmiðunarkerfi S og S'



$$\vec{\mathbf{r}}_{PS} = \vec{\mathbf{r}}_{PS'} + \vec{\mathbf{r}}_{S'S}$$

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$$\vec{\mathbf{v}}_{PS} = \vec{\mathbf{v}}_{PS'} + \vec{\mathbf{v}}_{S'S}$$

ef $\bar{a}_{ss'} = 0$ $\rightarrow$ $\vec{\mathbf{a}}_{PS} = \vec{\mathbf{a}}_{PS'}$

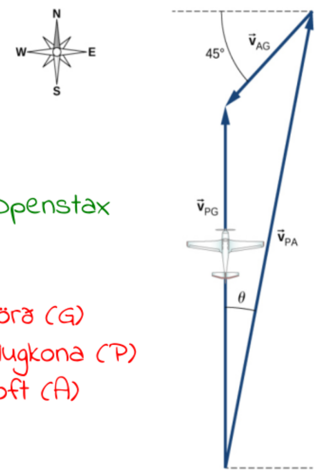
Figure 4.26 The positions of particle P relative to frames S and S' are $\vec{\mathbf{r}}_{PS}$ and $\vec{\mathbf{r}}_{PS'}$, respectively.

8

EXAMPLE 4.14

Flying a Plane in a Wind

A pilot must fly his plane due north to reach his destination. The plane can fly at 300 km/h in still air. A wind is blowing out of the northeast at 90 km/h. (a) What is the speed of the plane relative to the ground? (b) In what direction must the pilot head her plane to fly due north?



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Jörð (G)
Flugkona (P)
Loft (A)

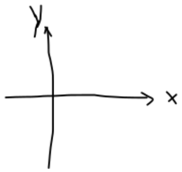
$\bar{v}_{AG}$: vindhraði miðað við jörð

$\bar{v}_{PG}$: hraði vélar miðað við jörð

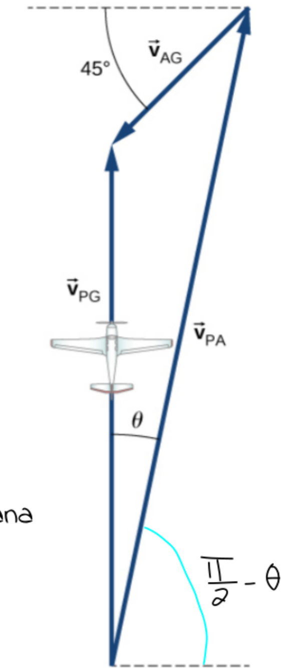
$\bar{v}_{PA}$: hraði vélar miðað við loft

Þekkjum ekki θ og $|\bar{v}_{PG}|$
en vitum $|\bar{v}_{PA}| = 300$ km/s

9



notum kartísku hnitin til að leggja saman hraðavigrana



$$\overline{v}_{PG} = \overline{v}_{PA} + \overline{v}_{AG}$$

$$\overline{v}_{PG} = (0, v_{PG})$$

$$\overline{v}_{PA} = v_{PA}\left(\cos\left(\tfrac{\pi}{2}-\theta\right), \sin\left(\tfrac{\pi}{2}-\theta\right)\right)$$

$$\overline{v}_{AG} = v_{AG}\left(\cos\left(\tfrac{5\pi}{4}\right), \sin\left(\tfrac{5\pi}{4}\right)\right)$$

10

því umskrifast

$$\overline{v}_{PG} = \overline{v}_{PA} + \overline{v}_{AG}$$

sem

$$(0, v_{PA}) = \left(v_{PA}\cos\left(\tfrac{\pi}{2}-\theta\right) + v_{AG}\cos\left(\tfrac{5\pi}{4}\right),\; v_{PA}\sin\left(\tfrac{\pi}{2}-\theta\right) + v_{AG}\sin\left(\tfrac{5\pi}{4}\right)\right)$$

notum

$$\cos\left(\tfrac{\pi}{2}-\theta\right) = \sin\theta, \qquad \sin\left(\tfrac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\sin\left(\tfrac{\pi}{2}-\theta\right) = \cos\theta, \qquad \cos\left(\tfrac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

11

þá fæst fyrir x-hnitið

$$0 = v_{PA}\sin\theta - \frac{v_{AG}}{\sqrt{2}} \qquad (1)$$

og fyrir y-hnitið

$$v_{PG} = v_{PA}\cos\theta - \frac{v_{AG}}{\sqrt{2}} \qquad (2)$$

Við þekkjum $v_{PA} = 300$ km/klst og $v_{AG} = 90$ km/klst, en viljum finna hornið θ og ferðina v_{PG}

12

$$(1) \longrightarrow \sin\theta = \left(\frac{v_{AG}}{v_{PA}}\frac{1}{\sqrt{2}}\right)$$

$$\longrightarrow \theta = \arcsin\left(\frac{v_{AG}}{v_{PA}}\frac{1}{\sqrt{2}}\right) \approx 0{,}2138 \text{ rad}$$

$$\approx 12{,}2^\circ$$

austur af norður

$$(2) \longrightarrow v_{PG} = v_{PA}\cos\theta - \frac{v_{AG}}{\sqrt{2}}$$

$$\approx 230 \text{ km/klst}$$

1

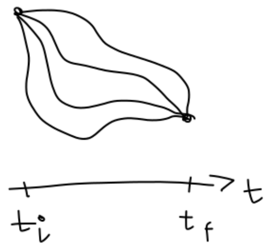
Lögmál Newtons -- aflfræði

Höfum hreyfilýsingu, hvernig x(t), v(t), og a(t) tengjast.
Viljum skilja hvað veldur hreyfingu og hvernig hún þróast í tíma og rúmi.

Við munum nota aflfræði Newtons -- hreyfing undir áhrifum krafta
--> hreyfifjöfnur

Um svipað leyti og Newton setti fram sín lögmál varð til önnur aðferð:
byggð á hnikareikningi:

$$S = \int_{t_i}^{t_f} L\,dt \;,\quad L = K - U \;,\quad \delta S = 0$$



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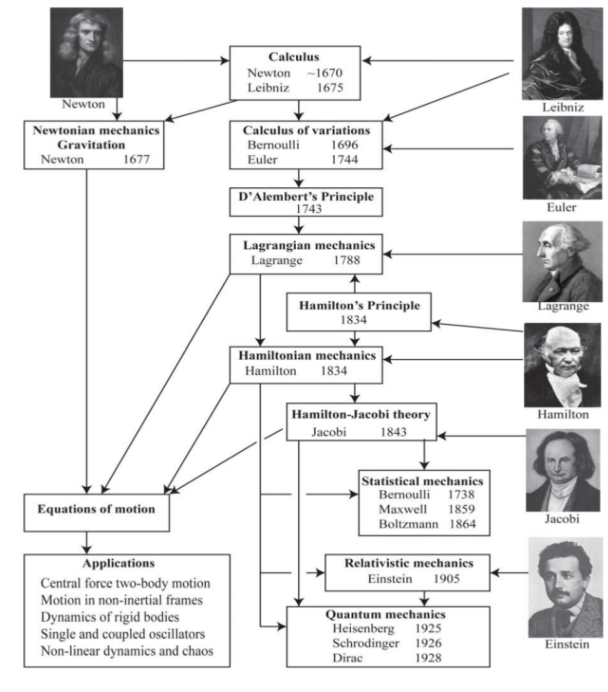

Figure 1.1: Chronological roadmap of the parallel development of the Newtonian and Variational-principles approaches to classical mechanics.

þróun aflfræðinnar
úr bókinni:
Variational Principles
in Classical Mechnics

Douglas Cline

frjáls á vefnum:

http//classicalmechanics
.lib.rochester.edu

3

Kraftar

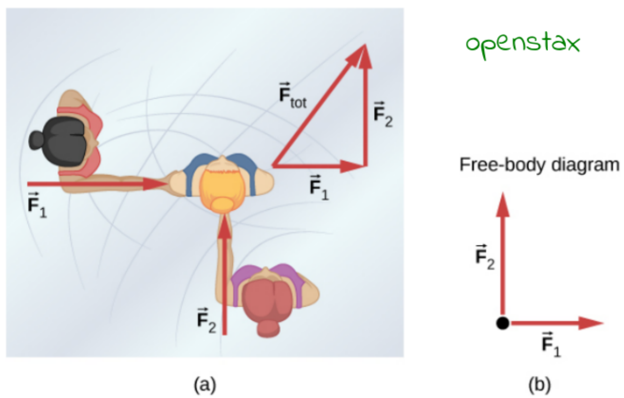

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Figure 5.3 (a) An overhead view of two ice skaters pushing on a third skater. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. (b) A free-body diagram representing the forces acting on the third skater.

Figure 5.3(b) is our first example of a **free-body diagram**, which is a sketch showing all external forces acting on an object or system. The object or system is represented by a single isolated point (or free body), and only those forces acting *on* it that originate outside of the object or system—that is, **external forces**—are shown. (These forces are the only ones shown because only external forces acting on the free body affect its motion. We can ignore any internal forces within the body.) The forces are represented by vectors extending outward from the free body.

4

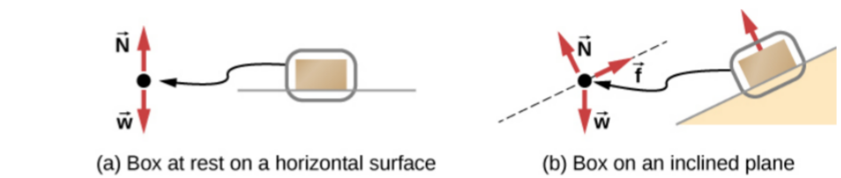

Figure 5.4 In these free-body diagrams, $\vec{N}$ is the normal force, $\vec{w}$ is the weight of the object, and $\vec{f}$ is the friction.

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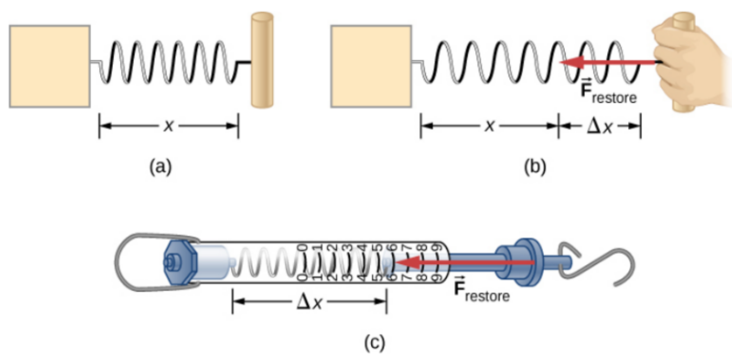

Mæling krafta

regla Hooks fyrir gorm

$$F = -k\,\Delta x$$

kraftstuðull k

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5

Fyrsta lögmál Newtons

Newton's First Law of Motion

A body at rest remains at rest or, if in motion, remains in motion at constant velocity unless acted on by a net external force.

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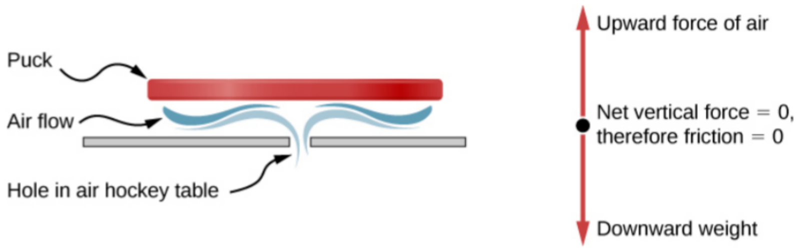
Tregðulögmálið



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6

Tregðukerfi

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Inertial Reference Frame

A reference frame moving at constant velocity relative to an inertial frame is also inertial. A reference frame accelerating relative to an inertial frame is not inertial.

Kerfi sem snýst miðað við tregðukerfi er ekki tregðukerfi ....

svo strangt tiltekið er yfirborð jarðar ekki tregðukerfi, áhrif þess sjást vel í veðurkerfum ...

7

Annað lögmál Newtons

Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system and is inversely proportion to its mass. In equation form, Newton's second law is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}_{\text{net}}}{m},$$

where $\vec{\mathbf{a}}$ is the acceleration, $\vec{\mathbf{F}}_{\text{net}}$ is the net force, and m is the mass. This is often written in the more familiar form

$$\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}, \qquad 5.3$$

but the first equation gives more insight into what Newton's second law means. When only the magnitude of force and acceleration are considered, this equation can be written in the simpler scalar form:

$$F_{\text{net}} = ma. \qquad 5.4$$

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Ytri kraftar orsaka hröðun kerfis

8

(Hreyfijöfnur)

Við skoðum aðallega, $F = F(t)$

$$\to \quad m\bar{a} = \bar{F}(t)$$

$$\to \quad \boxed{m\frac{d^2x}{dt^2} = \bar{F}(t)}$$

Einföld hreyfijafna, afleiðujafna sem leysa má með upphafsskilyrðum og beinni heildun

oft er kraftur háður staðsetningu $F = F(x)$

$$\to \quad \boxed{m\frac{d^2\bar{x}}{dt^2} = \bar{F}(\bar{x})}$$

Annars stigs afleiðujöfnuhneppi sem venjulega verður ekki leyst með beinni heildun. Leysist sem afleiðujöfnuhneppi með greini eða tölulegum aðferðum eftir að upphafsgildi eru fest

þyngdarkraftur, gormkraftur ....

9

Skriðþungi

Skilgreinum skriðþunga

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

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Fyrsta lögmálið er þá um varðveislu skriðþunga, og annað lögmálið lýsir hvernig ytri kraftur getur breytt skriðþunganum

$$\vec{\mathbf{F}}_{\text{net}} = \frac{d\vec{\mathbf{p}}}{dt}$$

$$\vec{\mathbf{F}}_{\text{net}} = \frac{d\vec{\mathbf{p}}}{dt} = \frac{d\left(m\vec{\mathbf{v}}\right)}{dt}$$

$$\vec{\mathbf{F}}_{\text{net}} = m\frac{d\left(\vec{\mathbf{v}}\right)}{dt} = m\vec{\mathbf{a}}$$

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10

Þyngd

Eining krafts er "Newton"

$$1\,\mathrm{N} = 1\,\frac{\mathrm{kg\,m}}{\mathrm{s}^2}, \quad (F = ma)$$

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Weight

The gravitational force on a mass is its weight. We can write this in vector form, where $\vec{\mathbf{w}}$ is weight and m is mass, as

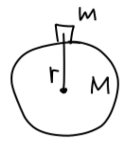
$$\vec{\mathbf{w}} = m\vec{\mathbf{g}}. \tag{5.8}$$

In scalar form, we can write

$$w = mg. \tag{5.9}$$

vegna þyngdarkrafts Newtons

$$|\vec{F}| = G\frac{mM}{r^2}$$



$$\rightarrow mg = G\frac{mM}{r^2}$$

$$\rightarrow g = \frac{GM}{r^2}$$

11

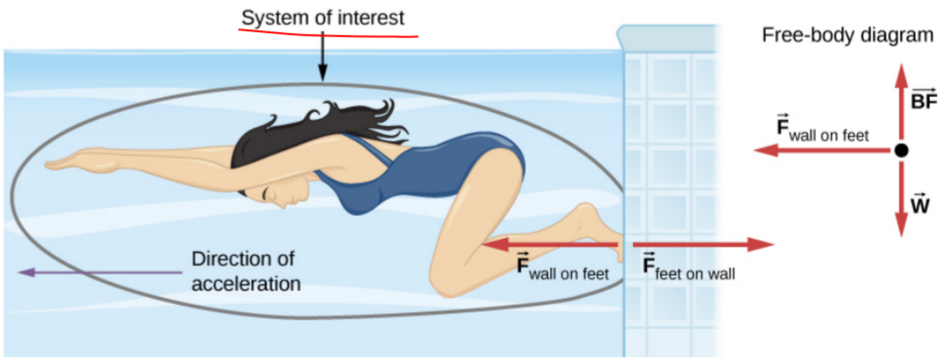
Þriðja lögmál Newtons

Newton's Third Law of Motion

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts. Mathematically, if a body A exerts a force $\vec{\mathbf{F}}$ on body B, then B simultaneously exerts a force $-\vec{\mathbf{F}}$ on A, or in vector equation form,

$$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}. \tag{5.10}$$

12



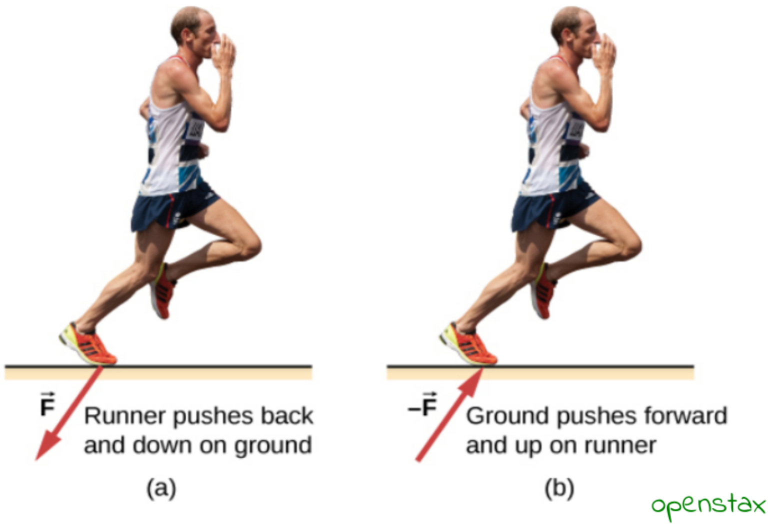
Hér þarf að athuga vel hvaða "kerfi" við erum að skoða, og hverjir eru ytri kraftarnir á það

Skoðum betur með dæmum næst

1

Using the Newtons laws

Defining the "system" in Newtons third law



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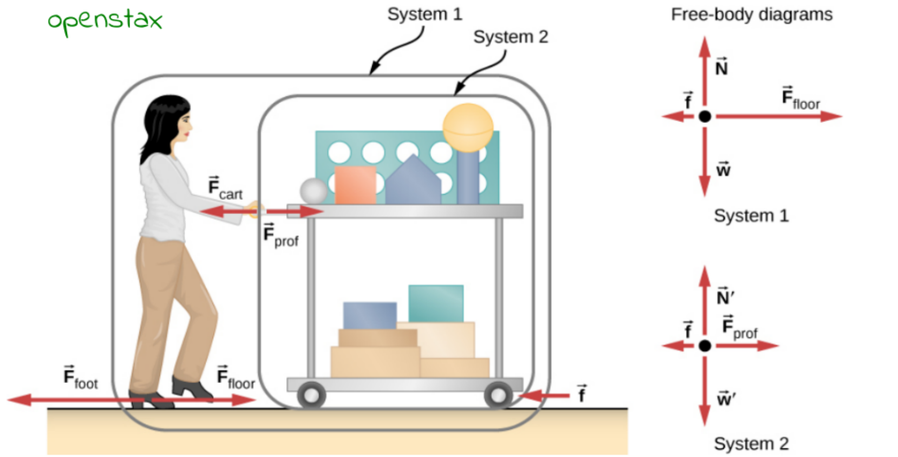


Figure 5.20 A professor pushes the cart with her demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for $\vec{f}$, because it is too small to drawn to scale). System 1 is appropriate for this example, because it asks for the acceleration of the entire group of objects. Only $\vec{F}_{floor}$ and $\vec{f}$ are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for the next example so that $\vec{F}_{prof}$ is an external force and enters into Newton's second law. The free-body diagrams, which serve as the basis for Newton's second law, vary with the system chosen.

3

Mass of professor	$M_p = 65$ Kg
Mass of chart	$M_c = 12$ Kg
Mass of equipment	$M_e = 7$ Kg
Friction force	$f = 24$ N
Force of her foot on the floor	$F_{Foot} = 150$ N

What is the acceleration of "system 1", (P + C + E)?

The only external force: $\bar{F}_{net} = \bar{F}_{Floor} - \bar{f}$

$$\rightarrow \quad \bar{a} = \frac{\bar{F}_{net}}{M_T} = \frac{\bar{F}_{Floor} - \bar{f}}{M_p + M_c + M_e} = \frac{(150-24)\,\text{N}}{84\,\text{Kg}}$$

$$\approx 1{,}5\ \text{m/s}^2$$

4

The force on the chart (Ex. 5.11)

Now the relevant system is "system 2"

$$F_{net} = F_p - F \qquad \bar{a}\ \text{"2"}$$

known

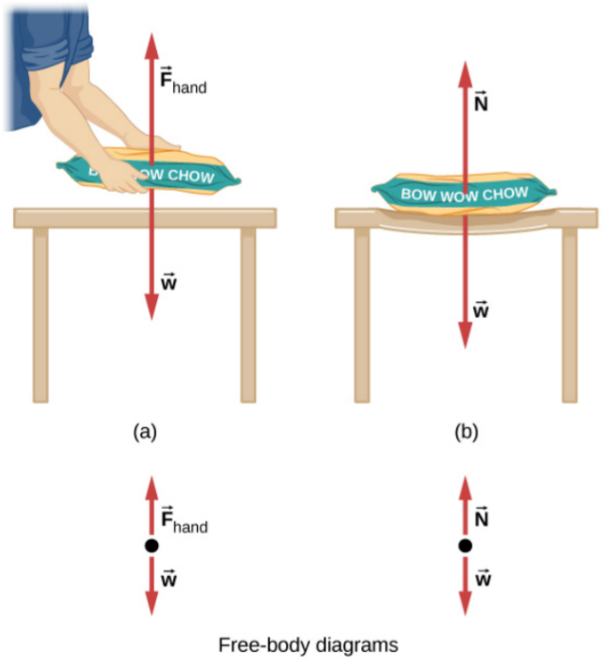
$$\rightarrow \quad F_p = F_{net} + f\,, \quad F_{net} = (M_c + M_e)\,a$$

$$\rightarrow \quad F_p = (M_c + M_e)\,a + f$$

$$\approx 29\,\text{N} + 24\,\text{N} = 53\,\text{N}$$

She pushes with much larger force on the floor, than the chart, the difference goes into her own acceleration!

Weight and normal force (þyngd og normalkraftur)



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Weight on an incline, (Ex. 5.12)

Find a if f = 45 N

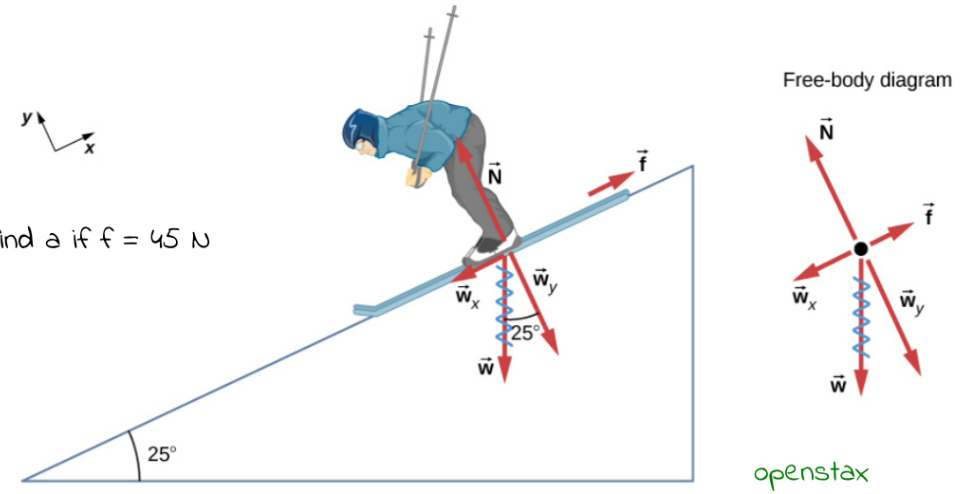

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Figure 5.22 Since the acceleration is parallel to the slope and acting down the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular to it (axes shown to the left of the skier). $\vec{N}$ is perpendicular to the slope and $\vec{f}$ is parallel to the slope, but $\vec{w}$ has components along both axes, namely, w_y and w_x. Here, $\vec{w}$ has a squiggly line to show that it has been replaced by these components. The force $\vec{N}$ is equal in magnitude to w_y, so there is no acceleration perpendicular to the slope, but f is less than w_x, so there is a downslope acceleration (along the axis parallel to the slope).

We find the components of w along the axes of the coordinate system

$$W_x = -W\sin\theta \qquad \theta = 25^\circ$$
$$W_y = -W\cos\theta \qquad m = 60\ \text{kg}$$

x: $(F_{net})_x = W_x + f$

y: $(F_{net})_y = W_y + N = 0$, no acc. along y

$$(F_{net})_x = -W\sin\theta + f = ma_x$$

$$a_x = \frac{-W\sin\theta + f}{m} \approx -3{,}39\ \text{m/s}^2$$

Tension - togkraftur

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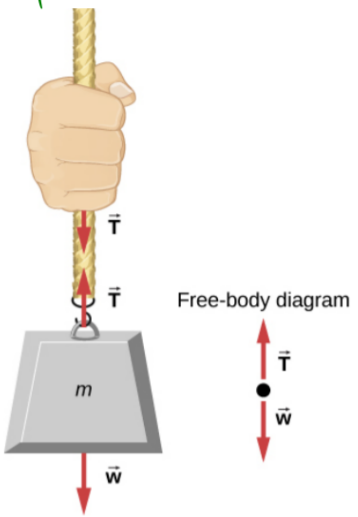


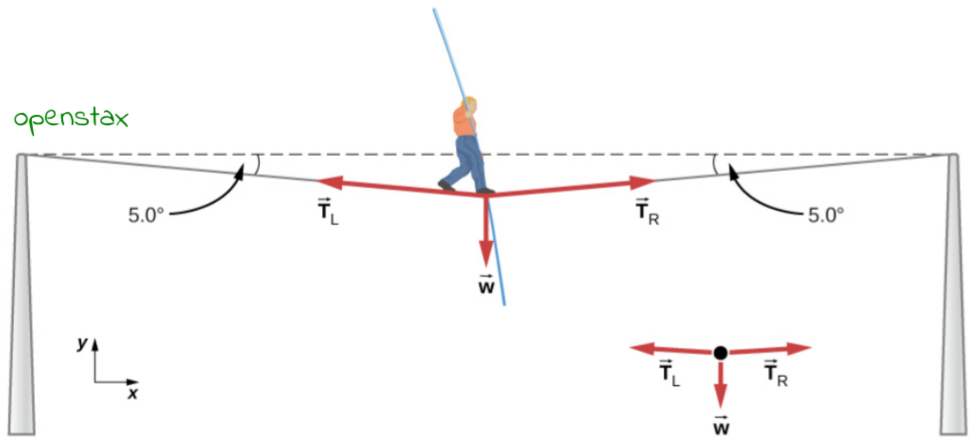
If the mass m is not accelerated we have the condition

$$\boxed{F_{net} = T - W = 0}$$

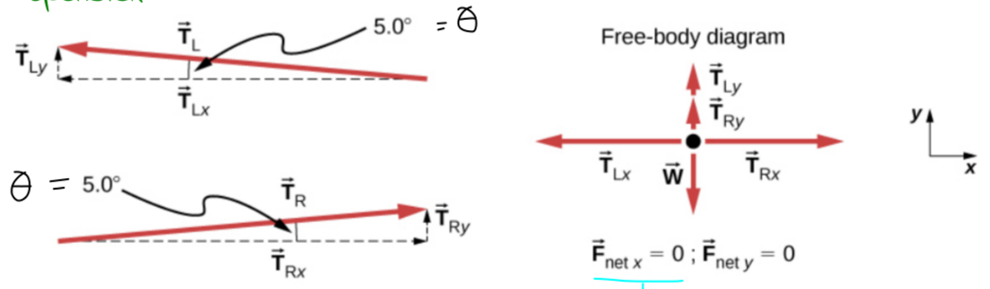
T is the tension in the rope, and here we have

$$T = mg, \text{ or } T = W$$

Tension in a tightrope, (Ex. 5.13)



we need to find the components of the forces along the x- and y-axes



X:
$$(F_{net})_x = (T_R)_x - (T_L)_x = 0$$

$$\rightarrow \quad (T_L)_x = (T_R)_x \quad \rightarrow \quad T_L \cos\theta = T_R \cos\theta$$

$$\rightarrow \quad T_L = T_R$$

y:
$$(F_{net})_y = (T_L)_y + (T_R)_y - w = 0$$

$$\rightarrow \quad 0 = T\sin\theta + T\sin\theta - w$$

$$0 = 2T\sin\theta - w$$

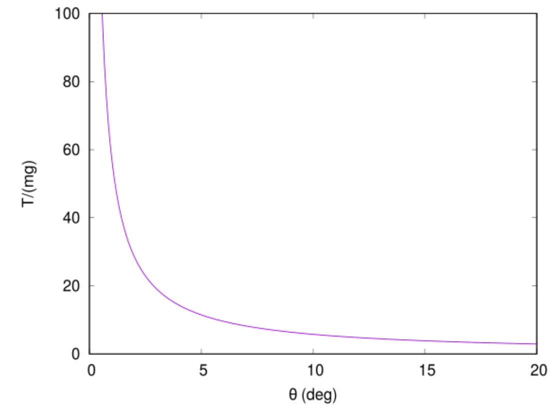
$$\rightarrow \quad T = \frac{w}{2\sin\theta} = \frac{mg}{2\sin\theta}$$

If $m = 70$ kg
$g = 9.81$ m/s²
$\theta = 5°$

$\rightarrow \quad T = 3930$ N

but $w \approx 687$ N

view in a graph,
singularity
(properties of a rope)



Possible usage, (think about T, the rope, savety)

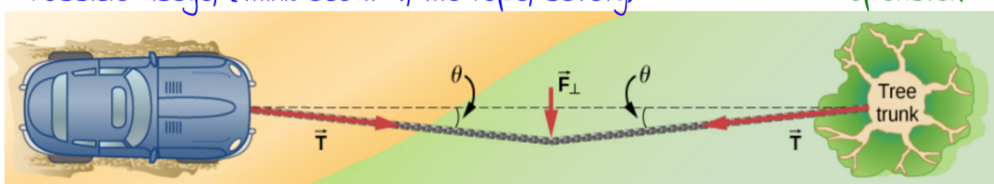
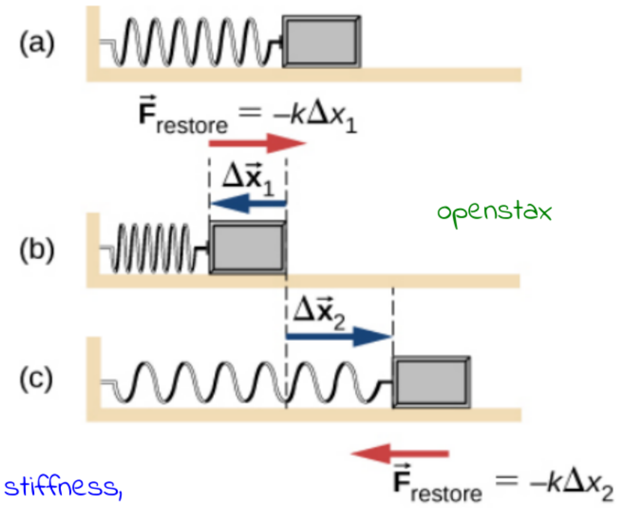


Figure 5.28 We can create a large tension in the chain—and potentially a big mess—by pushing on it perpendicular to its length, as shown.

Spring force, (Hookes law)



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$$\boxed{\bar{F} = -k\,\overline{\Delta x}}$$

spring constant, measure of stiffness,
fjaðurfasti, gormfasti
experimental observation

Pseudo forces in noninertial frames
Gerfikraftar í ekki-tregðukerfum

Coriolis forces -- centrifugal forces ...

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1

Applications of Newtons laws

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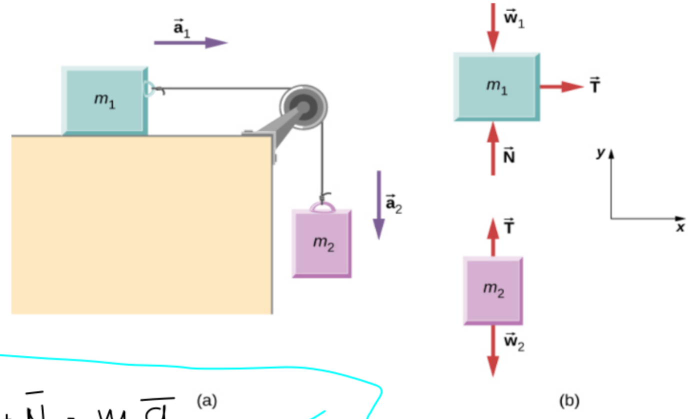
Ex. 6.4



No friction,
find a and T, knowing
m_1, m_2, and g

m_1: $\bar{T} + \bar{w}_1 + \bar{N} = m_1 \bar{a}_1$

m_2: $\bar{T} + \bar{w}_2 = m_2 \bar{a}_2$

$|\bar{T}| = |\bar{T}|$

2

1x: $T = m_1 a_{1x}$

2y: $T - m_2 g = m_2 a_{2y}$, $a_{1x} = -a_{2y} = a$

① $T = m_1 a$, ② $T - m_2 g = -m_2 a$

Two linear equations with two unknowns, T and a, solve together

① - ② $\rightarrow$ $0 + m_2 g = m_1 a + m_2 a = (m_1 + m_2) a$

$\rightarrow$ $(m_1 + m_2) a = m_2 g$ $\rightarrow$ $a = \left(\frac{m_2}{m_1 + m_2}\right) g$

3

Use the result for a in ①

$\hookrightarrow$ $T = \frac{m_1 m_2}{(m_1 + m_2)} g$

The system is accelerated

$T \neq m_2 g$

Friction - viðnámskraftar

Friction

Friction is a force that opposes relative motion between systems in contact.

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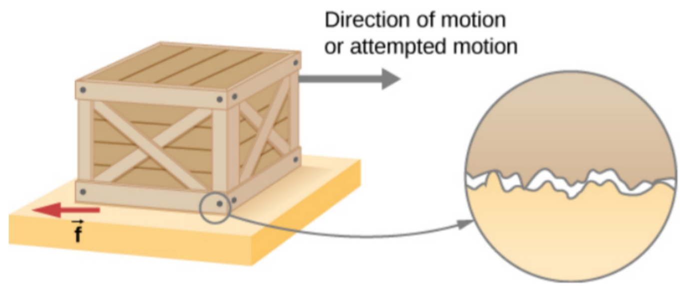
Static and Kinetic Friction

If two systems are in contact and stationary relative to one another, then the friction between them is called **static friction**. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**.

4



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Magnitude of Static Friction

The magnitude of static friction f_s is

$$f_s \leq \mu_s N, \qquad (6.1)$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force.

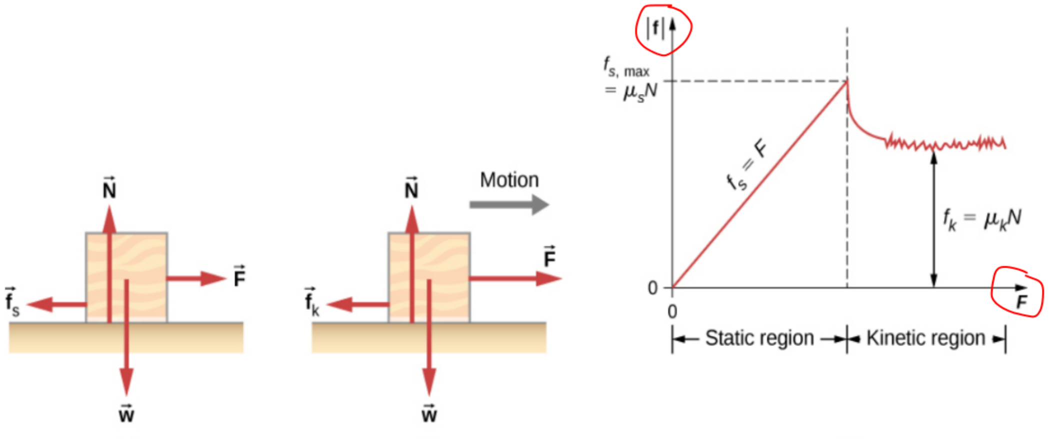
Magnitude of Kinetic Friction

The magnitude of kinetic friction f_k is given by

$$f_k = \mu_k N, \qquad (6.2)$$

where μ_k is the coefficient of kinetic friction.

5



(a) Impending motion (b) Object moves (c)

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Comparison of static and dynamic friction

6

System	Static Friction μ_s	Kinetic Friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.5-0.7	0.3-0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

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7

Ex. 6.10

$M = 20\,kg$, $\mu_k = 0{,}600$

$\mu_s = 0{,}700$

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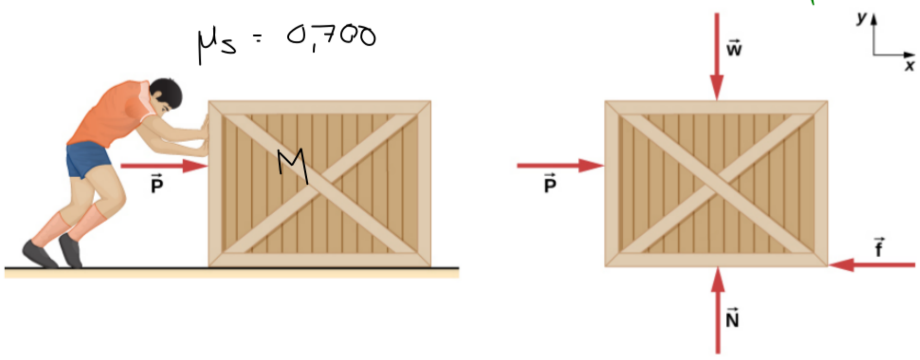

(a) (b)

Finna f, a

y: $-mg + N = 0 = ma_y$

$\rightarrow N = mg$

8

x: $P - f = Ma_x \rightarrow a_x = \frac{P-f}{M}$

$N = w = mg \simeq 20 \cdot 9{,}81\ \frac{km}{s^2} \simeq 196\,N$

$f_s \leq \mu_s N = 0{,}700 \cdot 196\,N \simeq 137\,N$

$f_k = \mu_k N = 0{,}600 \cdot 196 \simeq 118\,N$

a) $P = 20\,N \rightarrow f_s = 20\,N$

b) $P = 30\,N \rightarrow f_s = 30\,N$

c) $P = 120\,N \rightarrow f_s = 120\,N$

d) $P = 180\,N$

$\downarrow$

$f_k = 118\,N$

$a_x = \frac{P - f_k}{M} = 3{,}1\,\frac{m}{s^2}$

9

Centripetal force – miðsóknarkraftur

For steady circular motion we had

$$a_c = \frac{v^2}{r} = r\omega^2$$

radial inward directed acceleration needed to maintain the motion

$$\to \quad F_c = m a_c = m\frac{v^2}{r} = m r \omega^2$$

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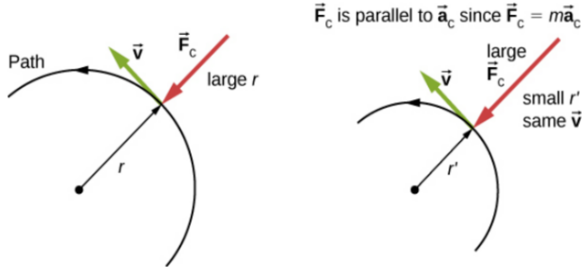

10

Ex. 6.15

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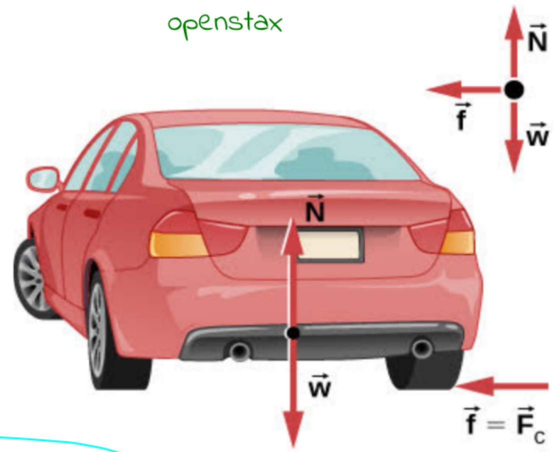
Car $M = 900$ kg

500 m - radius curve

at 25 m/s

Find needed μ_s for no slip



$$F_c = mv^2/r$$

$$F_c = f = \mu_s N = \mu_s m g \;\to\; \frac{mv^2}{r} = \mu_s m g$$

$$\to \quad \mu_s = \frac{v^2}{rg}$$

11

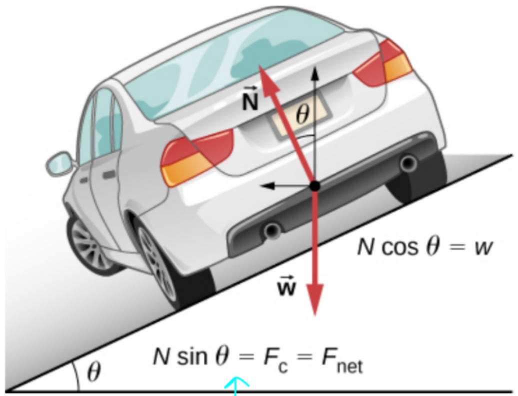
$$\mu_s = \frac{v^2}{rg} = \frac{25^2}{500 \cdot 9{,}81} = 0{,}13$$

this is lower than usually the real coefficient for tire and asphalt, so OK, but it is better so ...
The mass cancels!

Banked curve, why?

Ideal banking

the needed Fc comes from the banking



12

We have

$$N\sin\theta = \frac{mv^2}{r}$$

$$N\cos\theta = mg \;\to\; N = \frac{mg}{\cos\theta}$$

$$mg\tan\theta = \frac{mv^2}{r}$$

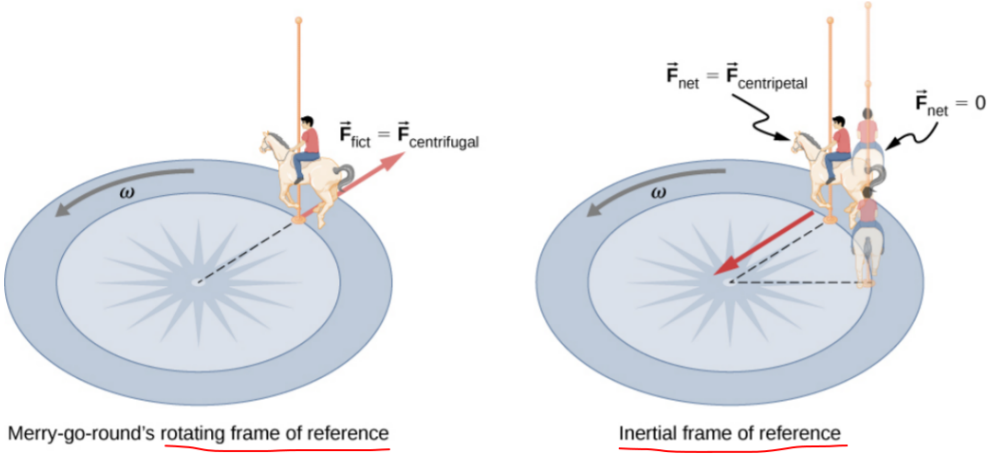
$$\to \quad \tan\theta = \frac{v^2}{rg}$$

$$\to \quad \boxed{\theta = \arctan\left\{\frac{v^2}{rg}\right\}}$$

no m

13

Pseudoforce -- centrifugal force in noninertial systems



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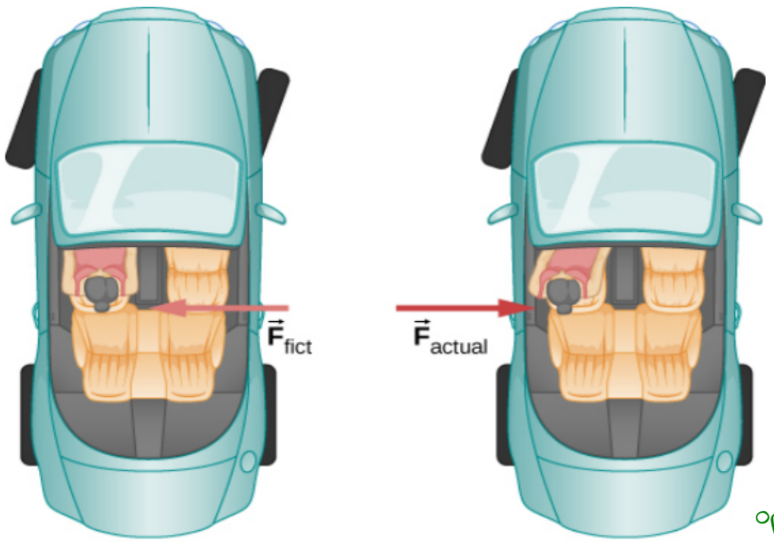

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rotating system

inertial system

15

Pseudoforce - Coriolis force (noninertial system)

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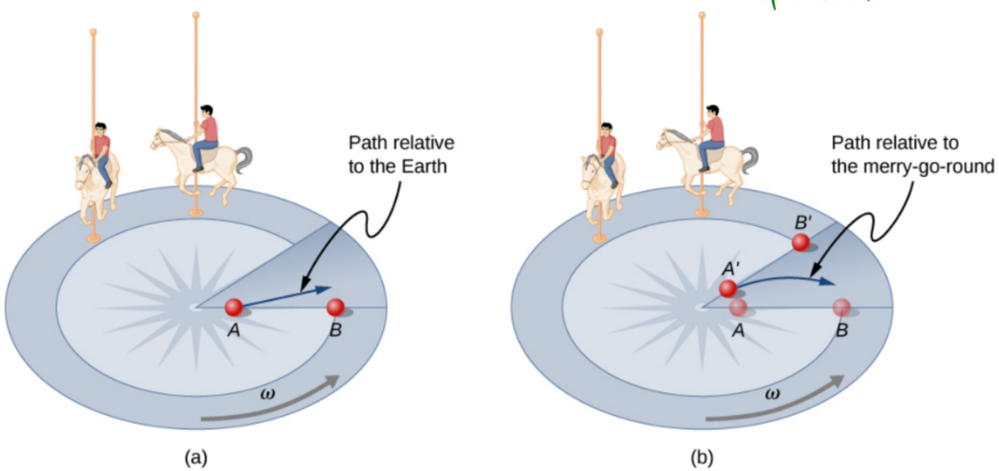

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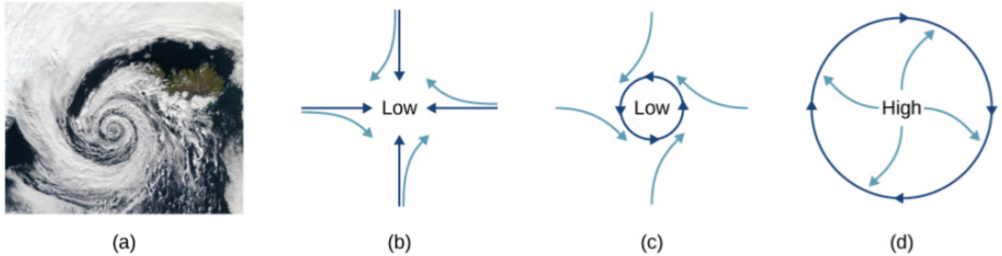

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1

Drag forces -- loftmótstaða -- vökvamótstaða

Empirically or by low order approximations to fluid dynamics it is known that for large objects of high speed in not very dense fluids one has

> **Drag Force**
>
> Drag force F_D is proportional to the square of the speed of the object. Mathematically,
>
> $$F_D = \frac{1}{2} C \rho A v^2,$$
>
> where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

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F_D is proportional to v^2

only an approximation -- empirical fact -- reynslulögmál....

2

Object	C
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram Pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90

Object	C
Skydiver (horizontal)	1.0
Circular flat plate	1.12

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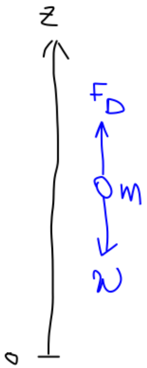
3

Terminal velocity - markhraði



we look at free fall in gravitational field

Newtons second law:

$$ma = m\frac{d^2z}{dt^2} = -mg + F_D$$

If the object reaches a constant velocity (terminal velocity) then

$$a = \left(\frac{d^2z}{dt^2}\right) = 0 = -mg + F_D$$

$$\rightarrow F_D = mg \quad \rightarrow \frac{1}{2} C \rho A v^2 = mg$$

4

$$\rightarrow v^2 = \frac{2mg}{\rho C A} \quad \rightarrow \boxed{v = \sqrt{\frac{2mg}{\rho C A}} \equiv v_T}$$

markhraði -- terminal velocity

Skydiver

$$\left.\begin{aligned} M &= 75 \text{ kg} \\ \rho &= 1{,}21 \text{ kg/m}^3 \\ A &= 0{,}18 \text{ m}^2 \\ C &= 0{,}7 \end{aligned}\right\} \rightarrow v_T \simeq 98 \text{ m/s}$$

$$\simeq 350 \text{ km/hr}$$

5

For small spherical object at low speed in dense fluid

Stokes' Law

For a spherical object falling in a medium, the drag force is

$$F_s = 6\pi r\eta v, \qquad \textbf{6.6}$$

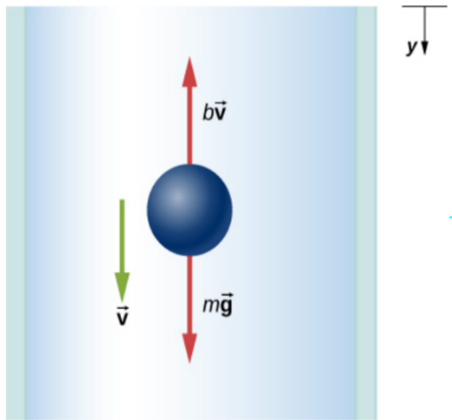
where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

F_s is proportional to v

Again, an approximation to fluid dynamics -- empirical law

6

Equation of motion -- integration -- solution



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Using the coordinat system in the Figure:

Assume: $f_R = -bv$

$$\rightarrow \quad ma = m\frac{d^2y}{dt^2} = m\frac{dv}{dt}$$

$$= mg - bv$$

use the equation of motion

with variables t and v $\rightarrow$ $m\frac{dv}{dt} = mg - bv$

7

we can find the terminal velocity when dv/dt = 0

$$\rightarrow \quad 0 = mg - bv \quad \rightarrow \quad \boxed{v_T = \frac{mg}{b}}$$

note how it increases with m and g increasing but decreases as b increases. very plausible behavior.

we can use separation of variables

$$m\frac{dv}{dt} = mg - bv \quad \rightarrow \quad \underbrace{\frac{m\,dv}{mg - bv}}_{\text{only v and constants}} = \underbrace{dt}_{\text{only t}}$$

8

Initial values: v(t=0) = 0, y(t=0) = 0

we integrate

$$\int_0^v \frac{m\,dv'}{mg - bv'} = \int_0^t dt'$$

prime variables are dummy integration variables

Limits have to correspond

$$\rightarrow \quad -\frac{m}{b}\ln\left[mg - bv'\right]\Big|_0^v = t - 0$$

$$\rightarrow \quad -\frac{m}{b}\left[\ln\left[mg - bv\right] - \ln\left[mg\right]\right] = t$$

9

use $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

to obtain

$$-\frac{m}{b}\ln\left[1-\frac{bv}{mg}\right] = t$$

$$\rightarrow \quad -\frac{m}{b}\ln\left\{1-\frac{v}{v_T}\right\} = t$$

$$\rightarrow \quad \underbrace{\ln\left\{1-\frac{v}{v_T}\right\} = -\frac{bt}{m}}_{\text{dimensionless...}}$$

10

take exponential of both sides

$$\rightarrow \quad 1-\frac{v}{v_T} = \exp\left\{-\frac{bt}{m}\right\}$$

$$\rightarrow \quad \frac{v}{v_T} = -\exp\left\{-\frac{bt}{m}\right\} + 1$$

$$\rightarrow \quad \boxed{v(t) = v_T\left[1-\exp\left\{-\frac{bt}{m}\right\}\right]}$$

we see two characteristic scales:

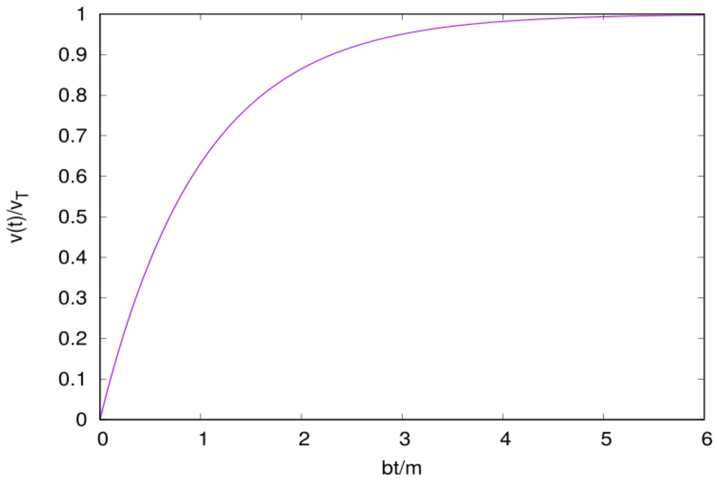
$$v_T, \quad [v_T] = \frac{L}{T}, \qquad \frac{b}{m}, \quad \left[\frac{b}{m}\right] = \frac{1}{T}$$

11

and indeed:

$$v(t) \longrightarrow v_T \quad \text{when} \quad \frac{tb}{m} \gg 1$$

or "$t \rightarrow \infty$"



markhraai - terminal velocity

Dimensionless variables on the axes

12

Position

$$v(t) = \frac{dy}{dt} = v_T\left[1-e^{-\frac{bt}{m}}\right]$$

$$\rightarrow \quad dy = v_T\left[1-e^{-bt/m}\right]dt$$

integrate $$\int_0^y dy' = \int_0^t v_T\left[1-e^{-bt'/m}\right]dt'$$

$$\rightarrow \quad y - 0 = v_T \cdot t + \frac{m v_T}{b}\left[e^{-\frac{bt}{m}} - 1\right]$$

13

so we get

$$\boxed{y = v_T\left[t + \frac{m}{b}\left(e^{-\frac{bt}{m}} - 1\right)\right]}$$

and we note that the dimensions add up

When $t \ggg \frac{m}{b}$ we get

$$y(t) \longrightarrow \underline{v_T t}$$

①

Work -- vinna

$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \left|\vec{\mathbf{F}}\right| \left|d\vec{\mathbf{r}}\right| \cos\theta. \quad 7.1$$

Then, we can add up the contributions for infinitesimal displacements, along a path between two positions, to get the total work.

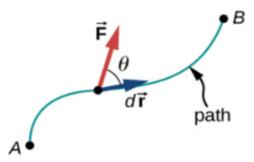
Work Done by a Force

The work done by a force is the integral of the force with respect to displacement along the path of the displacement:

$$W_{AB} = \int_{\text{path } AB} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}. \quad 7.2$$

The vectors involved in the definition of the work done by a force acting on a particle are illustrated in Figure 7.2.

Any force



only the component
along the path matters
Inner product

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②

Work done by a force can be negative, 0, positive
We will see this corresponds to the force taking energy out,
or supplying to the system
We ask about the work of any force, with no focus on the net force

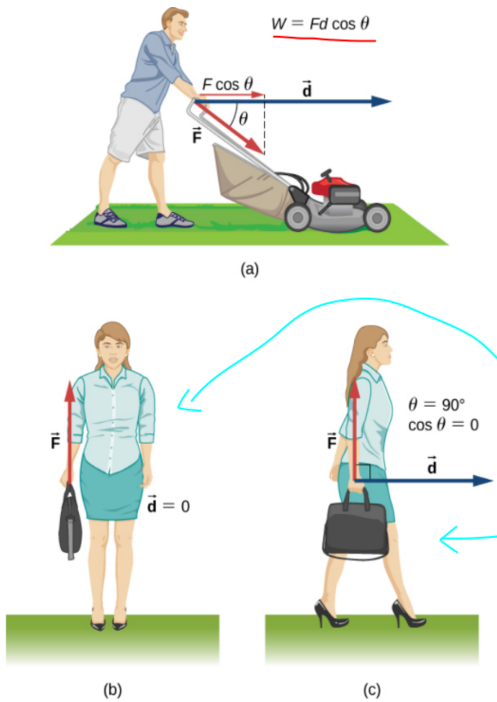
Constant force (special case)

$$W_{AB} = \int_A^B \bar{F} \cdot d\bar{r} = \int_A^B (F_x dx + F_y dy + F_z dz)$$

$$= F_x \int_A^B dx + F_y \int_A^B dy + F_z \int_A^B dz = F_x(x_B - x_A) + F_y(y_B - y_A) + F_z(z_B - z_A)$$

$$= \bar{F} \cdot (\bar{r}_B - \bar{r}_A)$$

③



$\bar{F} \cdot \bar{d} \neq 0$

$\bar{d} \neq 0$

$dW_N = \bar{d} \cdot \bar{N} = 0$

$W = 0$

$\bar{d} = 0$

$\bar{F} \cdot \bar{d} = 0$

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④

The friction force does negative work on the lawn mower

$$W_{fr} = \int_A^B \vec{\mathbf{f}}_k \cdot d\vec{\mathbf{r}} = -f_k \int_A^B |dr| = -f_k\, |l_{AB}|,$$

$\bar{F}_k \cdot d\bar{r} < 0$

It depends on the length of the total path

Moving a couch horizontally

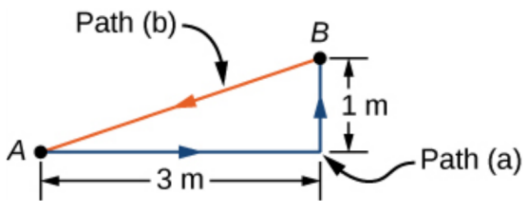

Figure 7.4 Top view of paths for moving a couch.

Two paths:
Path (a) A --> B, open path
Path (b) A --> B --> A, closed

$\mu_k = 0{,}6$

$|N| = 1\,kN$

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(5)

work done by the friction force

a: $W = -0{,}6(1kN)\cdot(3m+1m) = \underline{-2{,}4\,kJ}$

b: $W = -0{,}6(1kN)\cdot(3m+1m+\sqrt{10}\,m) = \underline{-4{,}3\,kJ}$

$$\boxed{\oint \bar{F}_k \cdot d\bar{r} \neq 0}$$

as the force of friction is not conservative, ekki geyminn, see more soon...

It is dissipative, takes energy out of the system (open systems)

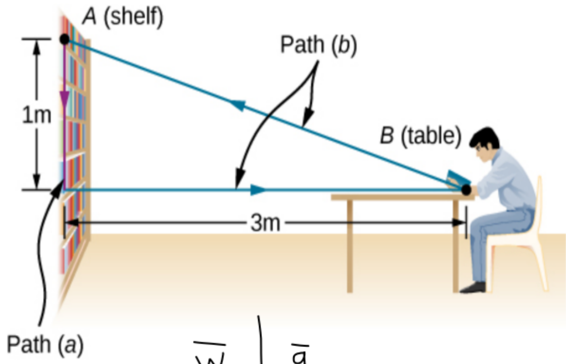
(6)

Shelving a book $\qquad W = \int_A^B \bar{F}\cdot d\bar{r}$

Constant force of gravity

it's work on the book

$$W_{AB} = -mg(y_B - y_A)$$
$$= \underline{mg(y_A - y_B) > 0}$$



$\bar{W} \downarrow \bar{g}$

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$$\underline{W_{ABA} = 0}$$

Gravity is conservative force (geyminn), only the endpoints of the path matter

(7)

variable force

Path: $y = (0{,}5\,m^{-1})x^2$

$$\bar{F} = \left(5\tfrac{N}{m}\right)y\,\hat{i} + \left(10\tfrac{N}{m}\right)x\,\hat{j}$$
$$= \left(5\tfrac{N}{m}y,\ 10\tfrac{N}{m}x\right)$$

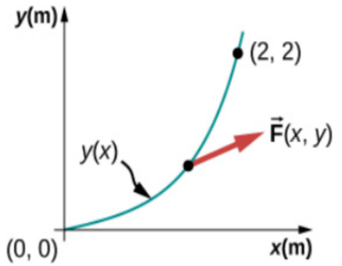

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Figure 7.6 The parabolic path of a particle acted on by a given force.

Parametrize the path (stika)

$$\bar{r} = (x,y) = (x,\ 0{,}5x^2)$$
$$\rightarrow d\bar{r} = (dx,\ x\,dx)$$

$$dy = \left(\frac{dy}{dx}\right)dx$$

$$dW = \bar{F}\cdot d\bar{r} = 5y\,dx + 10x\,x\,dx = \frac{5}{2}x^2dx + 10x^2dx$$
$$\rightarrow W = \int_0^2\left\{\frac{5}{2}x^2dx + 10x^2dx\right\} = \int_0^2\left\{\frac{25}{2}x^2\right\}dx$$

(8)

$$W = \int_0^2 \frac{25}{2}x^2dx = \frac{25}{2\cdot 3}x^3\Big|_0^2 = \frac{25\cdot 8}{6}\,Nm \simeq \underline{33{,}3\,J}$$

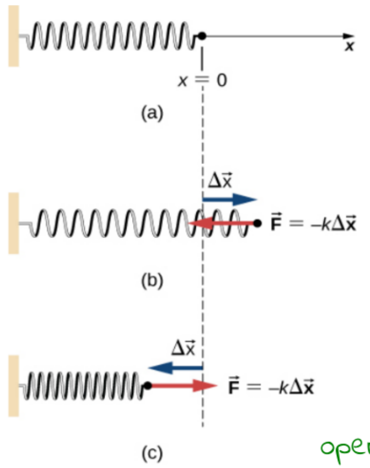
work done by a spring

$$W_{\text{spring},AB} = \int_A^B F_x\,dx = -k\int_A^B x\,dx = -k\frac{x^2}{2}\Big|_A^B = -\frac{1}{2}k\left(x_B^2 - x_A^2\right).$$

If $x_A = 0$

$\rightarrow \underline{W_{\text{spring},AB} < 0}$

as stretching or compressing the spring from the equilibrium requires external work



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9

Hreyfiorka - kinetic energy

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Kinetic Energy

The kinetic energy of a particle is one-half the product of the particle's mass m and the square of its speed v:

$$K = \frac{1}{2}mv^2. \qquad 7.6$$

$$\bar{p} = m\bar{v} \quad \rightarrow \quad \boxed{K = \frac{(mv)^2}{2m} = \frac{p^2}{m}}$$

Notice specially that the kinetic energy is always positive and grows as the square of the velocity

10

Work - energy

The net work done on a particle

$$dW_{net} = \bar{F}_{net}\cdot d\bar{r}\ , \qquad \bar{F}_{net} = m\frac{d\bar{v}}{dt}$$

$$= m\left(\frac{d\bar{v}}{dt}\right)\cdot d\bar{r} = m\left(\frac{d\bar{v}}{dt}\right)dt\cdot\underbrace{\frac{d\bar{r}}{dt}}_{=\bar{v}}$$

$$= m\,d\bar{v}\cdot\bar{v} = m\bar{v}\cdot d\bar{v}$$

$$\rightarrow W_{net,AB} = \int_A^B m\bar{v}\cdot d\bar{v} = \int_A^B \left[mv_x dv_x + mv_y dv_y + mv_z dv_z\right]$$

$$= \frac{m}{2}\left|v_x^2 + v_y^2 + v_z^2\right|_A^B = \frac{1}{2}m\left|v^2\right|_A^B$$

11

Work-Energy Theorem

The net work done on a particle equals the change in the particle's kinetic energy:

$$W_{net} = K_B - K_A. \qquad 7.9$$

Power - afl

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Power

Power is defined as the rate of doing work, or the limit of the average power for time intervals approaching zero,

$$P = \frac{dW}{dt}. \qquad 7.11$$

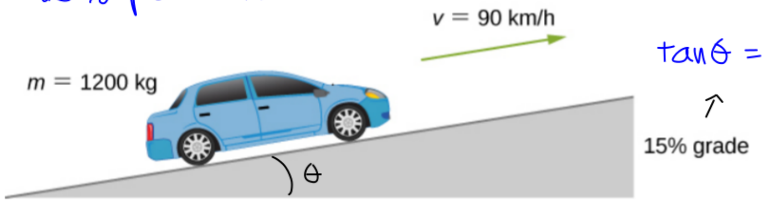
$$P = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\left(\frac{d\vec{r}}{dt}\right) = \vec{F}\cdot\vec{v}$$

general equation even though it may look as a particular result

12

Ex. 7.12

25 % power to friction



$\tan\theta = 0{,}15$

Figure 7.15 We want to calculate the power needed to move a car up a hill at constant speed.

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Constant v --> ΔK = 0, power against gravity and friction

→ 75% of P against gravity

$$P = m\bar{g}\cdot\bar{v} = mgv\sin\theta$$

$$\rightarrow 0{,}75\cdot P = mgv\sin\left[\arctan(0{,}15)\right]$$

$$P = \frac{1200\cdot 9{,}81\left(\frac{90}{3{,}6}\,m/s\right)\sin(8{,}53°)}{0{,}75} = 58\,kW$$

Potential energy - energy conservation, stöðuorka - orkuvarðveisla

As the football falls toward Earth, the work done on the football is now positive, because the displacement and the gravitational force both point vertically downward. The ball also speeds up, which indicates an increase in kinetic energy. Therefore, energy is converted from gravitational potential energy back into kinetic energy.

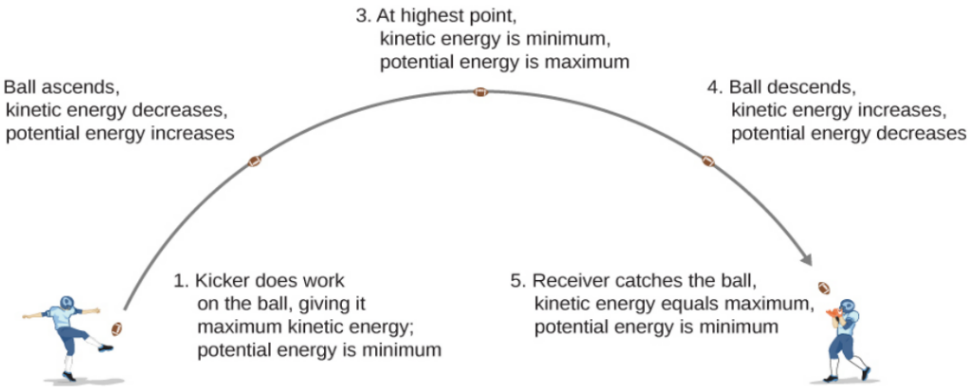

Figure 8.2 As a football starts its descent toward the wide receiver, gravitational potential energy is converted back into kinetic energy.

Based on this scenario, we can define the difference of potential energy from point A to point B as the negative of the work done:

$$\Delta U_{AB} = U_B - U_A = -W_{AB}. \qquad (8.1)$$

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1

Define potential energy function $U(\vec{r})$

$$\boxed{\Delta U = U(\vec{r}) - U(\vec{r}_0)}$$

with reference point $\vec{r}_0$, stöðuorkufall - mættisorka - mættisorkufall

If no friction or air resistance, (closed system)

$$\boxed{\Delta K_{AB} = -\Delta U_{AB}}$$

valid for a system of particles

Different types of potential energy
Gravitational
Electrical
Spring .....

2

Typical scales of energy of phenomenas

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Annual world energy use	4.0×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
1 barrel crude oil	5.9×10^{9}
1 metric ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
Tennis ball at 100 km/h	22
Mosquito (10^{-2} g at 0.5 m/s)	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

Table 8.1 Energy of Various Objects and Phenomena

Power in Iceland (2014)

Hydropower stations 1984 MW
Geothermal 665 MW
Fuel 117 MW

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3

Conservative and nonconservative forces - geymnir og ógeymnir kraftar

Conservative Force

The work done by a conservative force is independent of the path; in other words, the work done by a conservative force is the same for any path connecting two points:

$$W_{AB,\text{path-1}} = \int_{AB,\text{path-1}} \vec{\mathbf{F}}_{\text{cons}} \cdot d\vec{\mathbf{r}} = W_{AB,\text{path-2}} = \int_{AB,\text{path-2}} \vec{\mathbf{F}}_{\text{cons}} \cdot d\vec{\mathbf{r}}. \qquad (8.8)$$

The work done by a non-conservative force depends on the path taken.

Equivalently, a force is conservative if the work it does around any closed path is zero:

$$W_{\text{closed path}} = \oint \vec{\mathbf{F}}_{\text{cons}} \cdot d\vec{\mathbf{r}} = 0. \qquad (8.9)$$

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Globally stated

Locally stated (in 2D) $\vec{F} \cdot d\vec{r} = F_x dx + F_y dy$

has to be exact differencial: $\boxed{\frac{dF_x}{dy} = \frac{dF_y}{dx}}$

4

(5)

Ex. 8.5

a) $\bar{F} = a(xy^3, yx^3) \rightarrow \frac{dF_x}{dy} = 3axy^2, \ \frac{dF_y}{dx} = 3ax^2y$

$\rightarrow$ not conservative

b) $\bar{F} = a\left(\frac{y^2}{x}, 2y\ln\left(\frac{x}{b}\right)\right) \rightarrow \frac{dF_x}{dy} = 2a\frac{y}{x}$

$\frac{dF_y}{dx} = 2ay \cdot \frac{1}{x/b} \cdot \frac{1}{b} = 2a\frac{y}{x}$ $\rightarrow$ Conservative

c) $\bar{F} = \frac{a}{x^2+y^2}(x, y) \rightarrow \frac{dF_x}{dy} = -\frac{ax\,2y}{(x^2+y^2)^2}$

(wXwaxwa) $\frac{dF_y}{dx} = -\frac{ay\,2x}{(x^2+y^2)^2}$ $\rightarrow$ Conservative

(6)

Potential energy can only be found for conservative forces

we defined the increase in potential energy as the negative work done by the force

$$dU = -\bar{F} \cdot d\bar{\ell} = -F_\ell \, d\ell$$

$$\rightarrow F_\ell = -\frac{dU}{d\ell}$$

partial derivative .. hlutafleiða

gradient, stigull

For 2D we thus have $\boxed{\bar{F} = -\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}\right) = -\bar{\nabla} U}$

..and of course

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \quad \text{as} \quad \frac{\partial^2 U}{\partial x \partial y} = \frac{\partial^2 U}{\partial y \partial x}$$

(7)

Check

$U = \frac{1}{2}kx^2 \rightarrow F = -\frac{\partial U}{\partial x} = -kx$

Hookes law

$U = \frac{1}{2}k\{x^2 + y^2\} \rightarrow \bar{F} = -k(x, y)$

Potential bowl, quantum dot

$U = \frac{1}{4}cx^4 \rightarrow F = -cx^3$

(8)

Conservation of energy

single particle at the moment

Conservation of Energy

The mechanical energy E of a particle stays constant unless forces outside the system or non-conservative forces do work on it, in which case, the change in the mechanical energy is equal to the work done by the non-conservative forces:

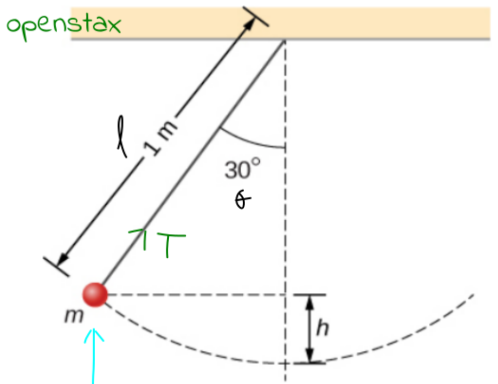
$$W_{\mathrm{nc},AB} = \Delta(K + U)_{AB} = \Delta E_{AB}. \qquad 8.12$$

External conservative forces can add energy to, or extraxt energy from a system by the work done on it, positive or negative. In a closed system with no external forces the energy is conserved.

9

Ex. 8.7

The system is the pendulum and the gravitational force



$v = 0$ (i)

$h = 0$ (f)

$$\Delta\left[K + U\right] = 0$$

Tensiion T does no work, always perpendicular to the path

$$\bar{T} \cdot d\bar{r} = 0$$

$$K = \frac{1}{2} m v^2$$

$$U = mgh$$

10

$K_i = 0$ as $v_i = 0$

$U_f = 0$ as $h_f = 0$

$$U_i = mgh_i = K_f = \frac{1}{2} m v_f^2$$

$$\rightarrow mgh_i = \frac{1}{2} m v_f^2 \rightarrow gh_i = \frac{1}{2} v_f^2$$

$$\rightarrow v_f = \sqrt{2gh_i}$$

Note

$$\ell = h + \ell\cos\theta$$

$$\rightarrow h = \ell - \ell\cos\theta$$

$$\rightarrow U = mgh = mg\ell(1 - \cos\theta), \rightarrow F_\theta = -\frac{\partial U}{\ell\,\partial\theta} = -mg\sin\theta$$

11

arclength

$$s = \ell\theta$$

$$v = \frac{ds}{dt} = \ell\frac{d\theta}{dt} \rightarrow a = \ell\frac{d^2\theta}{dt^2}$$

$$ma = \ell\frac{d^2\theta}{dt^2} = \ell\ddot{\theta}, \quad F = -mg\sin\theta$$

one variable, θ, --> Equation of motion for the pendulum is

$$ma = F \rightarrow m\ell\ddot{\theta} = -mg\sin\theta$$

$$\rightarrow \boxed{\ddot{\theta} + \frac{g}{\ell}\sin\theta = 0}$$ nonlinear

For small θ (in radians)

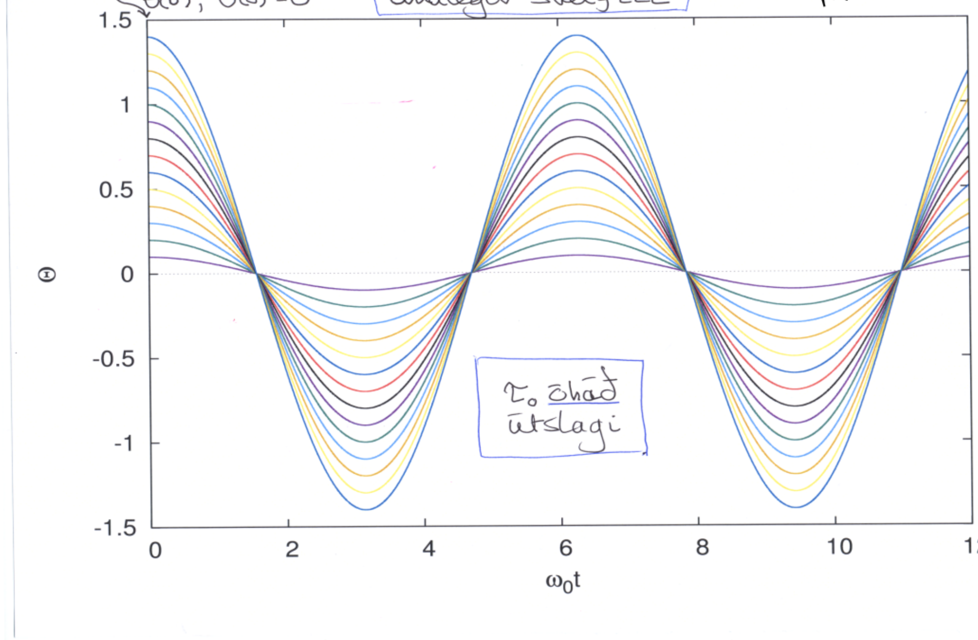
$$\sin\theta \simeq \theta - \frac{\theta^3}{6} + \dots$$

linear

$$\boxed{\ddot{\theta} + \frac{g}{\ell}\theta \simeq 0}$$

12

$\theta(0)$, $\dot{\theta}(0) = 0$ — Línulegur sveifill — $\omega_0 = \sqrt{\frac{g}{\ell}}$



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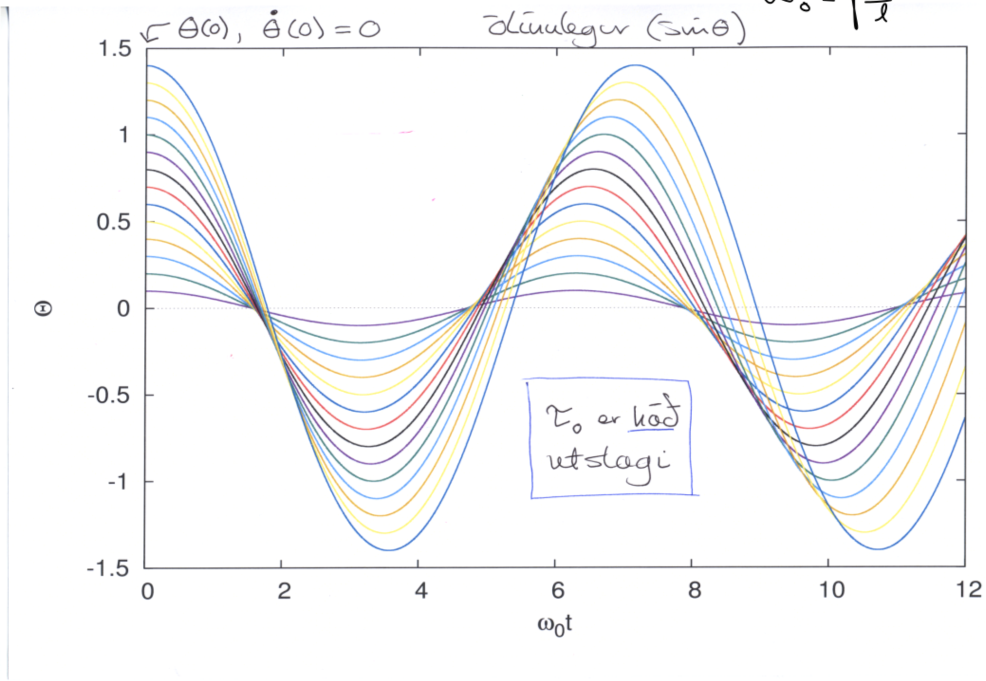
$\omega_0 = \sqrt{\frac{g}{\ell}}$

$\theta(0)$, $\dot{\theta}(0) = 0$ Ólínulegur ($\sin\theta$)



τ_0 er háð útslagi

14

1D motion

$$E = K + U(x)$$

$$K = \frac{m}{2}v^2 = E - U(x)$$

$$\rightarrow \quad v = \frac{dx}{dt} = \sqrt{\frac{2(E - U(x))}{m}}$$

$$\rightarrow \quad dt = \frac{dx}{\sqrt{\frac{2(E-U(x))}{m}}}$$

Integrate

$$t = \int_0^t dt' = \int_{x_0}^{x} \frac{dx'}{\sqrt{\frac{2(E-U(x'))}{m}}}$$

Solution for the path found from energy conservation. often difficult to invert to x(t), and not good for numerical calculations but can still give information

15

Potential energy -- stability (1D)

Free fall

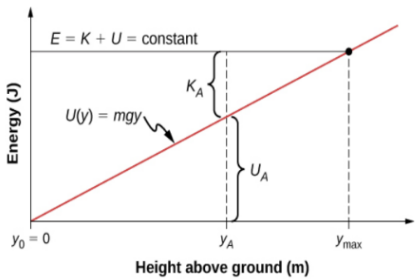
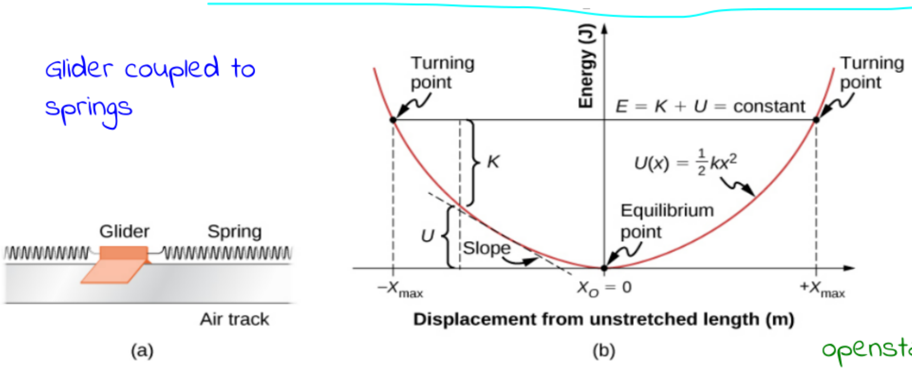


Figure 8.10 The potential energy graph for an object in vertical free fall, with various quantities indicated.

Glider coupled to springs



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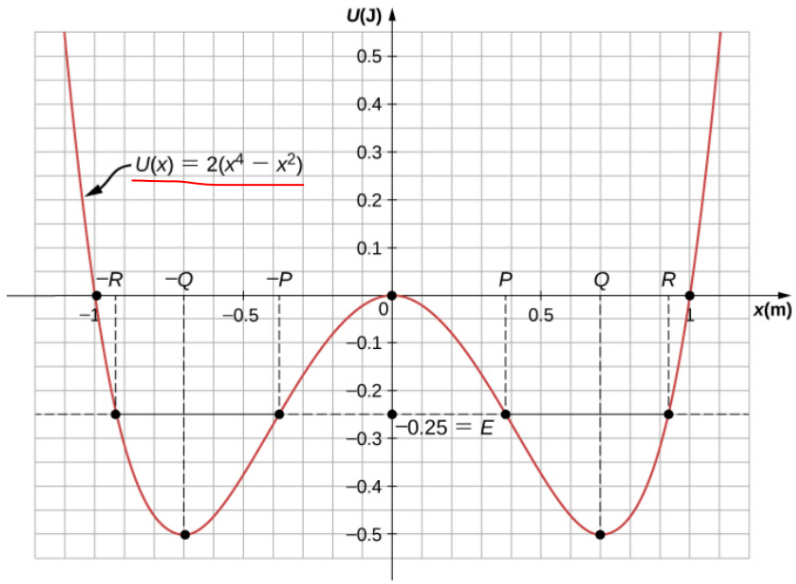

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1

Atlag - impulse, skriðþungi - momentum

Momentum

The momentum p of an object is the product of its mass and its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}. \qquad 9.1$$

Figure 9.3 This supertanker transports a huge mass of oil; as a consequence, it takes a long time for a force to change its (comparatively small) velocity. (credit: modification of work by "the_tahoe_guy"/Flickr)

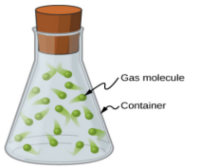

Figure 9.4 Gas molecules can have very large velocities, but these velocities change nearly instantaneously when they collide with the container walls or with each other. This is primarily because their masses are so tiny.

bæði massinn m og hraðinn v skipta máli þegar skriðþunginn er reiknaður

$$\bar{p} = m\bar{v}$$

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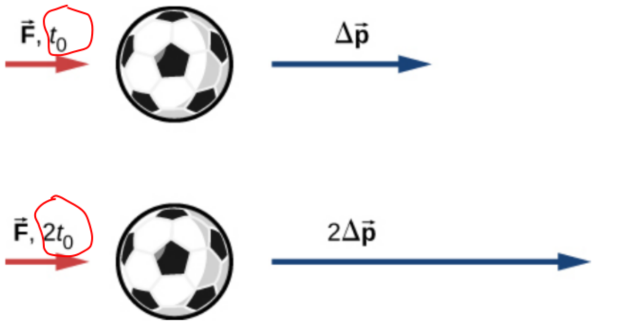
2

Atlag - impulse



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Impulse

Let $\vec{\mathbf{F}}(t)$ be the force applied to an object over some differential time interval dt (Figure 9.6). The resulting impulse on the object is defined as

$$d\vec{\mathbf{J}} \equiv \vec{\mathbf{F}}(t)dt. \qquad 9.2$$

3

$$\vec{\mathbf{J}} = \int_{t_i}^{t_f} d\vec{\mathbf{J}} \text{ or } \vec{\mathbf{J}} \equiv \int_{t_i}^{t_f} \vec{\mathbf{F}}(t)dt$$

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$$\bar{F}_{ave} = \frac{1}{\Delta t}\int_{t_i}^{t_f} \bar{F}(t)\,dt \rightarrow \boxed{\bar{J} = \bar{F}_{ave}\,\Delta t}$$

$$\bar{J} = \int_{t_i}^{t_f} \bar{F}(t)\,dt = m\int_{t_i}^{t_f} \bar{a}(t)\,dt = m\int_{t_i}^{t_f} \frac{d\bar{v}}{dt}\,dt$$

$$= m\left[\bar{v}(t_f) - \bar{v}(t_i)\right] = m\Delta\bar{v}$$

4

$$\bar{J} = m\Delta\bar{v}$$

Impulse-Momentum Theorem

An impulse applied to a system changes the system's momentum, and that change of momentum is exactly equal to the impulse that was applied:

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}. \qquad 9.7$$

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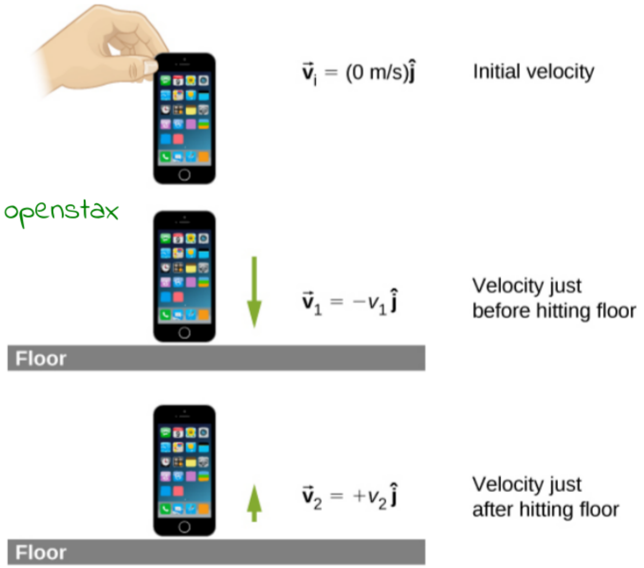
Fall síma, Ex. 9.4

Sími fellur úr kyrrstöðu, h = 1.5m, hvaða kraftar verka á hann?

Ekki bara w = -mg, hvað getum við sagt um kraft gólfsins á hann?

5



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$$\bar{J} = \bar{F}_{ave}\,\Delta t$$

$$\bar{F}_{ave} = \frac{\bar{J}}{\Delta t}$$

$$\bar{J} = \Delta\bar{p} = m\Delta\bar{v}$$

$$\bar{F}_{ave} = \frac{m\Delta\bar{v}}{\Delta t}$$

Ef $\bar{v}_2 = 0$ $\rightarrow$ $m\Delta\bar{v} = m(\bar{v}_2 - \bar{v}_1) = m(0-(-v_1\hat{j}))$

6

$$m\Delta\bar{v} = +mv_1\hat{j}$$

eftir fall úr kyrrstöðu $\quad v_1 = \sqrt{2hg}$

$$\rightarrow \bar{F} = \frac{\Delta\bar{p}}{\Delta t} = \frac{m\Delta\bar{v}}{\Delta t} = \frac{mv_1\hat{j}}{\Delta t} = \frac{m\sqrt{2gh}\,\hat{j}}{\Delta t}$$

$m = 0{,}172\ \text{kg}$

$g = 9{,}81\ \text{m/s}^2$

$h = 1{,}5\ \text{m}$

$\Delta t = 0{,}026\ \text{s}$

$= (36\ \text{N})\hat{j}$

en $mg = 1{,}68\ \text{N}$

stefna upp

Árekstrartími metinn frá lengd síma 0.14m $= L$
og lokafallferð $v_1 = 5.4$ m/s $\quad \Delta t \simeq \frac{L}{v_1}$

7

Reiknandi atlagið fengum við

$$\bar{F}_{ave} = \frac{\Delta\bar{p}}{\Delta t}$$

en fyrir samfellda lýsingum var komið

$$\bar{F} = \frac{d\bar{p}}{dt} = \frac{d(m\bar{v})}{dt}\ m\frac{d\bar{v}}{dt} = m\bar{a}$$

ef m er fastur. því fæst:

Newton's Second Law of Motion in Terms of Momentum

The net external force on a system is equal to the rate of change of the momentum of that system caused by the force:

$$\boxed{\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}.}$$

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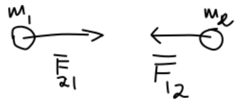
8

Varðveisla skriðþunga

Tveir hlutir víxlverkast, engir aðrir kraftar, 3. lögmál Newtons



Lokað kerfi

$$\bar{F}_{21} = -\bar{F}_{12} \rightarrow m_1\bar{a}_1 = -m_2\bar{a}_2$$

$$\rightarrow \frac{d}{dt}\left[m_1\bar{v}_1\right] = -\frac{d}{dt}\left[m_2\bar{v}_2\right]$$

$$\rightarrow \frac{d\bar{p}_1}{dt} + \frac{d\bar{p}_2}{dt} = 0 \rightarrow \frac{d}{dt}\left[\bar{p}_1 + \bar{p}_2\right] = 0$$

$$\rightarrow \bar{p}_1 + \bar{p}_2 = \text{fasti}$$

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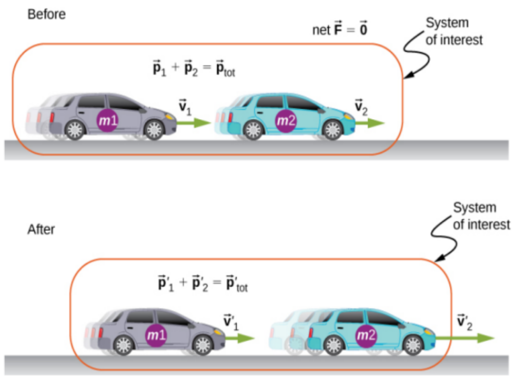
Law of Conservation of Momentum

The total momentum of a closed system is conserved:

$$\boxed{\sum_{j=1}^{N} \vec{\mathbf{p}}_j = \text{constant.}}$$



Við munum ekki fara frekar í flokkun árekstra og þá aðferðafræði sem heppileg er til að greina þá

Þurfum að nefna fyrir hlut eða kerfi agna

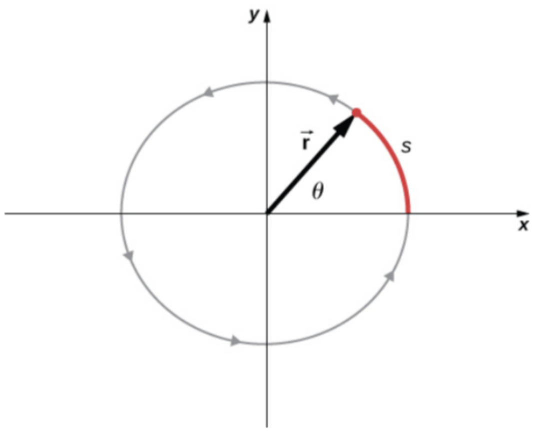
Massamiðja

$$\bar{r}_{CM} = \frac{1}{M}\sum_{j=1}^{N} m_j \bar{r}_j \,, \quad \bar{r}_{CM} = \frac{1}{M}\int \bar{r}\, dm$$

$$\bar{v}_{CM} = \frac{1}{M}\sum_{j=1}^{N} m_j \bar{v}_j \,, \quad \boxed{\bar{F}_{ext} = \sum_{j=1}^{N} \frac{d\bar{p}_j}{dt} \quad \text{eða} \quad \bar{F} = \frac{d\bar{p}_{CM}}{dt}}$$

Aðeins ytri kraftar hafa áhrif á hreyfingu massamiðjunnar

Hraði í hringhreyfingu



Viljum endurbæta lýsingu hringhreyfingar. Byrjum á að skilgreina hornhraðavigur

Áður var komið að

$$s = r\theta$$

allt skalarstærðir

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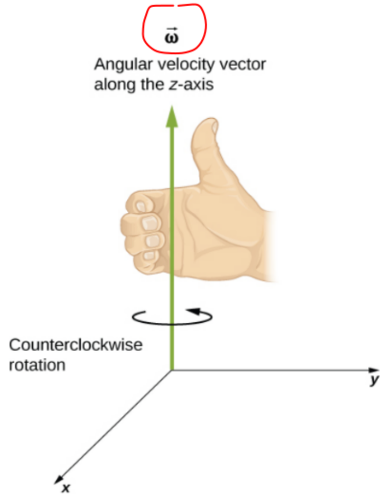
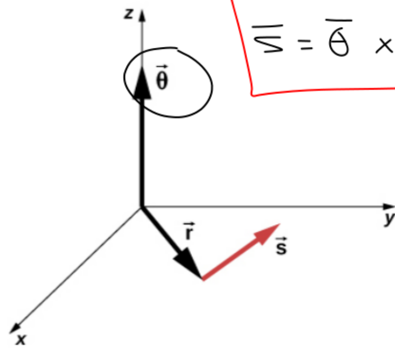
Skiptum yfir í vigurstærðir (vectors)

Hringhreyfing í x-y-sléttu, (andsælis)





$$\boxed{\bar{s} = \bar{\theta} \times \bar{r}}$$

$$\boxed{\bar{v} = \bar{\omega} \times \bar{r}}$$

$$\omega = \frac{d\theta}{dt}$$

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(13)

snertilferð (tangential speed)

$$v_t = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = \theta\underbrace{\frac{dr}{dt}}_{=0} + r\frac{d\theta}{dt} = r\frac{d\theta}{dt} = r\omega$$

Hornhröðun

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Snertilhröðun

$$\overline{a}_t = \overline{\alpha} \times \overline{r}$$

$$a_t = r\alpha$$

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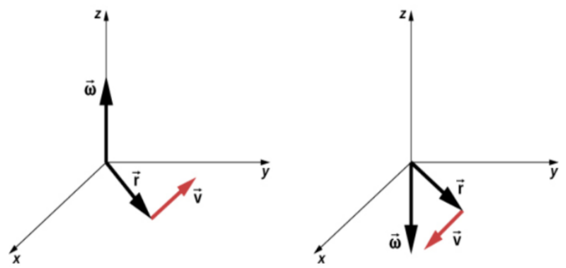

(14)

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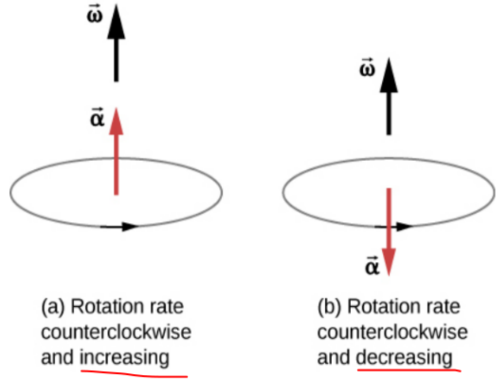

Figure 10.7 The rotation is counterclockwise in both (a) and (b) with the angular velocity in the same direction. (a) The angular acceleration is in the same direction as the angular velocity, which increases the rotation rate. (b) The angular acceleration is in the opposite direction to the angular velocity, which decreases the rotation rate.

(15)

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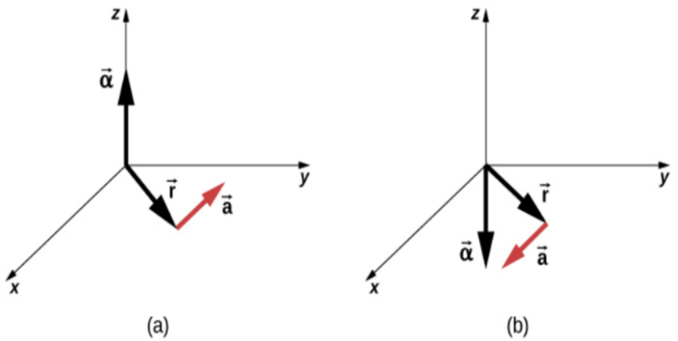

Figure 10.8 (a) The angular acceleration is the positive *z*-direction and produces a tangential acceleration in a counterclockwise sense. (b) The angular acceleration is in the negative *z*-direction and produces a tangential acceleration in the clockwise sense.

$$\boxed{\overline{a}_t = \overline{\alpha} \times \overline{r}}$$

1

Hringsnúningur með fastri hröðun, (sértilfelli)

Notum einfaldlega samsvörunina við lýsingu línulegar hreyfingar með fastri hröðun

Angular displacement from average angular velocity	$\theta_f = \theta_0 + \bar{\omega} t$
Angular velocity from angular acceleration	$\omega_f = \omega_0 + \alpha t$
Angular displacement from angular velocity and angular acceleration	$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
Angular velocity from angular displacement and angular acceleration	$\omega_f^2 = \omega_0{}^2 + 2\alpha(\Delta\theta)$

Table 10.1 Kinematic Equations

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	Linear	Rotational
Position	x	θ
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

Rotational	Translational
$\theta_f = \theta_0 + \bar{\omega} t$	$x = x_0 + \bar{v} t$
$\omega_f = \omega_0 + \alpha t$	$v_f = v_0 + a t$
$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x_f = x_0 + v_0 t + \frac{1}{2} a t^2$
$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$	$v_f^2 = v_0^2 + 2a(\Delta x)$

Table 10.2 Rotational and Translational Kinematic Equations

2

Hreyfiorka í hringhreyfingu -- rotational kinetic energy
Hverfitregða -- moment of inertia

Snúningur um fastan ás

Hugsum hlutsem snýst sem samsettan úr fjölda massa

$$\boxed{K} = \sum_j \frac{1}{2} m_j v_j^2 = \sum_j \frac{1}{2} m_j (r_j \omega_j)^2$$

$\rightarrow$ $\omega_j = \omega$ fyrir alla massana

$$\rightarrow \quad K = \frac{1}{2}\left[\sum_j m_j r_j^2\right]\omega^2 = \boxed{\frac{1}{2} I \omega^2}$$

$$I = \sum_j m_j r_j^2 \rightarrow \int r^2 dm$$

hverfitregða fyrir safn punktmassa eða hlut

3

Hverfitregða nokkurra hluta um fastan ás

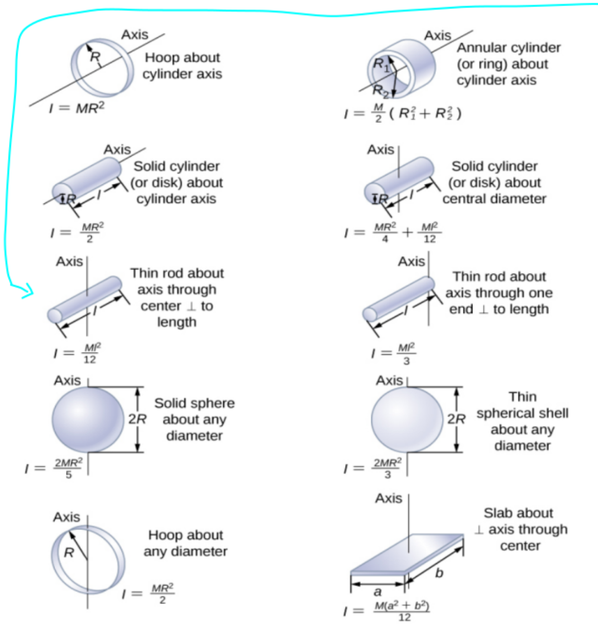
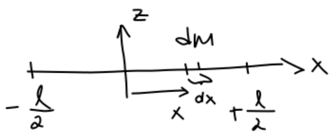


Figure 10.20 Values of rotational inertia for common shapes of objects.

Reynum



$$dm = \frac{M}{\ell} dx$$

$$I = \int r^2 dm$$

$$= \int x^2 \frac{M}{\ell} dx$$

$$= \frac{M}{\ell} \int_{-\ell/2}^{\ell/2} x^2 dx$$

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$$I = \frac{M}{\ell}\int_{-\ell/2}^{\ell/2} x^2 dx = \frac{M}{\ell} \left.\frac{x^3}{3}\right|_{-\ell/2}^{\ell/2} = \frac{M}{3\ell}\left\{\left(\frac{\ell}{2}\right)^3 - \left(-\frac{\ell}{2}\right)^3\right\}$$

$$= \frac{M}{3\ell}\frac{\ell^3}{8} 2 = \frac{1}{12} M\ell^2$$

Ef snúningsásinn væri í gegnum annan endann fæst $I = \frac{1}{3} M\ell^2$ og almennar

Parallel-Axis Theorem

Let m be the mass of an object and let d be the distance from an axis through the object's center of mass to a new axis. Then we have

$$I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2. \quad (10.20)$$

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Huygens - Steiner

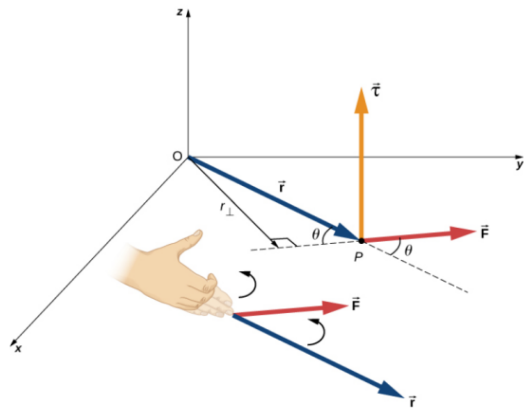
5

vægi -- torque

Torque

When a force $\vec{\mathbf{F}}$ is applied to a point P whose position is $\vec{\mathbf{r}}$ relative to O (Figure 10.32), the torque $\vec{\tau}$ around O is

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}. \qquad (10.22)$$



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6

Annað lögmál Newtons fyrir hringhreyfingu um fastan ás

Newton's Second Law for Rotation

If more than one torque acts on a rigid body about a fixed axis, then the sum of the torques equals the moment of inertia times the angular acceleration:

$$\sum_i \tau_i = I\alpha. \qquad (10.25)$$

Samanborið við $\boxed{\bar{F} = m\bar{a}}$

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en eigum enn eftir að sjá framsetningu sem hægt er að bera saman við

$$\boxed{\bar{F} = \frac{d}{dt}\bar{P}}$$

7

Vinna og afl fyrir hringheyfingu um fastan ás

$$\bar{S} = \bar{\theta} \times \bar{r}, \quad d\bar{s} = d(\bar{\theta} \times \bar{r}) = d\bar{\theta} \times \bar{r}$$

$$W = \int \sum \bar{F} \cdot d\bar{s} = \int \sum \bar{F} \cdot (d\bar{\theta} \times \bar{r}) = \int d\bar{\theta} \cdot (\bar{r} \times \sum \bar{F})$$

þ.s. $\bar{a} \cdot (\bar{b} \times \bar{c}) = \bar{b} \cdot (\bar{c} \times \bar{a})$

Notum $\bar{r} \times \sum \bar{F} = \sum \bar{\tau}$

$$\longrightarrow \quad W = \int \sum \bar{\tau} \cdot d\bar{\theta}$$

Tökum betur saman

8

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Work-Energy Theorem for Rotation

The work-energy theorem for a rigid body rotating around a fixed axis is

$$W_{AB} = K_B - K_A \qquad (10.29)$$

where

$$K = \frac{1}{2} I \omega^2$$

and the rotational work done by a net force rotating a body from point A to point B is

$$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta. \qquad (10.30)$$

Afl - power

$$\boxed{P = \frac{dW}{dt}} = \frac{d}{dt}(\tau\theta) = \tau\frac{d\theta}{dt} = \boxed{\tau\omega}$$

Vinna fæst út úr kerfinu með vægi (eða sett í kerfið)

9

velta án skriks

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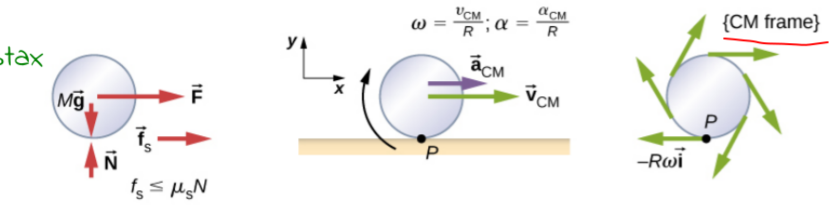
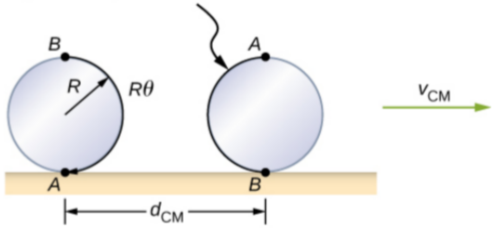


(a) Forces on the wheel

(b) Wheel rolls without slipping

(c) Point P has velocity vector in the negative direction with respect to the center of mass of the wheel

Arc length AB maps onto wheel's surface



$$\bar{v}_P = -R\omega\hat{i} + \bar{v}_{CM}\hat{i} = 0$$

$$\rightarrow \quad v_{CM} = R\omega$$
$$a_{CM} = R\alpha$$
$$d_{CM} = R\theta$$

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velta niður halla

$\sum F_x = ma_x$; $\sum F_y = ma_y$.

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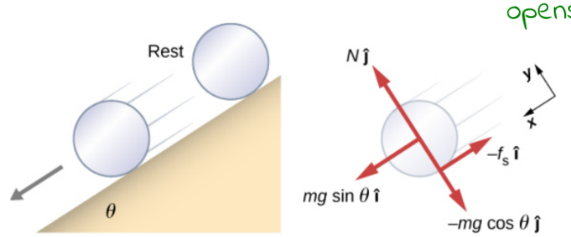

Rolling without slipping

Free-body diagram

x: $\quad mg\sin\theta - f_s = m(a_{CM})_x = ma_{CM}$

y: $\quad N - mg\cos\theta = 0$

$\rightarrow \quad a_{CM} = g\sin\theta - \frac{f_s}{m}$ (3)

$\rightarrow \quad f_s = \frac{I_{CM}\alpha}{r}$

Annað lögmál Newtons fyrir snúning

$$\sum \tau_{CM} = I_{CM}\alpha \quad \rightarrow \quad f_s r = I_{CM}\alpha \quad (4)$$

11

$$a_{CM} = r\alpha$$

(4) $\rightarrow \quad f_s = \frac{I_{CM}\alpha}{r} = \frac{I_{CM}a_{CM}}{r^2}$

(3) $\rightarrow \quad a_{CM} = g\sin\theta - \frac{I_{CM}a_{CM}}{mr^2}$

$\rightarrow \quad a_{CM}\left[1 + \frac{I_{CM}}{mr^2}\right] = g\sin\theta$

$\rightarrow \quad a_{CM} = \frac{mg\sin\theta}{m + \frac{I_{CM}}{r^2}}$

Sívalningur

$$I_{CM} = \frac{mr^2}{2} \quad \rightarrow \quad a_{CM} = \frac{mg\sin\theta}{m + \frac{mr^2}{2r^2}} = \frac{2}{3}g\sin\theta$$

12

Síðan fæst

$$f_s = \frac{I_{CM}\alpha}{r} = \frac{mgI_{CM}\sin\theta}{mr^2 + I_{CM}}$$

$$f_s \le \mu_s N = \mu_s mg\cos\theta$$

$$\rightarrow \quad \mu_s \geq \frac{\tan\theta}{1 + \frac{mr^2}{I_{CM}}} = \frac{1}{3}\tan\theta$$

Hröðun sívalningsins niður hallann er minni en hlutar sem rinni niður án viðnáms þar sem sívalningurinn hefur massa og hverfitregðu

Fyrir vissan halla getum við metið hvaða μ_s þarf til að hann skriki ekki

13

Orkuvarðveisla í veltu

Heildarorkan er

$$\boxed{E_T = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2 + mgh}$$

Hverfiþungi - angular momentum

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Angular Momentum of a Particle

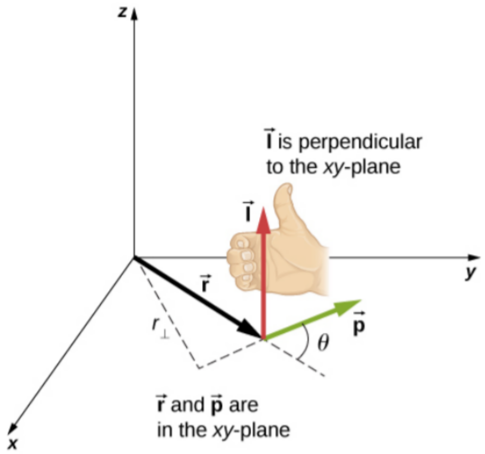
The **angular momentum** $\vec{\mathbf{l}}$ of a particle is defined as the cross-product of $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$, and is perpendicular to the plane containing $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$:

$$\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}. \qquad 11.5$$

Táknum

$$\boxed{\bar{L} = \bar{r} \times \bar{p}}$$

14



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$$\bar{L} = \bar{r} \times \bar{p}$$

$$\frac{d\bar{L}}{dt} = \frac{d\bar{r}}{dt} \times \bar{p} + \bar{r} \times \frac{d\bar{p}}{dt}$$

$$= \underbrace{\bar{v} \times m\bar{v}}_{=0} + \bar{r} \times \frac{d\bar{p}}{dt}$$

$$= \bar{r} \times \bar{F} = \bar{\tau}$$

$$\rightarrow \boxed{\frac{d\bar{L}}{dt} = \sum \bar{\tau}}$$

Hreyfijafna fyrir hverfiþunga

15

Varðveisla hverfiþunga

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Law of Conservation of Angular Momentum

The angular momentum of a system of particles around a point in a fixed inertial reference frame is conserved if there is no net external torque around that point:

$$\frac{d\vec{\mathbf{L}}}{dt} = 0 \qquad 11.10$$

or

$$\vec{\mathbf{L}} = \vec{\mathbf{l}}_1 + \vec{\mathbf{l}}_2 + \cdots + \vec{\mathbf{l}}_N = \text{constant}. \qquad 11.11$$

Alltaf þarf að taka fram hvaða viðmiðunarpunkt er átt við fyrir hverfiþunga

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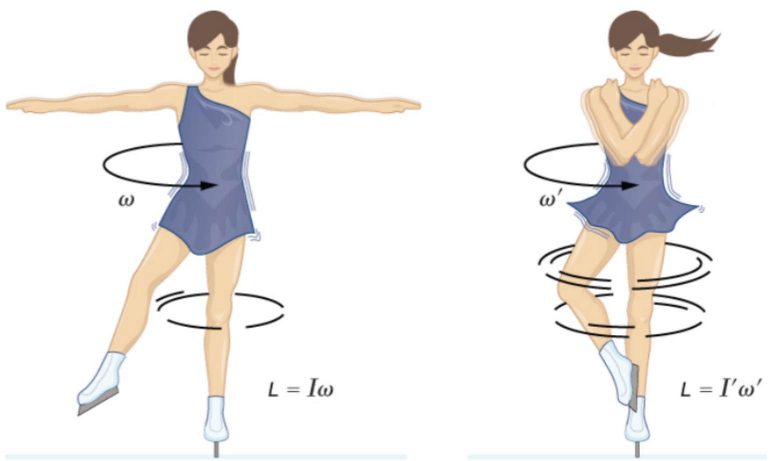

Varðveisla hverfiþungans leiðir til þess að hornferðin breytist þegar stúlkan breytir hverfiþunganum

(1)

vökvar - fluids

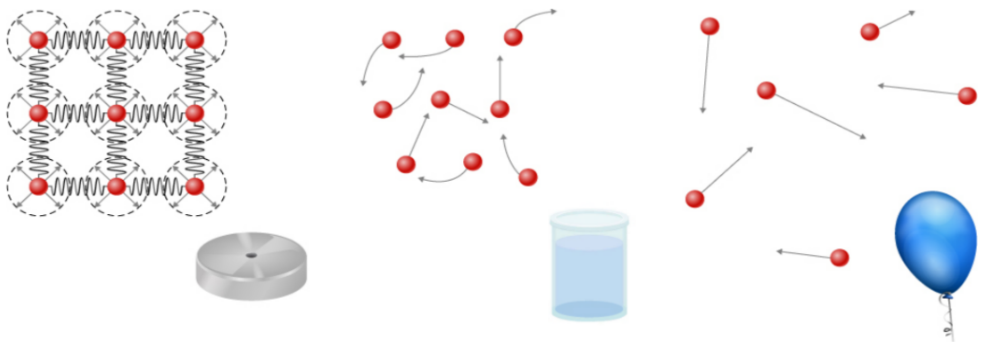

Figure 14.2 (a) Atoms in a solid are always in close contact with neighboring atoms, held in place by forces represented here by springs. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between the atoms strongly resist attempts to compress the atoms. (c) Atoms in a gas move about freely and are separated by large distances. A gas must be held in a closed container to prevent it from expanding freely and escaping.

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(2)

þéttleiki - density

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Density

The average density of a substance or object is defined as its mass per unit volume,

$$\rho = \frac{m}{V} \qquad 14.1$$

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume.

Solids (0.0°C)		Liquids (0.0°C)		Gases (0.0°C, 101.3 kPa)	
Substance	**ρ(kg/m^3)**	**Substance**	**ρ(kg/m^3)**	**Substance**	**ρ(kg/m^3)**
Aluminum	2.70×10^3	Benzene	8.79×10^2	Air	1.29×10^0
Bone	1.90×10^3	Blood	1.05×10^3	Carbon dioxide	1.98×10^0
Brass	8.44×10^3	Ethyl alcohol	8.06×10^2	Carbon monoxide	1.25×10^0

(3)

Solids (0.0°C)		Liquids (0.0°C)		Gases (0.0°C, 101.3 kPa)	
Concrete	2.40×10^3	Gasoline	6.80×10^2	Helium	1.80×10^{-1}
Copper	8.92×10^3	Glycerin	1.26×10^3	Hydrogen	9.00×10^{-2}
Cork	2.40×10^2	Mercury	1.36×10^4	Methane	7.20×10^{-2}
Earth's crust	3.30×10^3	Olive oil	9.20×10^2	Nitrogen	1.25×10^0
Glass	2.60×10^3			Nitrous oxide	1.98×10^0
Gold	1.93×10^4			Oxygen	1.43×10^0
Granite	2.70×10^3				
Iron	7.86×10^3				
Lead	1.13×10^4				
Oak	7.10×10^2				
Pine	3.73×10^2				
Platinum	2.14×10^4				
Polystyrene	1.00×10^2				
Tungsten	1.93×10^4				
Uranium	1.87×10^3 – 4				

Table 14.1 Densities of Some Common Substances

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(4)

getur verið mjög háð hitastigi

Substance	ρ(kg/m^3)
Ice (0°C)	9.17×10^2
Water (0°C)	9.998×10^2
Water (4°C)	1.000×10^3
Water (20°C)	9.982×10^2
Water (100°C)	9.584×10^2
Steam (100°C, 101.3 kPa)	1.670×10^2
Sea water (0°C)	1.030×10^3

Table 14.2 Densities of Water

$$\text{Specific gravity} = \frac{\text{Density of material}}{\text{Density of water}}$$

getur verið breytilegt í misleitum vökva

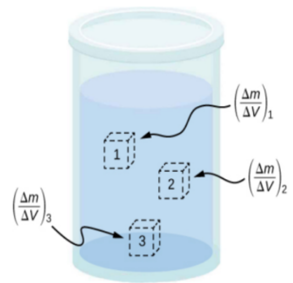

Figure 14.4 Density may vary throughout a heterogeneous mixture. Local density at a point is obtained from dividing mass by volume in a small volume around a given point.

Local density can be obtained by a limiting process, based on the average density in a small volume around the point in question, taking the limit where the size of the volume approaches zero,

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} \qquad 14.2$$

where ρ is the density, m is the mass, and V is the volume.

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5

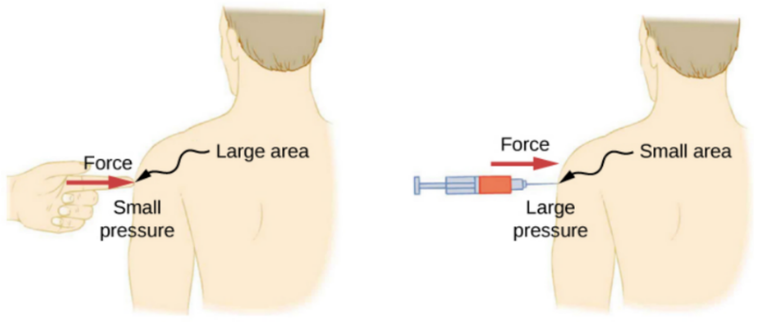
þrýstingur - pressure

Pressure

Pressure (p) is defined as the normal force F per unit area A over which the force is applied, or

$$p = \frac{F}{A}. \quad 14.3$$

To define the pressure at a specific point, the pressure is defined as the force dF exerted by a fluid over an infinitesimal element of area dA containing the point, resulting in $p = \frac{dF}{dA}$.



(a) (b)

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6

þrýstingur sem fall af dýpt

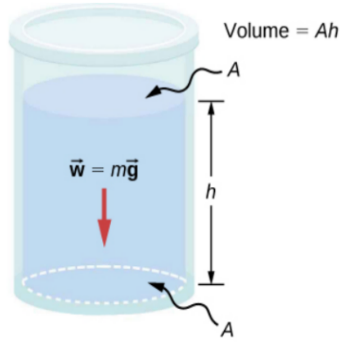

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Figure 14.6 The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), so the bottom must support it all.

Á dýpi h vegur vökvasúlan

$$w = mg = [\rho V]g = [\rho A h]g$$

Því er þrýstingur á dýpi h

$$p(h) = \frac{F}{A} = p_0 + \rho h g$$

$$p(0) = p_0$$

7

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Pressure at a Depth for a Fluid of Constant Density

The pressure at a depth in a fluid of constant density is equal to the pressure of the atmosphere plus the pressure due to the weight of the fluid, or

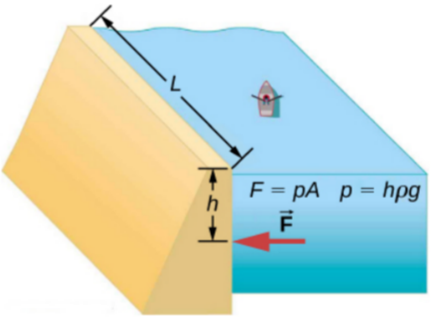
$$p = p_0 + \rho h g, \quad 14.4$$

Where p is the pressure at a particular depth, p_0 is the pressure of the atmosphere, ρ is the density of the fluid, g is the acceleration due to gravity, and h is the depth.

Ex, 14.1

$L = 500\,m$

$h = 80{,}0\,m$



Meðal þrýstingur á garð

$$\langle p \rangle = \langle h \rangle \rho g = 40\,m \left(1000\frac{kg}{m^3}\right)\left(9{,}80\frac{m}{s^2}\right)$$

$$= 3{,}92 \cdot 10^5 \frac{N}{m^2}$$

$$F = \langle p \rangle A = \langle h \rangle \rho g A$$

$$= 1{,}57 \cdot 10^{10}\,N$$

8

þrýstingur vökva í jafnvægi í föstum þyngdarkrafti

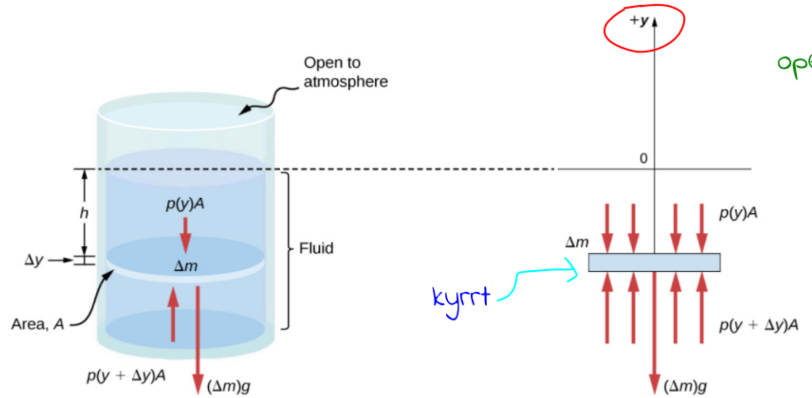


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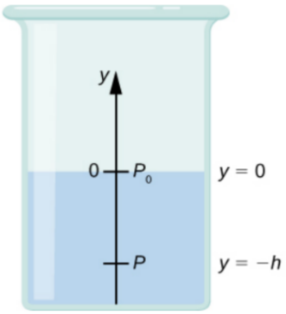
Figure 14.8 Forces on a mass element inside a fluid. The weight of the element itself is shown in the free-body diagram.

$$p(y+\Delta y)A - p(y)A - g\Delta m = 0, \quad \Delta m = |\rho A \Delta y| = -\rho A \Delta y$$

$$\rightarrow \frac{p(y+\Delta y) - p(y)}{\Delta y} = -\rho g \rightarrow \boxed{\frac{dp}{dy} = -\rho g}$$

9

Reynum



$$\frac{dp}{dy} = -\rho g$$

$$\rightarrow \quad dp = -\rho g\, dy$$

heildum

$$\int_{p_0}^{p} dp' = -\int_{0}^{-h} \rho g\, dy$$

$$\rightarrow \quad \{p - p_0\} = -\rho g(-h) = \rho g h$$

$$\rightarrow \quad \boxed{p = p_0 + \rho g h}$$

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10

..en í andrúmslofti í jafnvægi?

kjörgas - ideal gas: $pV = nRT$

$$\rightarrow \quad p = \frac{nRT}{V} = \frac{nRm}{V}\,\frac{T}{m} = \frac{n m N_A}{V}\,\frac{k_B T}{m}$$

$$= \rho \frac{k_B T}{m}$$

massi sameindar

En höfðum líka

$$\frac{dp}{dy} = -\rho g = -g\left[\frac{pm}{k_B T}\right]$$

n: fjöldi móla

N_A: Tala Avogadrosar

k_B: fasti Boltzmanns

$$\rightarrow \quad \frac{dp}{dy} = -p\left[\frac{mg}{k_B T}\right] = -\alpha p$$

11

aðgreinum breytistærðir

$$\frac{dp}{p} = -\alpha\, dy \quad \rightarrow \quad \int_{p_0}^{p(y)} \frac{dp}{p} = -\alpha \int_0^y dy$$

$$\rightarrow \quad \ln\left\{\frac{p(y)}{p_0}\right\} = -\alpha y \quad \rightarrow \quad \boxed{p(y) = p_0 e^{-\alpha y}}$$

$$\alpha = \frac{mg}{k_B T} = \frac{4{,}8\cdot 10^{-26}\,\text{kg} \cdot 9{,}81\,\text{m/s}^2}{1{,}38\cdot 10^{-23}\,\text{J/K} \cdot 300\,\text{K}} \approx \frac{1}{8800\,\text{m}}$$

fyrir N_2

12

Stefna þrýsings

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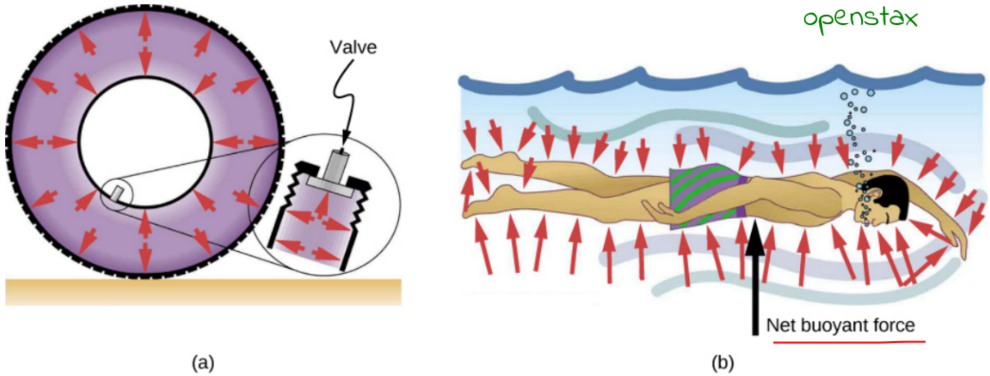

Figure 14.10 (a) Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows represent directions and magnitudes of the forces exerted at various points. (b) Pressure is exerted perpendicular to all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force. The net vertical force on the swimmer is equal to the sum of the buoyant force and the weight of the swimmer.

13

þrýstingur mældur

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Absolute Pressure

The absolute pressure, or total pressure, is the sum of gauge pressure and atmospheric pressure:

$$p_{abs} = p_g + p_{atm} \qquad 14.11$$

where p_{abs} is absolute pressure, p_g is gauge pressure, and p_{atm} is atmospheric pressure.

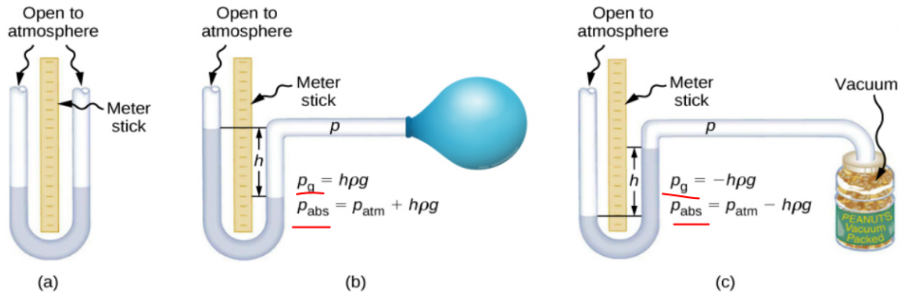

Figure 14.12 An open-tube manometer has one side open to the atmosphere. (a) Fluid depth must be the same on both sides, or the pressure each side exerts at the bottom will be unequal and liquid will flow from the deeper side. (b) A positive gauge pressure $p_g = h\rho g$ transmitted to one side of the manometer can support a column of fluid of height h. (c) Similarly, atmospheric pressure is greater than a negative gauge pressure p_g by an amount $h\rho g$. The jar's rigidity prevents atmospheric pressure from being transmitted to the peanuts.

14

Loftvog

Unit	Definition
SI unit: the Pascal	$1\ \text{Pa} = 1\ \text{N/m}^2$
English unit: pounds per square inch (lb/in.^{2} or psi)	$1\ \text{psi} = 6.895 \times 10^3\ \text{Pa}$
Mismunandi einingar Other units of pressure	$1\ \text{atm} = 760\ \text{mmHg}$ $= 1.013 \times 10^5\ \text{Pa}$ $= 14.7\ \text{psi}$ $= 29.9$ inches of Hg $= 1013\ \text{mbar}$
	$1\ \text{bar} = 10^5\ \text{Pa}$
	$1\ \text{torr} = 1\ \text{mm Hg} = 133.3\ \text{Pa}$

Table 14.3 Summary of the Units of Pressure

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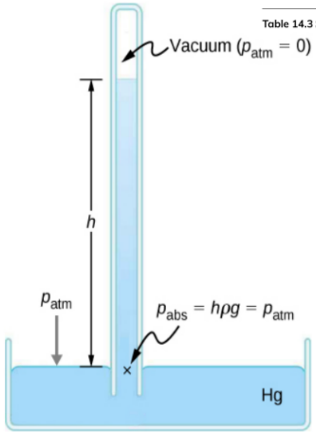

Figure 14.13 A mercury barometer measures atmospheric pressure. The pressure due to the mercury's weight, $h\rho g$, equals atmospheric pressure. The atmosphere is able to force mercury in the tube to a height h because the pressure above the mercury is zero.

15

Lögmál Pascals

$$\Delta p_{top} = \frac{Mg}{A}.$$

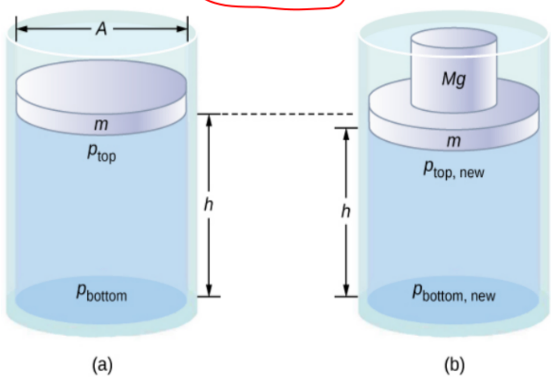

Figure 14.15 Pressure in a fluid changes when the fluid is compressed. (a) The pressure at the top layer of the fluid is different from pressure at the bottom layer. (b) The increase in pressure by adding weight to the piston is the same everywhere, for example, $p_{top\ new} - p_{top} = p_{bottom\ new} - p_{bottom}$.

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Vökva-kerfi - hydrolic systems

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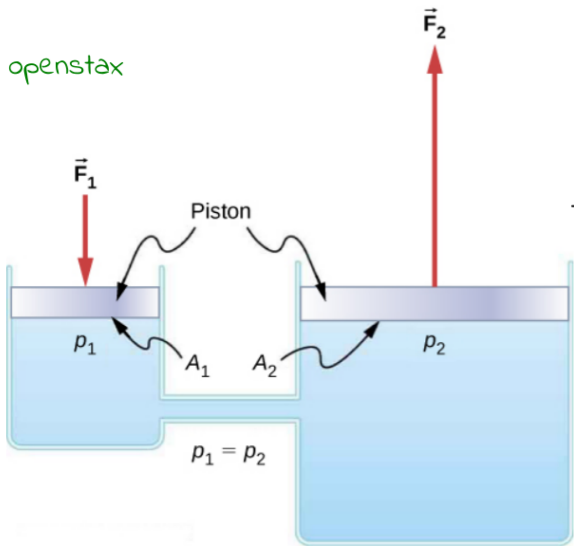


$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

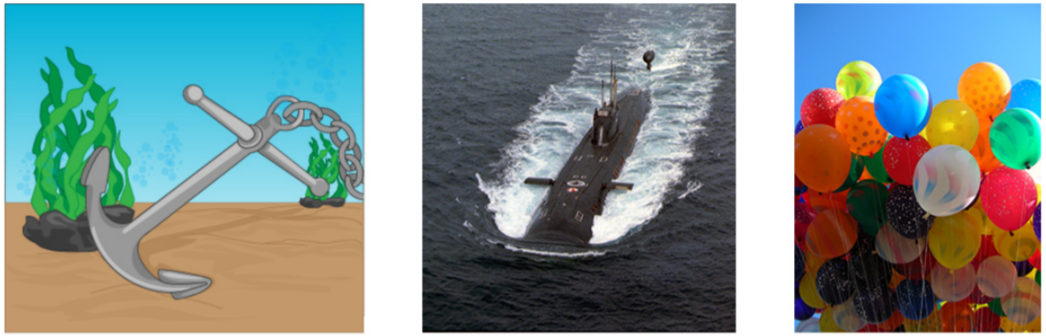
$$\rightarrow F_2 = \underbrace{\left(\frac{A_2}{A_1}\right)}_{} F_1$$

t.d.: > 1

Nýting
Lyftur (tjakkar)
Bremsukerfi
Stýri
...

1

Flotkraftar - buoyant forces



(a) (b) (c)

Figure 14.19 (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit b: modification of work by Allied Navy; credit c: modification of work by "Crystl"/Flickr)

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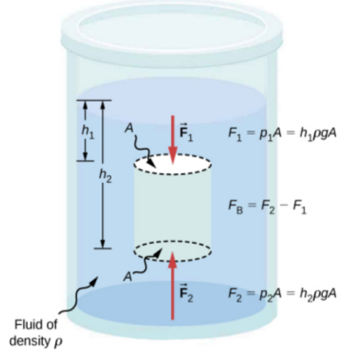
Buoyant Force

The buoyant force is the upward force on any object in any fluid.

2



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$$F_B = F_2 - F_1 = h_2\rho gA - h_1\rho gA$$

$$= (h_2 - h_1)\rho gA = \rho g\Delta hA = \rho gV$$

$$= w_{fL}$$

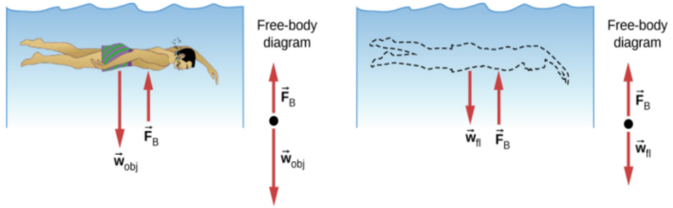
Archimedes' Principle

The buoyant force on an object equals the weight of the fluid it displaces. In equation form, **Archimedes' principle** is

$$\boxed{F_B = w_{fl},}$$

where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

This principle is named after the Greek mathematician and inventor Archimedes (ca. 287–212 BCE), who stated this principle long before concepts of force were well established.



(a) (b)

Figure 14.21 (a) An object submerged in a fluid experiences a buoyant force F_B. If F_B is greater than the weight of the object, the object

3

Vökvaaflfræði - fluid dynamics

Jafnt eða lagskipt flæði

Iðuflæði

(a) Laminar Flow (b) Turbulent Flow

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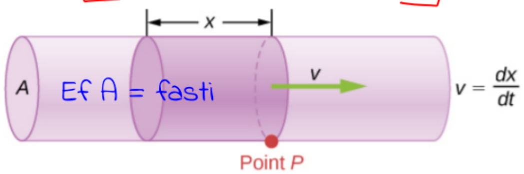
Figure 14.25 (a) Laminar flow can be thought of as layers of fluid moving in parallel, regular paths. (b) In turbulent flow, regions of fluid move in irregular, colliding paths, resulting in mixing and swirling.

$$\boxed{Q = \frac{dV}{dt} = \frac{d}{dt}(Ax) = A\frac{dx}{dt} = Av.}$$



$$Q = \frac{dV}{dt} = \frac{d}{dt}(Ax) = A\frac{dx}{dt} = Av$$

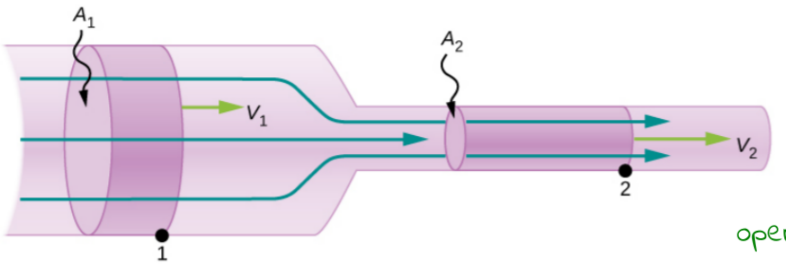
Flæði $\boxed{Q = \frac{dV}{dt}}$

$$[Q] = \frac{L^3}{T}$$

Eining m^3/s

4

Ósamþjappanlegur vökvi - incompressible fluid



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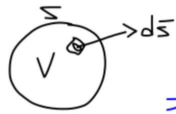
$$Q_1 = Q_2$$

$$\boxed{A_1v_1 = A_2v_2}$$

Sértilfelli af samfelldni jöfnunni

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$

$$\frac{\partial}{\partial t}M_V + \oint_S \vec{J}\cdot d\vec{S} = 0$$



straumþéttleiki

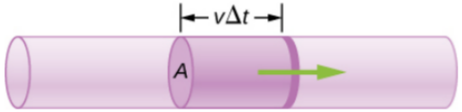
$\vec{J} = \rho\vec{v}$

Nett flæði "efnis" inn í V verður til að massinn þar breytist, varðveislulögmál

$$M_V = \int_V \rho\, dV'$$

5

Varðveisla massa



$m = \rho V = \rho A x$ ef ρ er fasti

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$$\frac{dm}{dt} = \frac{d}{dt}(\rho A x) = \rho A \frac{dx}{dt} = \rho A v$$

Massinn út úr einhverju rúmmáli verður að vera jafn massanum inn (sístætt ástand). Ef þéttleikinn í þverskurði gæti breyst:

$$\left(\frac{dm}{dt}\right)_1 = \left(\frac{dm}{dt}\right)_2 \rightarrow \boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

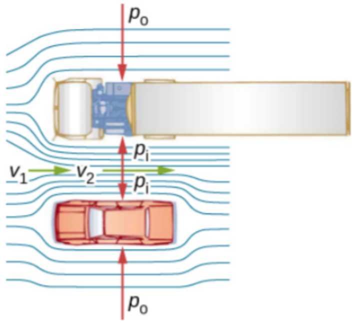
Ef vökvinn er ósamþjappanlegur $A_1 v_1 = A_2 v_2$

6

Jafna Bernoullis

Þurfum að huga að orkuvarðveislu í flæði. Athugum ósamþjappanlegan vökva án flæðisviðnáms

Hversdagsleg reynsla



Í engum hliðarvind er eins og kraftur myndist á litla bílinn þegar hinn fer framhjá. Krafturinn er að stóra bílnum og við eigum eftir að tengja hann við hraðabreytingu loftsins milli bílanna

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7

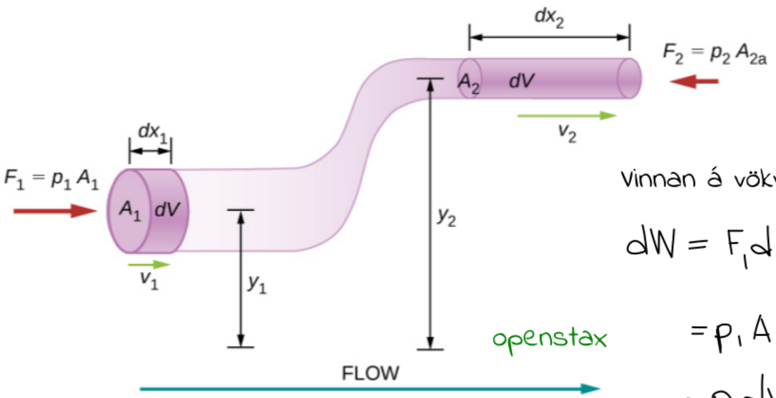

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Figure 14.30 The geometry used for the derivation of Bernoulli's equation.

Vinnan á vökvann

$$dW = F_1 dx_1 - F_2 dx_2$$
$$= p_1 A_1 dx_1 - p_2 A_2 dx_2$$
$$= p_1 dV - p_2 dV$$
$$= (p_1 - p_2) dV$$

Vinnan breytir hreyfiorku vökvans

$$dK = \tfrac{1}{2} m_2 v_2^2 - \tfrac{1}{2} m_1 v_1^2 = \tfrac{1}{2}\rho dV (v_2^2 - v_1^2)$$

breytingin í stöðuorku er

$$dU = mgy_2 - mgy_1 = \rho dV g (y_2 - y_1)$$

notuðum varðveislu massa

8

Orkuvarðveisla

$$dW = dK + dU$$

$$\rightarrow \quad (p_1 - p_2) dV = \tfrac{1}{2}\rho dV (v_2^2 - v_1^2) + \rho dV g (y_2 - y_1)$$

$$\rightarrow \quad (p_1 - p_2) = \tfrac{1}{2}\rho (v_2^2 - v_1^2) + \rho g (y_2 - y_1)$$

$$\rightarrow \quad p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\rightarrow \quad \boxed{p + \tfrac{1}{2}\rho v^2 + \rho g h = \text{Fasti}}$$

Jafna Bernoullis

9

Sértilfelli fyrir jöfnu Bernoullis

Vökvi án flæðis

$v_1 = v_2 = 0$

$P_1 + \rho g h_1 = P_2 + \rho g h_2$

$\rightarrow \boxed{P_2 = P_1 + \rho g h_1 \quad \text{ef } h_2 = 0}$

Enginn hæðarmunur, lögmál Bernoullis

$h_1 = h_2$

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2}$$

Munum, ósamþjappanlegur vökvi og ekkert viðnám við flæðinu

10

Notkun

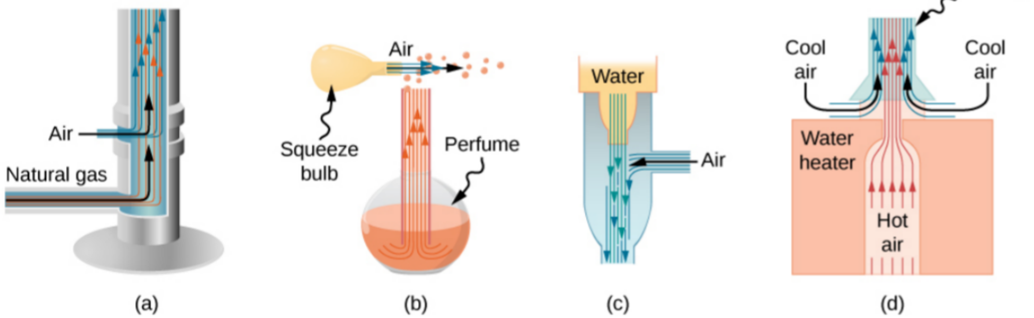

Figure 14.31 Entrainment devices use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling.

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11

Hraðamæling

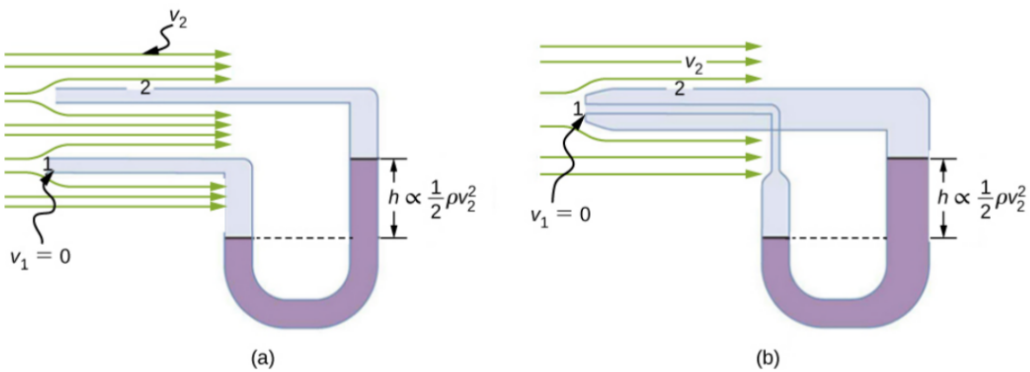

Figure 14.32 Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, so the fluid has a speed v across the opening; thus, pressure there drops. The difference in pressure at the manometer is $\frac{1}{2}\rho v_2^2$, so h is proportional to $\frac{1}{2}\rho v_2^2$. (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube.

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12

Ex. 14.7

Slanga $d = 6{,}40$ cm

Stútur $d_N = 3{,}00$ cm

$P_1 = 1{,}62 \cdot 10^6 \ \frac{N}{m^2}$

$Q = 40 \ l/s$

$h_2 = 10$ m

$h_1 = 0$

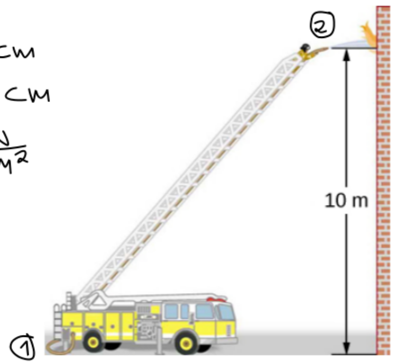

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Figure 14.33 Pressure in the nozzle of this fire hose is less than at ground level for two reasons: The water has to go uphill to get to the nozzle, and speed increases in the nozzle. In spite of its lowered pressure, the water can exert a large force on anything it strikes by virtue of its kinetic energy. Pressure in the water stream becomes equal to atmospheric pressure once it emerges into the air.

$$P_1 + \frac{1}{2}\rho v_1^2 + \underbrace{\rho g h_1}_{=0,\ h_1 = 0} = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

(13)

$$v_1 = \frac{Q_1}{A_1} = \frac{Q}{\pi\left(\frac{d}{2}\right)^2} = \frac{40\cdot 10^{-3}\ m^3/s}{\pi\,(3{,}2\cdot 10^{-2}\ m)^2} = 12{,}4\ m/s$$

$$v_2 = \frac{Q_2}{A_2} = \frac{Q}{\pi\left(\frac{d_N}{2}\right)^2} = 56{,}6\ m/s$$

$$p_2 = p_1 + \frac{1}{2}\rho\left\{v_1^2 - v_2^2\right\} - \rho g h_2$$

$$= p_1 + \frac{8\rho Q^2}{\pi^2}\left[\frac{1}{d^4} - \frac{1}{d_N^4}\right] - \rho g h_2$$

$$= 1{,}62\cdot 10^6\ \frac{N}{m^2} + \frac{(1000\ \frac{kg}{m^3})\, 8\, (40\cdot 10^{-3}\ \frac{m^3}{s})^2}{\pi^2}\left\{\frac{1}{(3{,}2\cdot 10^{-2}\ m)^4} - \frac{1}{(1{,}5\cdot 10^{-2}\ m)^4}\right\}$$

$$- 1000\ \frac{kg}{m^3}\ 9{,}80\ m/s^2 \cdot 10\ m \simeq 0$$

(14)

Seigla og iðustreymi

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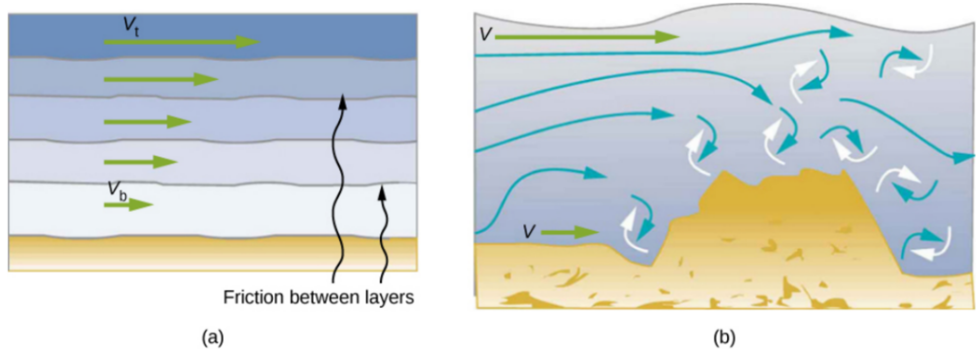
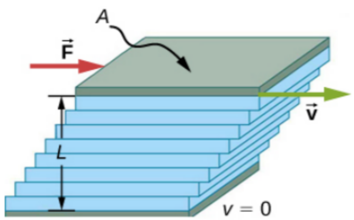


Figure 14.34 (a) Laminar flow occurs in layers without mixing. Notice that viscosity causes drag between layers as well as with the fixed surface. The speed near the bottom of the flow (v_b) is less than speed near the top (v_t) because in this case, the surface of the containing vessel is at the bottom. (b) An obstruction in the vessel causes turbulent flow. Turbulent flow mixes the fluid. There is more interaction, greater heating, and more resistance than in laminar flow.

(15)



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Seigja

$$F = \eta \frac{vA}{L}$$

$$\rightarrow \eta = \frac{FL}{vA}$$

$$[\eta] = \frac{M}{TL}$$

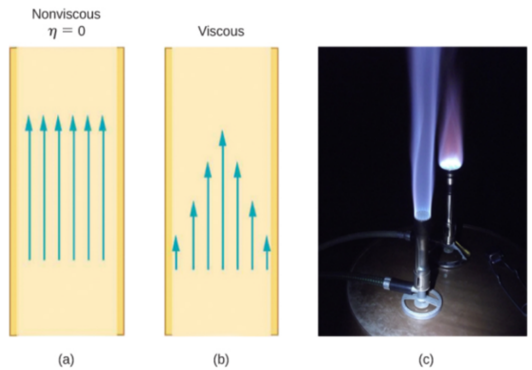
Eining Pa·s

Fluid	Temperature (°C)	Viscosity $\eta \times 10^3$
Air	0	0.0171
	20	0.0181
	40	0.0190
	100	0.0218
Ammonia	20	0.00974
Carbon dioxide	20	0.0147
Helium	20	0.0196
Hydrogen	0	0.0090
Mercury	20	0.0450
Oxygen	20	0.0203
Steam	100	0.0130
Liquid water	0	1.792
	20	1.002
	37	0.6947
	40	0.653
	100	0.282
Whole blood	20	3.015
	37	2.084
Blood plasma	20	1.810
	37	1.257
Ethyl alcohol	20	1.20
Methanol	20	0.584
Oil (heavy machine)	20	660
Oil (motor, SAE 10)	30	200
Oil (olive)	20	138
Glycerin	20	1500
Honey	20	2000–10000
Maple syrup	20	2000–3000
Milk	20	3.0
Oil (corn)	20	65

Table 14.4 Coefficients of Viscosity of Various Fluids

(16)

Lögmál Poiseuille



Lárétt flæði

$$\boxed{QR = P_2 - P_1}$$

R: viðnám við flæði

$$\boxed{R = \frac{8\eta l}{\pi r^4}}$$

lengd rörs: l

geisli rörs: r

$$\boxed{Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}}$$

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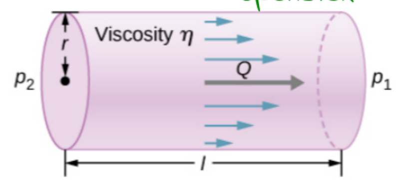

Reynoldstala fyrir flæði um rör

"Mæling" á iðustreymi, eða iðumyndun

$$\boxed{N_R = \frac{2\rho v r}{\eta}}$$

$$[N_R] = 1$$

víddarlaus fasti

Fyrir NR > 3000 er streymið orðið iðustreymi, fyrir 2000 < NR < 3000 er flæðið orðið óreiðukennt, ringlað streymi

1

Hitastig og varmi

Núllta lögmál varmafræðinnar

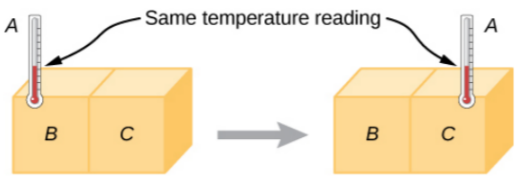

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Figure 1.2 If thermometer *A* is in thermal equilibrium with object *B*, and *B* is in thermal equilibrium with *C*, then *A* is in thermal equilibrium with *C*. Therefore, the reading on *A* stays the same when *A* is moved over to make contact with *C*.

Tveir hlutir í varmafræðilegu jafnvægi (jafn mikill varmi flýtur í hvora átt milli þeirra) eru með sama hitastig

Við munum síðar tengja hitastig við innri orku hluta og seinna sjá merkileg tengsl þess við óreiðu

2

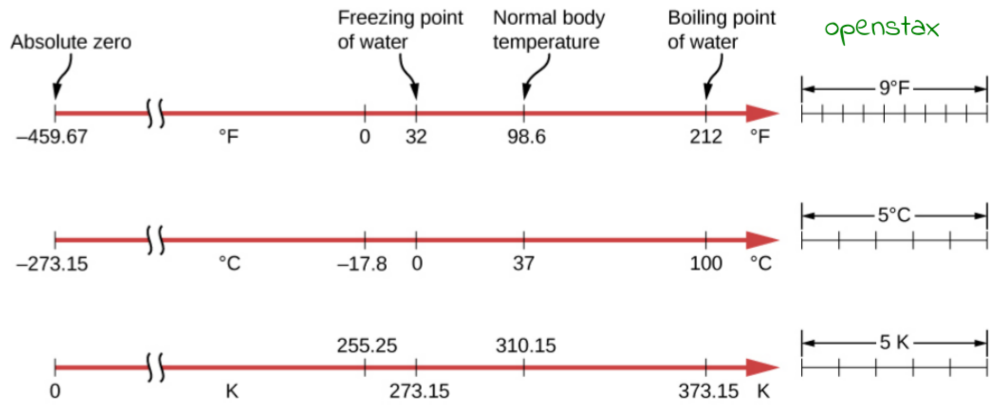

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Figure 1.4 Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales are shown. The relative sizes of the scales are also shown.

Munum sjá að Kelvin-kvarðinn fellur mjög vel að sígildri lýsingu á kjörgasi

3

Hitaþensla - thermal expansion

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Linear Thermal Expansion

According to experiments, the dependence of thermal expansion on temperature, substance, and original initial length is summarized in the equation

$$\frac{dL}{dT} = \alpha L \quad (1.1)$$

where $\frac{dL}{dT}$ is the instantaneous change in length per temperature, L is the length, and α is the **coefficient of linear expansion**, a material property that varies slightly with temperature. As α is nearly constant and also very small, for practical purposes, we use the linear approximation:

$$\Delta L = \alpha L \Delta T \quad (1.2)$$

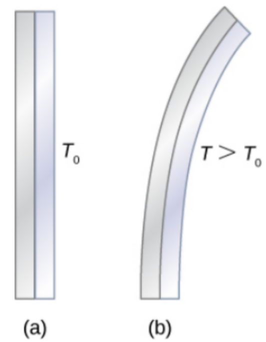
where ΔL is the change in length and ΔT is the change in temperature.

Línulega nálgunin getur góð fyrir smá bil í hitastigi, í raun getur α verið flókið fall af T....

4

Material	Coefficient of Linear Expansion α (1/°C)	Coefficient of Volume Expansion β (1/°C)
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	51×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or steel	12×10^{-6}	35×10^{-6}
Invar (nickel-iron alloy)	0.9×10^{-6}	2.7×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Silver	18×10^{-6}	54×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete, brick	$\sim 12 \times 10^{-6}$	$\sim 36 \times 10^{-6}$
Marble (average)	2.5×10^{-6}	7.5×10^{-6}
Liquids		
Ether		1650×10^{-6}
Ethyl alcohol		1100×10^{-6}
Gasoline		950×10^{-6}
Glycerin		500×10^{-6}
Mercury		180×10^{-6}
Water		210×10^{-6}

Material	Coefficient of Linear Expansion α (1/°C)	Coefficient of Volume Expansion β (1/°C)
Air and most other gases at atmospheric pressure		3400×10^{-6}



Mismunaþensla

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5

Thermal Expansion in Two Dimensions

For small temperature changes, the change in area ΔA is given by

$$\Delta A = 2\alpha A \Delta T \tag{1.3}$$

where ΔA is the change in area A, ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature. (The derivation of this equation is analogous to that of the more important equation for three dimensions, below.)

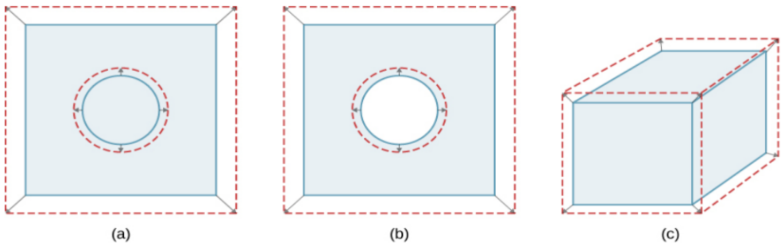

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Figure 1.7 In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place. (c) Volume also increases, because all three dimensions increase.

Thermal Expansion in Three Dimensions

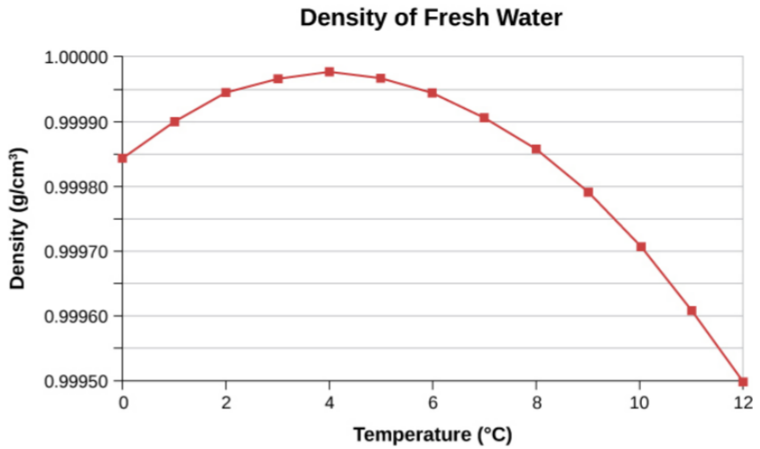
The relationship between volume and temperature $\frac{dV}{dT}$ is given by $\frac{dV}{dT} = \beta V$, where β is the **coefficient of volume expansion**. As you can show in Exercise 1.60, $\beta = 3\alpha$. This equation is usually written as

$$\Delta V = \beta V \Delta T. \tag{1.4}$$

Note that the values of β in Table 1.2 are equal to 3α except for rounding.

6

Dæmi um flóknari hegðun



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Skiptir miklu máli fyrir lífkerfi og önnur ....

7

Varmaflutningur, eðlisvarmi og varmamælingar

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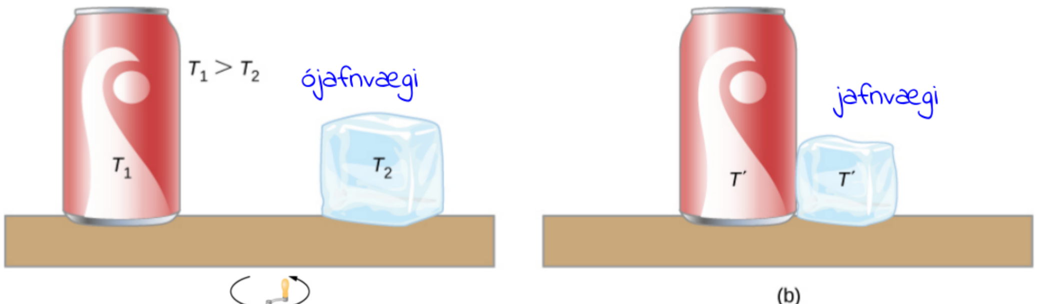
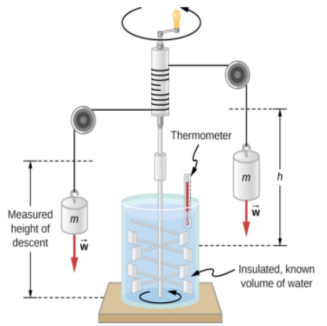


ójafnvægi

jafnvægi

Varmaflutningur --> breyting á innri orku



Jafngildi vélrænnar og varmaorku

$$\boxed{1{,}000 \text{ kcal} = 4186 \text{ J}}$$

Figure 1.10 Joule's experiment established the equivalence of heat and work. As the masses descended, they caused the paddles to do work, $W = mgh$, on the water. The result was a temperature increase, ΔT, measured by the thermometer. Joule found that ΔT was

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Varmarýmd - eðlisvarmi

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Heat Transfer and Temperature Change

A practical approximation for the relationship between heat transfer and temperature change is:

$$Q = mc\Delta T, \tag{1.5}$$

where Q is the symbol for heat transfer ("quantity of heat"), m is the mass of the substance, and ΔT is the change in temperature. The symbol c stands for the **specific heat** (also called "*specific heat capacity*") and depends on the material and phase. The specific heat is numerically equal to the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00 °C. The SI unit for specific heat is J/(kg × K) or J/(kg × °C). (Recall that the temperature change ΔT is the same in units of kelvin and degrees Celsius.)

Eðlisvarmarýmd $\boxed{c = \dfrac{C}{m}}$ þar sem C er varmarýmd hlutar með massa m

Í raun getur c verið flókið fall af hitastigi T: $c = c(T)$

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Substances	Specific Heat (c)	
Solids	J/kg · °C	kcal/kg · °C[2]
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, −50 °C to 0 °C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.40
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
Gases[3]		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)

Substances	Specific Heat (c)	
Steam (100 °C)	1520 (2020)	0.363 (0.482)

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$$c(T) = \frac{1}{m}\frac{dQ}{dT}$$

$$Q(T_2-T_1) = m\int_{T_1}^{T_2} c(T)\,dT$$

Því eru nákvæmar mælingar á c(T) mjög mikilvægar. Þær gefa upplýsingar um innri orku efna, fasabreytingar ....

10

Fasabreytingar

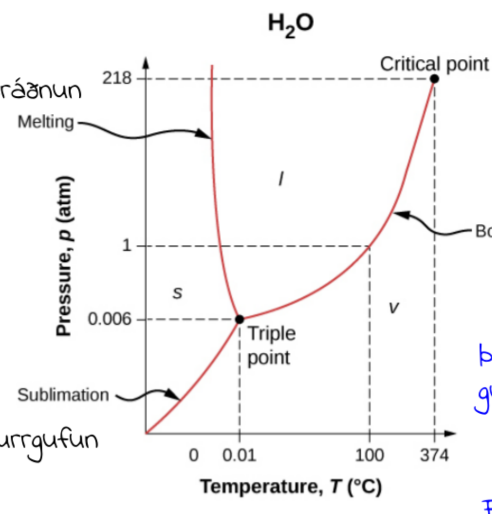
l: vökvi
s: fast efni
v: gufa - gas

Bráðnun

Markpunktur

Suða

Þurrgufun



þrípunktur, eini pT-punkturinn sem vökvi, gufa og ís geta verið í jafnvægi við

Fasaritið er mun flóknara þegar bætt er við öðrum kristallagerðum íss

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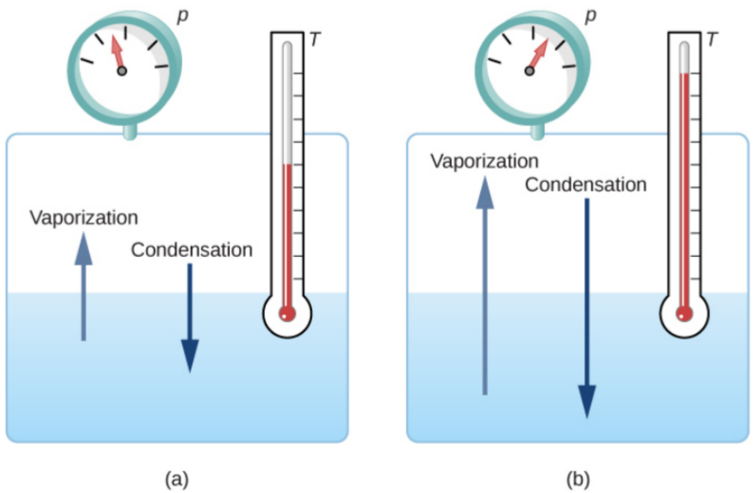

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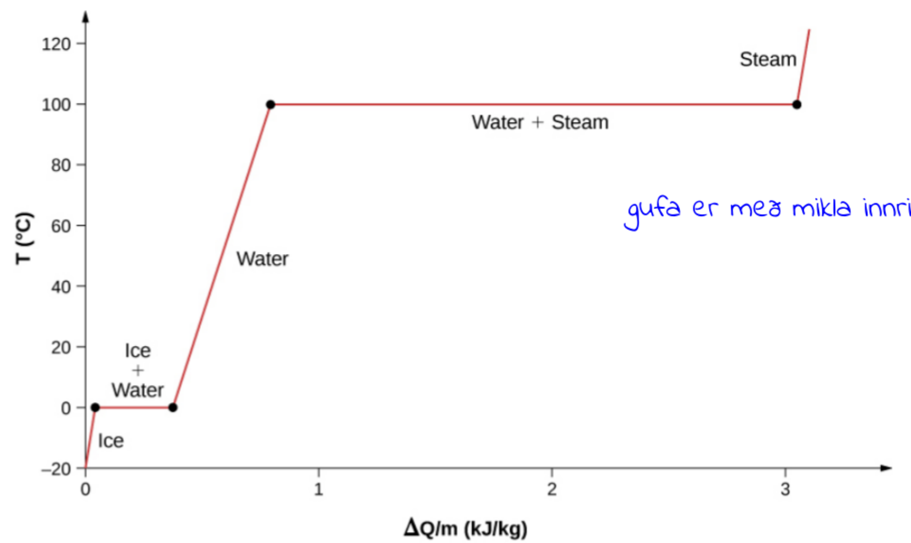
Lokað kerfi í jafnvægi. Við hærra hitastig er meira flæði sameinda úr og í fasana. Jafnvægi þýðir að flæðið verður að vera jafnt í báðar áttir

12

Fasabreytngar - hamskiptavarmi, Phase changes - latent heat



gufa er með mikla innri orku

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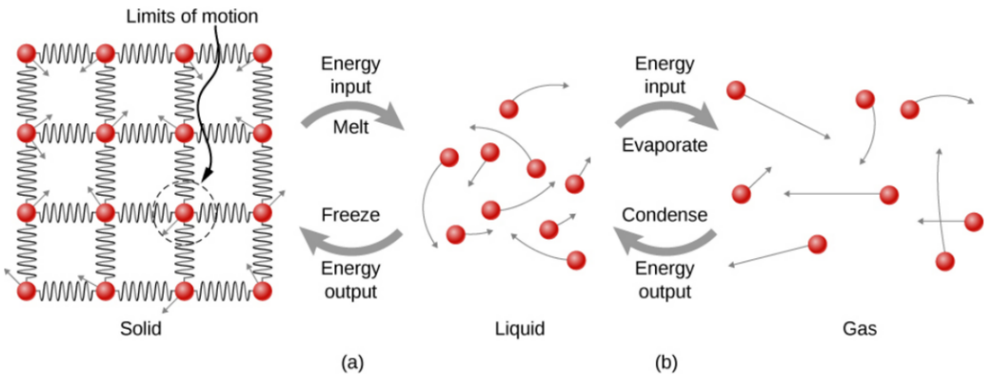

$$Q = mL_f \quad \text{bráðnun/frysting}$$

$$Q = mL_v \quad \text{suða/þétting}$$

(14)

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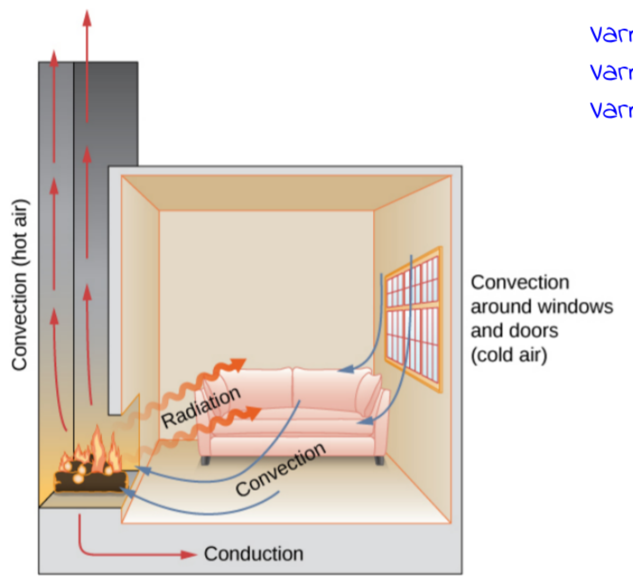
Substance	Melting Point (°C)	L_f kJ/kg	L_f kcal/kg	Boiling Point (°C)	L_v kJ/kg	L_v kcal/kg
Helium[2]	−272.2 (0.95 K)	5.23	1.25	−268.9 (4.2 K)	20.9	4.99
Hydrogen	−259.3 (13.9 K)	58.6	14.0	−252.9 (20.2 K)	452	108
Nitrogen	−210.0 (63.2 K)	25.5	6.09	−195.8 (77.4 K)	201	48.0
Oxygen	−218.8 (54.4 K)	13.8	3.30	−183.0 (90.2 K)	213	50.9
Ethanol	−114	104	24.9	78.3	854	204
Ammonia	−75	332	79.3	−33.4	1370	327
Mercury	−38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2256[3]	539[4]
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134

(15)

Varmaflutningur

varmaleiðni
varmaburður
varmageislun



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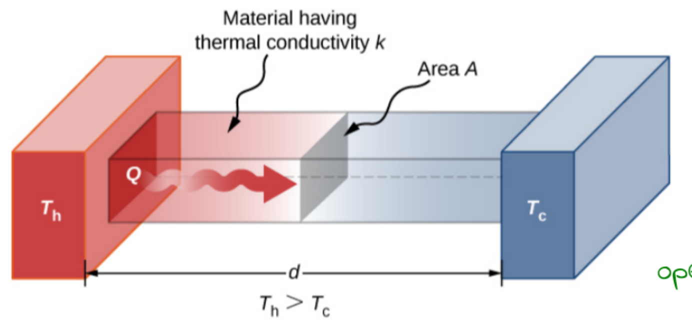
Varmaleiðni



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$$\boxed{P = \frac{dQ}{dt} = \frac{kA}{d}\left[T_h - T_c\right] \qquad P = -kA\frac{dT}{dx} \quad \text{almennar}}$$

Substance	Thermal Conductivity k (W/m · °C)
Diamond	2000
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14

Geislun

Svarthlutargeislun, e=1

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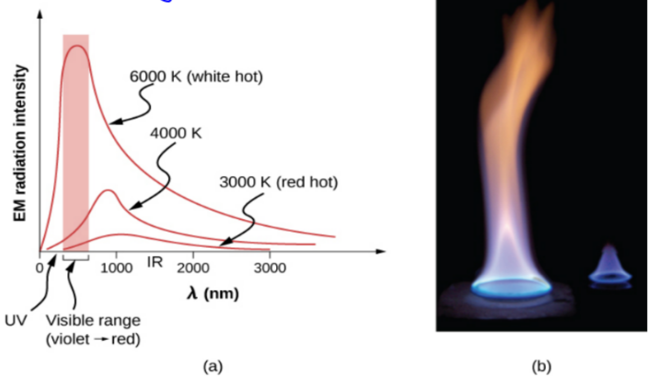
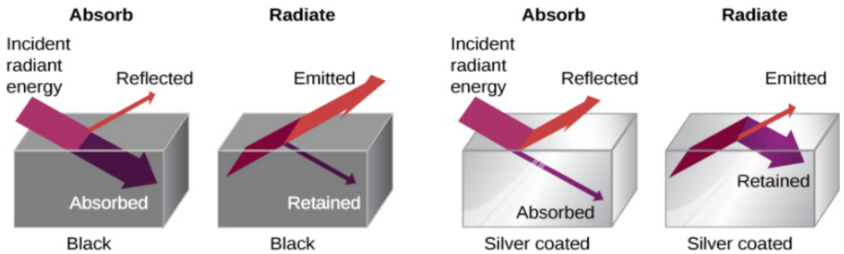
Ásamt varmarýmd
upphaf skammtafræði





Lögmál Stefans og Boltzmanns

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$$\boxed{P = \sigma A e T^4}$$

A = flötur hlutar
e = eðlisgeislun
$\sigma = 5.67\times10^{-8}$ J/s m^2 K^4

$$\boxed{P_{net} = \sigma e A\left[T_2^4 - T_1^4\right]}$$

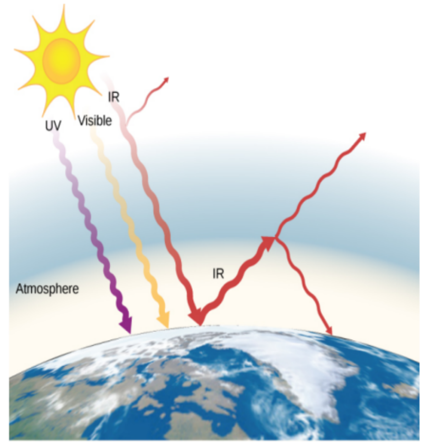

Figure 1.33 The greenhouse effect is the name given to the increase of Earth's temperature due to absorption of radiation in the atmosphere. The atmosphere is transparent to incoming visible radiation and most of the Sun's infrared. The Earth absorbs that energy and re-emits it. Since Earth's temperature is much lower than the Sun's, it re-emits the energy at much longer wavelengths, in the infrared. The atmosphere absorbs much of that infrared radiation and radiates about half of the energy back down, keeping Earth warmer than it would otherwise be. The amount of trapping depends on concentrations of trace gases such as carbon dioxide, and an increase in the concentration of these gases increases Earth's surface temperature.

1

Kvikfræði gass - kinetic theory of gases

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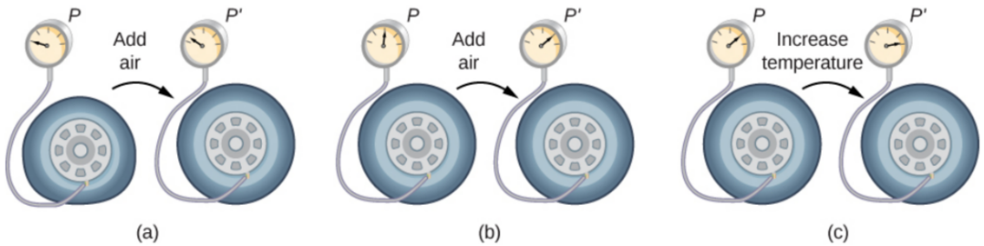

Figure 2.3 (a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion, and the pressure increases with more air. (c) Once the tire is inflated, its pressure increases with temperature.

Eitt mól $6{,}022 \times 10^{23}$ atóm eða sameindir --> fjöleindafræði - safneðlisfræði
Skoðum sígilda lýsingu

2

Rúmmál - þrýstingur, Robert Boyle

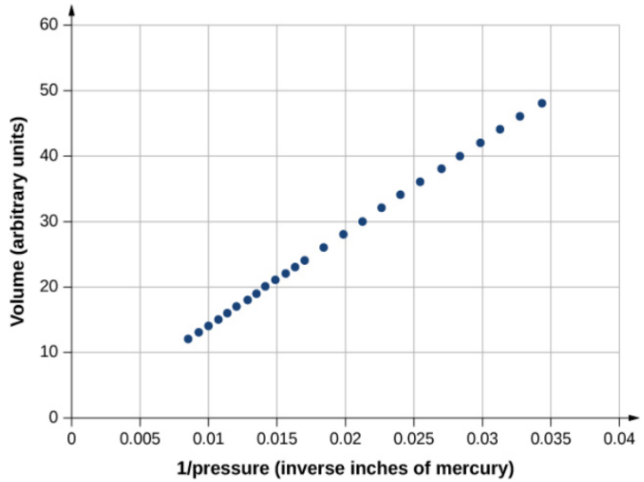

Figure 2.4 Robert Boyle and his assistant found that volume and pressure are inversely proportional. Here their data are plotted as V versus $1/p$; the linearity of the graph shows the inverse proportionality. The number shown as the volume is actually the height in inches of air in a cylindrical glass tube. The actual volume was that height multiplied by the cross-sectional area of the tube, which Boyle did not publish. The data are from Boyle's book *A Defence of the Doctrine Touching the Spring and Weight of the Air...*, p. 60.[1]

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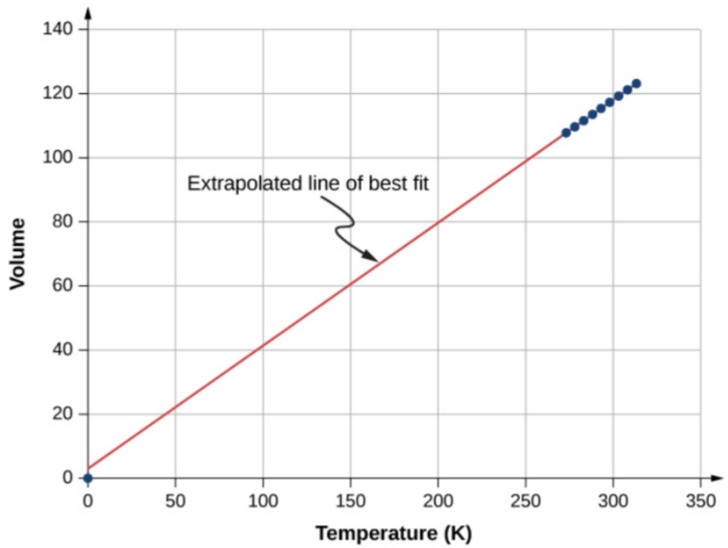
3



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Rúmmálið virðist hverfa við T = 0, (leiddi til upphafs Kelvin-kvarðans)

4

Kjörgas

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Ideal Gas Law

The ideal gas law states that

Ástandsjafna

$$\boxed{pV = Nk_BT,} \qquad (2.1)$$

where p is the absolute pressure of a gas, V is the volume it occupies, N is the number of molecules in the gas, and T is its absolute temperature.

The constant k_B is called the **Boltzmann constant** in honor of the Austrian physicist Ludwig Boltzmann (1844–1906) and has the value

$$k_B = 1.38 \times 10^{-23} \text{ J/K}.$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Upphafsmaður safneðlisfræði

5

Mól og tala Avogadrosar

$N_A = 6{,}02 \cdot 10^{23} \ ^1/_{\text{mol}}$

$N = N_A n$, n: fjöldi móla

$m_s = n M$

efnismassi

massi eins móls

$M = N_A m$, m: massi sameindar

$pV = N k_B T = \frac{N}{N_A}(N_A k_B) T$

Note that $n = N/N_A$ is the number of moles. We define the **universal gas constant** as $R = N_A k_B$, and obtain the ideal gas law in terms of moles.

Ideal Gas Law (in terms of moles)

In terms of number of moles n, the ideal gas law is written as

$$\boxed{pV = nRT.} \quad \text{Ástandsjafna} \qquad (2.3)$$

In SI units,

$$R = N_A k_B = \left(6.02 \times 10^{23}\ \text{mol}^{-1}\right)\left(1.38 \times 10^{-23}\ \frac{\text{J}}{\text{K}}\right) = 8.31\ \frac{\text{J}}{\text{mol} \cdot \text{K}}.$$

In other units,

$$R = 1.99 \frac{\text{cal}}{\text{mol} \cdot \text{K}} = 0.0821 \frac{\text{L} \cdot \text{atm}}{\text{mol} \cdot \text{K}}.$$

You can use whichever value of R is most convenient for a particular problem.

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6

Ástandsjafna Johannes van der Waals

Leiðrétting vegna veiks aðdráttarkrafts milli sameinda og endanlegs rúmmáls þeirra

$$\left[p + a\left(\frac{n}{V}\right)^2\right](V - nb) = nRT$$

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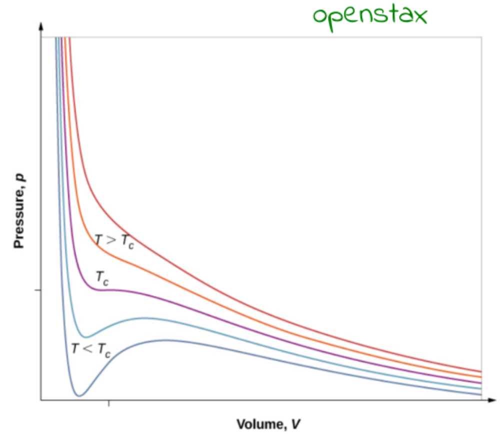

Figure 2.7 *pV* diagram for a Van der Waals gas at various temperatures. The red curves are calculated at temperatures above the critical temperature and the blue curves at temperatures below it. The blue curves have an oscillation in which volume (*V*) increases with increasing pressure (*P*), an impossible situation, so they must be corrected as in Figure 2.8. (credit: "Eman"/Wikimedia Commons)

Greinilegir eiginleikar gass, sem ekki er kjörgas

(ekki punkt eindir - sameindirnar víxlverkast)

Virial expansion - eflisliðun

7

Jafnhitaferlar

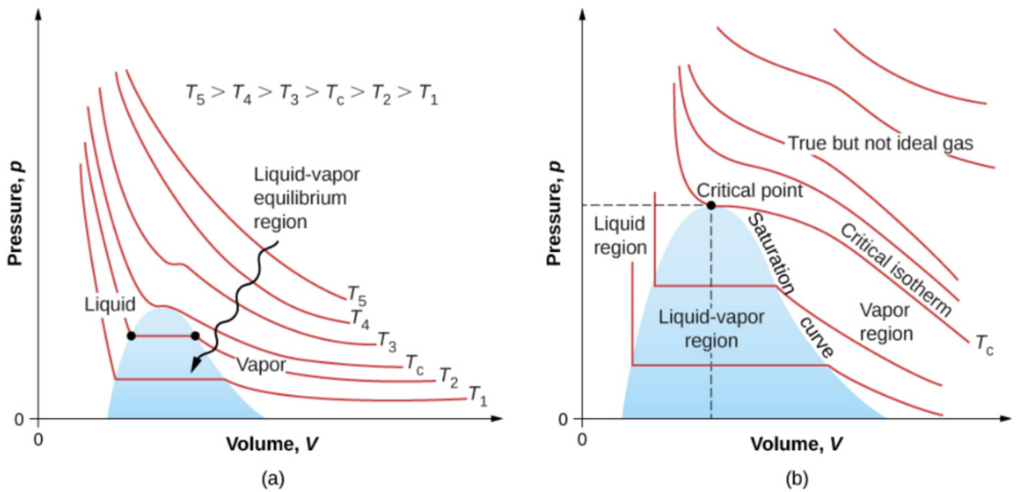

Figure 2.8 *pV* diagrams. (a) Each curve (isotherm) represents the relationship between *p* and *V* at a fixed temperature; the upper curves are at higher temperatures. The lower curves are not hyperbolas because the gas is no longer an ideal gas. (b) An expanded portion of the *pV* diagram for low temperatures, where the phase can change from a gas to a liquid. The term "vapor" refers to the gas phase when it exists at a temperature below the boiling temperature.

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8

Kvikfræði gass

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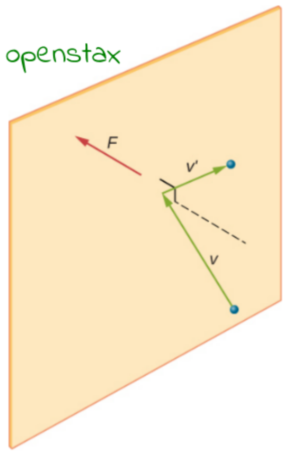

1. Mikill fjöldi sameinda, N_A
2. Lögmál Newtons
3. Mjög smáar sameindir
4. Fjaðrandi árekstrar
5. Markgildissetning tölfræðinnar

Þrýstingur á veggi vegna fjaðrandi árekstra

$\Delta m v = +m v_x - (-m v_x) = 2 m v_x$

$F_i = \frac{\Delta p_i}{\Delta t} = \frac{2 m v_{ix}}{\Delta t}$ meðaltími milli árekstra

$= \frac{2 m v_{ix}}{2l / v_{ix}} = \frac{m v_i^2}{l}$

$F = \sum_{i=1}^{N} F_i = \frac{m}{l} \sum_i^N v_{ix}^2 = N \frac{m}{l} \left[\frac{1}{N} \sum_{i=1}^{N} v_{ix}^2\right] = N \frac{m}{l} \langle v_x^2 \rangle$

l: hliðlengd kassa

(9)

$$\langle v^2\rangle = \langle v_x^2\rangle + \langle v_y^2\rangle + \langle v_z^2\rangle = 3\langle v_x^2\rangle$$

einsátta

→ $F = \dfrac{Nm\langle v^2\rangle}{3l}$, $p = \dfrac{F}{A} = N\dfrac{m\langle v^2\rangle}{3Al} = \dfrac{Nm\langle v^2\rangle}{3V}$

→ $pV = \frac{1}{3}Nm\langle v^2\rangle$, en líka $pV = Nk_BT$

ástandsjafnan

Average Kinetic Energy per Molecule

The average kinetic energy of a molecule is directly proportional to its absolute temperature:

$$\boxed{\bar{K} = \frac{1}{2}m\bar{v^2} = \frac{3}{2}k_BT.} \tag{2.6}$$

→ innri orka kjörgass: $\boxed{E_{int}(T) = N\langle K\rangle = \frac{3}{2}nRT}$

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(10)

RMS Speed of a Molecule

The **root-mean-square (rms) speed** of a molecule, or the square root of the average of the square of the speed $\bar{v^2}$, is

$$\boxed{v_{\text{rms}} = \sqrt{\bar{v^2}} = \sqrt{\frac{3k_BT}{m}}.} \tag{2.8}$$

T.d. N_2 við 20°C → $v_{rms} \approx 511$ m/s

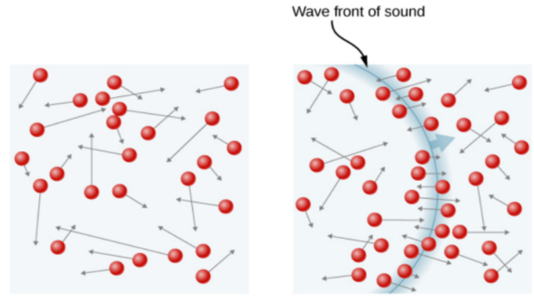

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Figure 2.11 (a) In an ordinary gas, so many molecules move so fast that they collide billions of times every second. (b) Individual molecules do not move very far in a small amount of time, but disturbances like sound waves are transmitted at speeds related to the molecular speeds.

(11)

Hlutþrýstingur

- Partial pressure is the pressure a gas would create if it existed alone
- Dalton's law states that the total pressure is the sum of the partial pressures of all of the gases present
- For any two gases (labeled 1 and 2) in equilibrium in a container

$$\frac{p_1}{n_1} = \frac{p_2}{n_2}$$

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- vapor pressure is the partial pressure of a vapor at which it is in equilibrium with the liquid (or solid, in the case of sublimation) phase of the same substance

Hlutþrýstingur vatns í lofti er alltaf lægri en gufuþrýstingur þess

(12)

T (°C)	Vapor Pressure (Pa)
0	610.5
3	757.9
5	872.3
8	1073
10	1228
13	1497
15	1705
18	2063
20	2338
23	2809
25	3167
30	4243
35	5623
40	7376

Table 2.2 Vapor Pressure of Water at Various Temperatures

Rakastig

$$\text{R.H.} = \frac{\text{Partial pressure of water vapor at } T}{\text{Vapor pressure of water at } T} \times 100\%.$$

Meðalspölur - mean free path

Meðalspölur er meðal vegalengd milli árekstra sameinda

$$\boxed{\lambda = \frac{V}{4\sqrt{2}\pi r^2 N} = \frac{k_BT}{4\sqrt{2}\pi r^2 p}}$$

Meðaltími (meðalævi)

$$\boxed{\tau = \frac{k_BT}{4\sqrt{2}\pi r^2 v_{rms}}}$$

r: árekstraþversnið

13

Varmarýmd og jafnskipting orku

Varmarýmd einsatóma kjörgass á mól við fast rúmmál

$$C_V = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)_V \quad \rightarrow \quad C_V = \frac{3}{2}R$$

$$\Delta Q = \Delta E_{int} = \frac{3}{2} n R \Delta T$$

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Equipartition Theorem

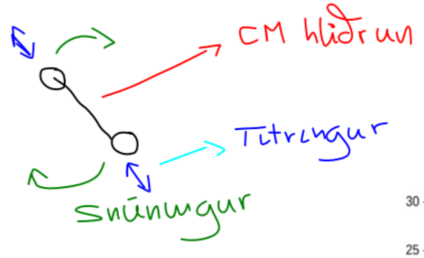
The energy of a thermodynamic system in equilibrium is partitioned equally among its degrees of freedom. Accordingly, the molar heat capacity of an ideal gas is proportional to its number of degrees of freedom, d:

$$C_V = \frac{d}{2}R. \qquad \textbf{2.14}$$

Einatóma gas: $d = 3$

14

Frelsisgráður H_2



Það þarf vissa innri orku til þess að mismunandi frelsisgráður örvist eða vakni (skammtafræði)

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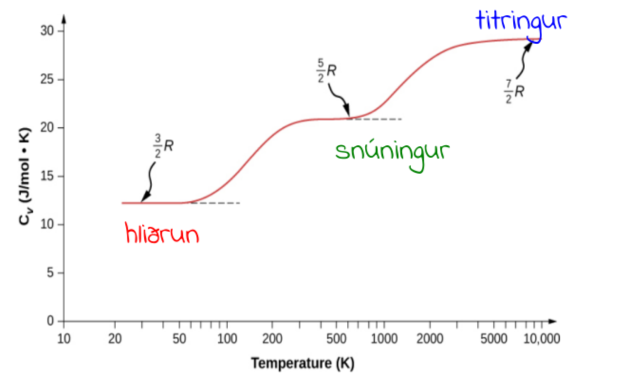


Figure 2.13 The molar heat capacity of hydrogen as a function of temperature (on a logarithmic scale). The three "steps" or "plateaus" show different numbers of degrees of freedom that the typical energies of molecules must achieve to activate. Translational kinetic energy corresponds to three degrees of freedom, rotational to another two, and vibrational to yet another two.

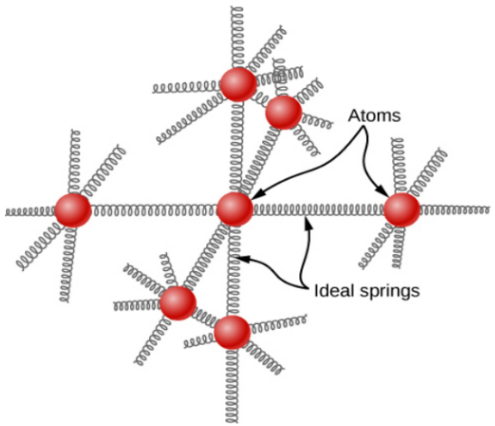
15

Fast efni

Í einföldu kristölluðu föstu efni þegar allir hljóðeindahættir eru virkjaðir við nógu hátt T fæst

$$d = 6$$

$$\rightarrow \quad C = 3R$$



Hraðadreifing Maxwells og Boltzmanns fyrir kjörgas

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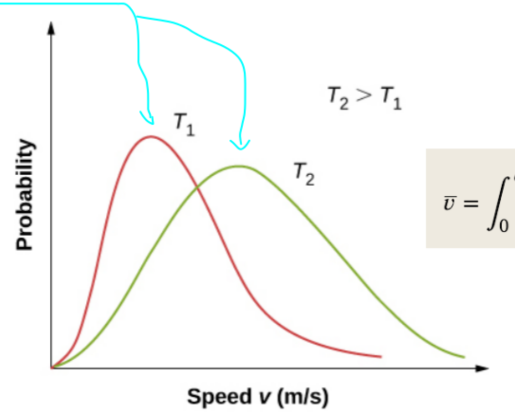
Maxwell-Boltzmann Distribution of Speeds

The distribution function for speeds of particles in an ideal gas at temperature T is

$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_B T}\right)^{3/2} v^2 e^{\left(-mv^2/\left(2k_B T\right)\right)}. \qquad \textbf{2.15}$$

16



The Maxwell-Boltzmann distribution is shifted to higher speeds and broadened at higher temperatures.

$$\bar{v} = \int_0^\infty v f(v) dv = \sqrt{\frac{8}{\pi}\frac{k_B T}{m}} = \sqrt{\frac{8}{\pi}\frac{RT}{M}}.$$

meðalhraðinn

$$v_p = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2RT}{M}},$$

líklegasti hraðinn

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1

Varmafræði

Sígild stórsæ kerfi - classical macroscopic systems

Kerfi - jaðar - umhverfi <---> opin eða lokuð kerfi
system - boundary - environment <---> open or closed systems
Jafnvægi - nærjafnvægi - ójafnvægi
(Stórsæ- smásæ kerfi)
(Tengsl við safneðlisfræði)

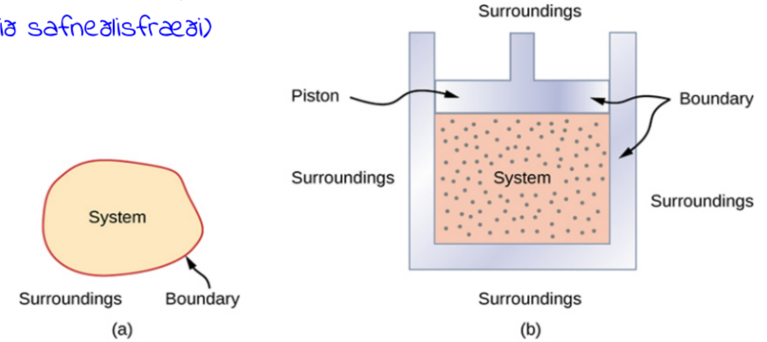

Figure 3.2 (a) A system, which can include any relevant process or value, is self-contained in an area. The surroundings may also have relevant information; however, the surroundings are important to study only if the situation is an open system. (b) The burning gasoline in the cylinder of a car engine is an example of a thermodynamic system.

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2

Ástands jafna - breytur

$$f(p,V,T) = 0$$

til dæmis fyrir kjörgas

$$F(p,V,T) = pV - nRT = 0$$

Magnbundnar breytur - extensive v.
Eðlisbundar breytur - intensive vari.

V, n

p, T

Vinna - varmi - innriorka

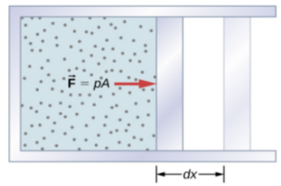

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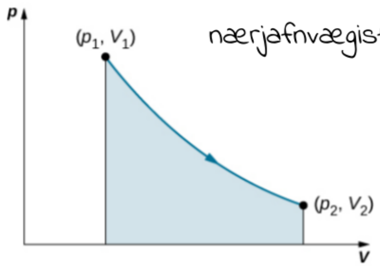
Figure 3.4 The work done by a confined gas in moving a piston a distance dx is given by $dW = F dx = p dV$.

$$dW = F\,dx = pA\,dx$$
$$= p\,dV$$
$$\rightarrow W = \int_{V_1}^{V_2} p\,dV$$

3

Kjörgas



When a gas expands slowly from V_1 to V_2, the work done by the system is represented by the shaded area under the pV curve.

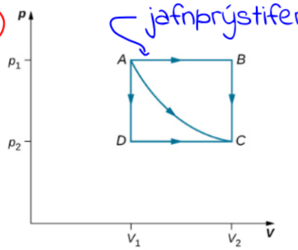
nærjafnvægisferli - quasi-static process

$$W_{AC} = \int_{V_1}^{V_2} p\,dV = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

Fast T - jafnhitaferli - isothermal process

$$W_{AC} = nRT \ln\left[\frac{V_2}{V_1}\right]$$

jafnþrýstiferli - isobaric p.



The paths *ABC*, *AC*, and *ADC* represent three different quasi-static transitions between the equilibrium states *A* and *C*.

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$$W_{AB} = p\int_{V_1}^{V_2} dV = p(V_2 - V_1)$$

$$W_{BC} = 0 \leftarrow \Delta V = 0$$

$$\rightarrow W_{ABC} \neq W_{AC}$$

Vinnan er háð ferli í ástands rúminu

4

Innriorka

$$E_{int} = \left\langle \sum_i (K_i + U_i) \right\rangle$$

Kjörgas

$$E_{int} = \left[\frac{3}{2} k_B T\right] n N_A = \frac{3}{2} nRT$$

einatóma

Fyrsta lögmál varmafræðinnar

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First Law of Thermodynamics

Associated with every equilibrium state of a system is its internal energy E_{int}. The change in E_{int} for any transition between two equilibrium states is

$$\Delta E_{int} = Q - W \qquad 3.7$$

where Q and W represent, respectively, the heat exchanged by the system and the work done by or on the system.

$$dE_{int} = dQ - dW$$

5

Thermodynamic Sign Conventions for Heat and Work

Process	Convention
Heat added to system	$Q > 0$
Heat removed from system	$Q < 0$
Work done by system	$W > 0$
Work done on system	$W < 0$

Table 3.1

$$\Delta E_{int} = Q - W$$

fyrir ferlið frá B --> C var $W = 0$, en ekkert var sagt um Q

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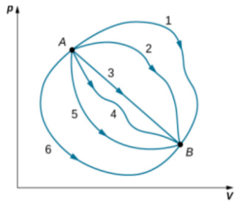


Figure 3.7 Different thermodynamic paths taken by a system in going from state *A* to state *B*. For all transitions, the change in the internal energy of the system $\Delta E_{int} = Q - W$ is the same.

Often the first law is used in its differential form, which is

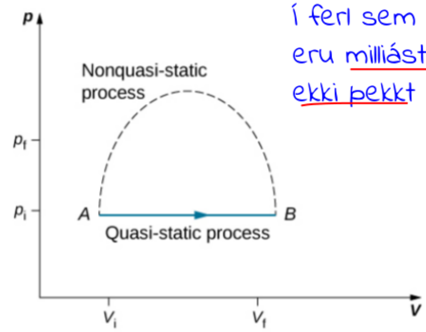
$$dE_{int} = dQ - dW. \quad 3.8$$

Here dE_{int} is an infinitesimal change in internal energy when an infinitesimal amount of heat dQ is exchanged with the system and an infinitesimal amount of work dW is done by (positive in sign) or on (negative in sign) the system.

Fyrir allar leiðir frá A til B er breytingin í innri orkunni sú sama, en dW og dQ eru breytilegar stærðir fyrir þær

6

Varmafræðileg ferli



Í ferli sem er ekki nærjafnvægisferli eru milliástöndin í ástandarúminu ekki þekkt

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Figure 3.8 Quasi-static and non-quasi-static processes between states *A* and *B* of a gas. In a quasi-static process, the path of the process between *A* and *B* can be drawn in a state diagram since all the states that the system goes through are known. In a non-quasi-static process, the states between *A* and *B* are not known, and hence no path can be drawn. It may follow the dashed line as shown in the figure or take a very different path.

Jafnhitaferli, $\Delta T = 0$

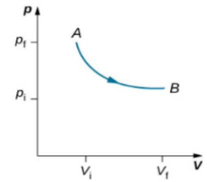


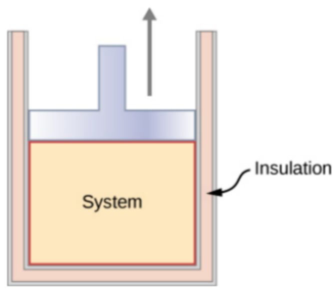
Figure 3.10 An isothermal expansion from a state labeled *A* to another state labeled *B* on a *pV* diagram. The curve represents the relation between pressure and volume in an ideal gas at constant temperature.

7

Óvermin ferli adiabatic process

$\Delta Q = 0$

Enginn varmi flæðir í eða úr kerfinu



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Hringferli - lotuferli cyclic process

$\Delta E_{int} = 0, \quad \Delta W = \Delta Q$ fyrir hverja lotu

Jafnþrýstiferli isobaric process

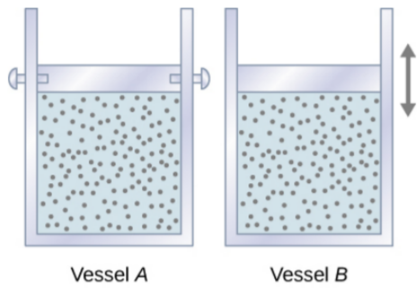
$\Delta p = 0$

jafnrúmmálsferli isochoric process

$\Delta V = 0$

8

Tvenns konar varmarýmd kjörgass



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A:
$$dE_{int} = dQ - dW = dQ$$
$$dQ = C_v n dT$$
$$\rightarrow dE_{int} = C_v n dT$$

C_v: mólar varmarýmd við fast rúmmál

B:
$$dQ = C_p n dT$$

C_p: mólar varmarýmd við fastan þrýsting

$$dW = p dV$$
$$d(pV) = d(RnT) = nRdT$$

$E_{int} = E_{int}(T)$

$$\rightarrow dE_{int} = dQ - pdV = (nC_p - nR)dT$$

$$\boxed{C_p = C_v + R}$$

9

Fyrir kjörgas

$$C_V = \frac{d}{2} R$$

frelsisgráður d

Molar Heat Capacities of Dilute Ideal Gases at Room Temperature

Type of Molecule	Gas	C_p (J/mol K)	C_V (J/mol K)	$C_p - C_V$ (J/mol K)
Monatomic	Ideal	$\frac{5}{2}R = 20.79$	$\frac{3}{2}R = 12.47$	$R = 8.31$
Diatomic	Ideal	$\frac{7}{2}R = 29.10$	$\frac{5}{2}R = 20.79$	$R = 8.31$
Polyatomic	Ideal	$4R = 33.26$	$3R = 24.94$	$R = 8.31$

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Óvermnir ferlar fyrir kjörgas

$\Delta Q = 0$

ekki nærjafnvægisferli

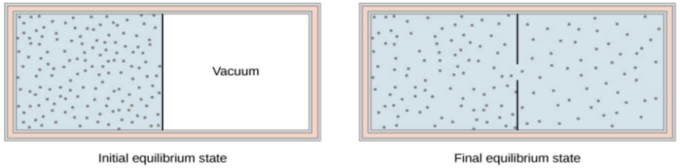

Figure 3.13 The gas in the left chamber expands freely into the right chamber when the membrane is punctured.

If the gas is ideal, the internal energy depends only on the temperature. Therefore, when an ideal gas expands freely, its temperature does not change.

10

Nærjafnvægis óvermið ferli

kjörgas

$$dQ = 0$$

$$dW = pdV$$

$$dE_{int} = C_V n dT$$

$$dE_{int} = dQ - pdV$$
$$= -pdV$$

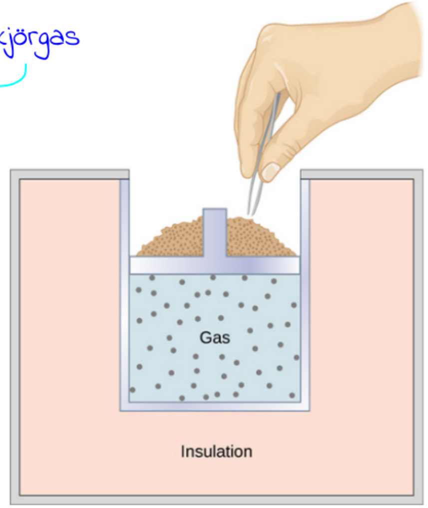

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Figure 3.14 When sand is removed from the piston one grain at a time, the gas expands adiabatically and quasi-statically in the insulated vessel.

$$C_V n dT = -pdV \to dT = -\frac{pdV}{nC_V}$$

11

$$dT = -\frac{pdV}{nC_V}, \quad pV = nRT \to d(pV) = d(nRT)$$

$$\to pdV + Vdp = nRdT$$

$$\to dT = \frac{pdV + Vdp}{nR}$$

$$\to -\frac{pdV}{nC_V} = \frac{pdV + Vdp}{nR}$$

$$\to -nRpdV = nC_V\left[pdV + Vdp\right]$$

12

$$\to nC_V Vdp + n\left[C_V + R\right]pdV = 0$$

$C_V + R = C_p$ og deilum með npV

$$\to C_V \frac{dp}{p} + C_p \frac{dV}{V} = 0$$

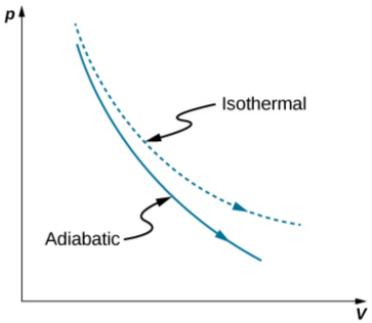
$$\to \frac{dp}{p} + \gamma \frac{dV}{V} = 0 \quad \text{með } \gamma = \frac{C_p}{C_V}$$

Heildum (óákveðið)

$$\int \frac{dp}{p} + \gamma \int \frac{dV}{V} = 0 \to \boxed{pV^{\gamma} = \text{fasti}}$$

Eins má leiða út

$$\boxed{\begin{aligned} p^{1-\gamma} T^{\gamma} &= \text{fasti} \\ T V^{\gamma-1} &= \text{fasti} \end{aligned}}$$



Quasi-static adiabatic and isothermal expansions of an ideal gas.

óvermið, hallatala: $\frac{dp}{dV} = -\gamma \frac{p}{V}$

Jafnhita, hallatala: $\frac{dp}{dV} = -\frac{p}{V}$

γ í næsta kafla

Eingengin ferli - irreversible processes, jafngengin ferli - reversible processes

1

$\Delta W = 0$
$\Delta Q = 0$

Kjörgas, $V \to 2V$, $pV = nRT \to p = p_0/2$, $\Delta E_{int} = 0$,

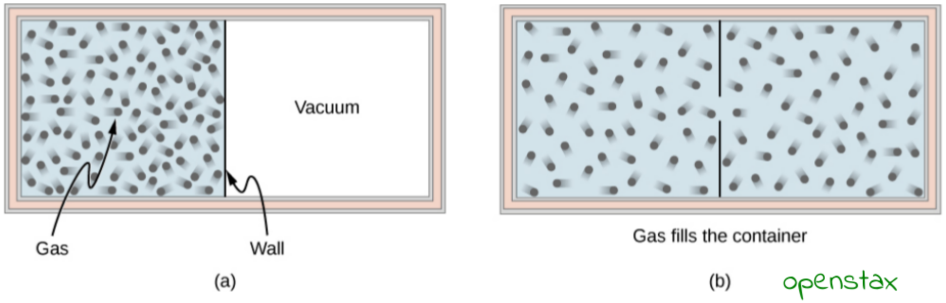


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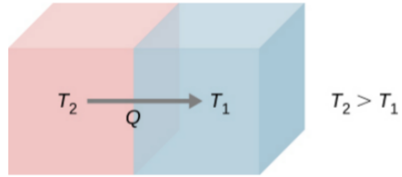
Figure 4.2 A gas expanding from half of a container to the entire container (a) before and (b) after the wall in the middle is removed.

Eingengt ferli, "ólíklegt" er að kerfið komist sjálfkrafa í upphafsástandið (vissulega er hægt að koma því með vinnu í upphafsástandið)

2



Spontaneous heat flow from an object at higher temperature T_2 to another at lower temperature T_1

Second Law of Thermodynamics (Clausius statement)

Heat never flows spontaneously from a colder object to a hotter object.

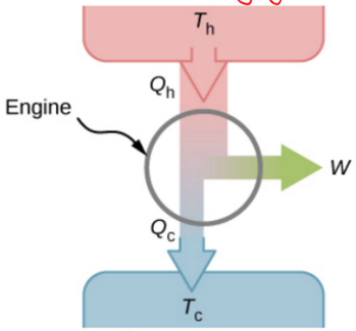
Reynslulögmál - tilraunaniðurstaða

Safneðlisfræði: ákaflega ákaflega ákaflega ólíklegt miðað við aldur alheimsins

3

Varmavélar

heitur varmageymir



kaldur varmageymir

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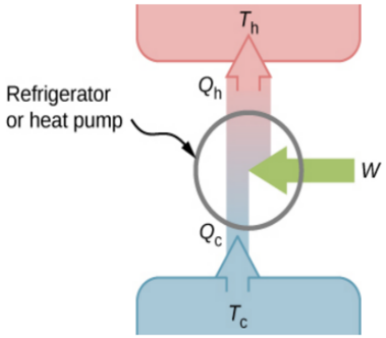
Einn hringur: $\Delta E_{int} = 0$

$$\boxed{W = Q - \Delta E_{int}} = (Q_h - Q_c) - 0$$
$$= \boxed{Q_h - Q_c}$$

Nýtni - efficiency: $\boxed{e = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h}}$

4

Kælivélar



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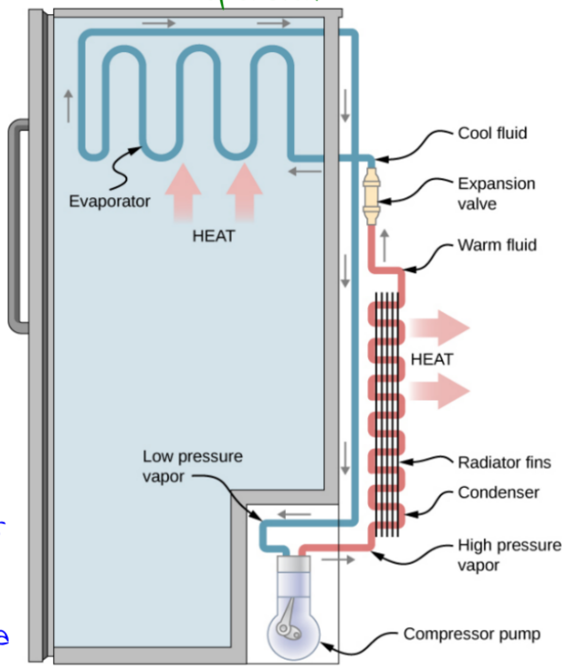

Afkastageta

$$K_R = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} \quad \text{kælir}$$

$$K_P = \frac{Q_h}{W} = \frac{Q_h}{Q_h - Q_c} \quad \text{dæla}$$

2. lögmálið á öðrum ham

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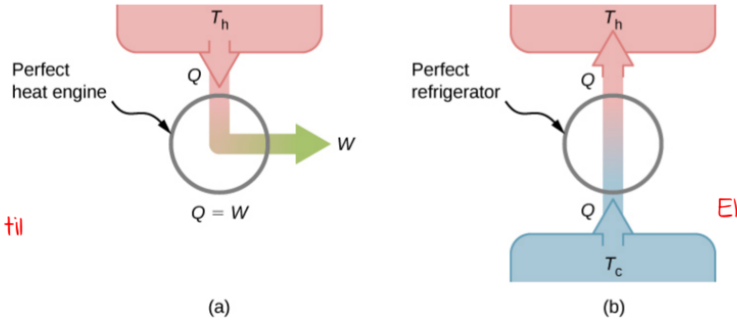
Second Law of Thermodynamics (Kelvin statement)

It is impossible to convert the heat from a single source into work without any other effect.



Ekki til

Ekki til

Figure 4.8 (a) A "perfect heat engine" converts all input heat into work. (b) A "perfect refrigerator" transports heat from a cold reservoir to a hot reservoir without work input. Neither of these devices is achievable in reality.

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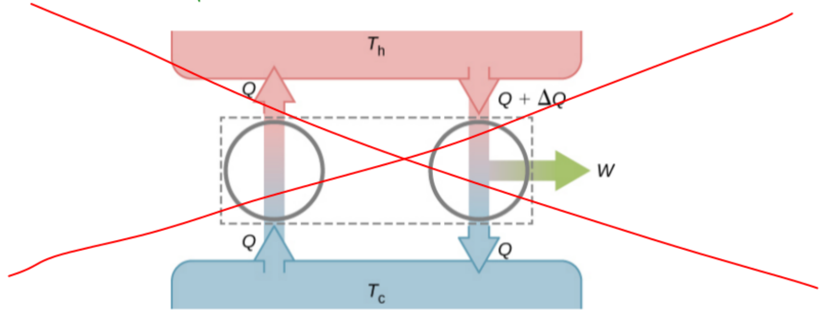

Combining a perfect refrigerator and a real heat engine yields a perfect heat engine because $W = \Delta Q$.

Skoðum

any reversible engine operating between two reservoirs has a greater efficiency than any irreversible engine operating between the same two reservoirs

all reversible engines operating between the same two reservoirs have the same efficiency

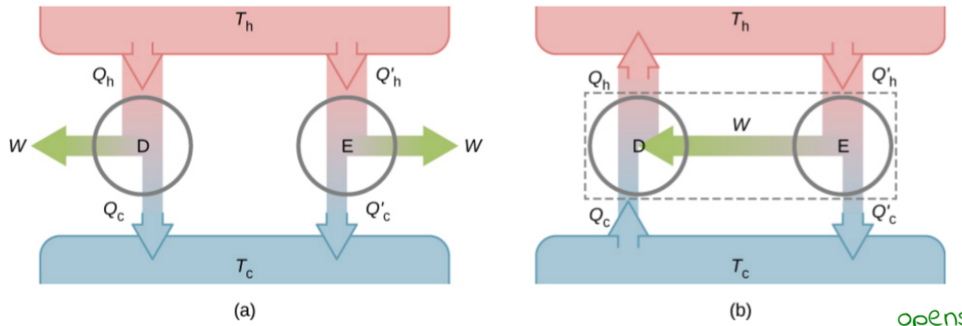

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Figure 4.10 (a) Two uncoupled engines D and E working between the same reservoirs. (b) The coupled engines, with D working in reverse.

* Ef D er jafngeng, E er eingeng með $e_E > e_D$, og $W_D = W_E = W$ -> $Q_h > Q'_h$, 1. Lögmál -> $Q_c > Q'_c$.

$e = 1 - \frac{Q_c}{Q_h}$ Snúum við D og tengjum -> (b), en

$Q_h > Q'_h$ og $Q_c > Q'_c$ -> varmi flutur úr c -> h,

ekki mögulegt -> $e_{irr} > e_{rev}$ ekki hægt

* Ef báðar jafngengar fæst á sama hátt að $e_D = e_E$

Hringur Carnots

Jafngengt ferli, hæsta möguleg nýtni

① Kjörgas, jafnhita

-> $\Delta E_{int} = 0$

$Q_h = W_1 = nRT_h \ln\left(\frac{V_N}{V_M}\right)$

② Óvermið

$T_h V_N^{\gamma-1} = T_c V_O^{\gamma-1}$

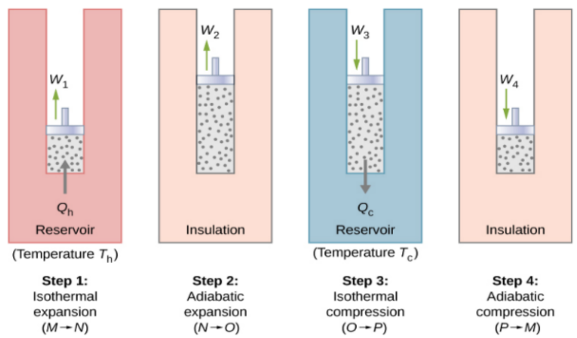
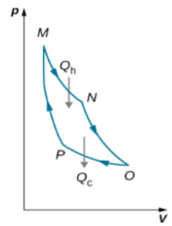


Figure 4.11 The four processes of the Carnot cycle. The working substance is assumed to be an ideal gas whose thermodynamic path *MNOPM* is represented in Figure 4.12.



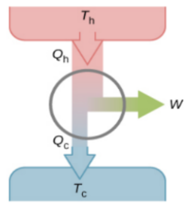


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Figure 4.12 The total work done by the gas in the Carnot cycle is shown and given by the area enclosed by the loop *MNOPM*.

9

③ Jafnhita

$$Q_c = nRT_c \ln\left(\frac{V_o}{V_P}\right)$$

④ Óvermið

$$T_c V_P^{\gamma-1} = T_h V_M^{\gamma-1}$$

$$\frac{Q_c}{Q_h} = \frac{T_c \ln\left(\frac{V_o}{V_P}\right)}{T_h \ln\left(\frac{V_N}{V_M}\right)}$$

og ② og ④ $\rightarrow$ $\frac{V_o}{V_P} = \frac{V_N}{V_M}$

$$\rightarrow \frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

Heildarvinna

$$W = W_1 + W_2 - W_3 - W_4$$

er flöturinn í pV-ritinu, hringur $\rightarrow \Delta E_{int} = 0$

$$\boxed{W} = Q - \Delta E_{int} = \{Q_h - Q_c\} - \Delta E_{int}$$
$$= Q_h - Q_c$$

$$\rightarrow \boxed{e = 1 - \frac{T_c}{T_h}}$$

10

Fyrir Carnot kælivél og varmadælu fæst á sama hátt

$$\boxed{K_R = \frac{T_c}{T_h - T_c}, \quad K_P = \frac{T_h}{T_h - T_c}}$$

Carnot's Principle

No engine working between two reservoirs at constant temperatures can have a greater efficiency than a reversible engine.

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óreiða - entropy

Byrjum með jafngengt ferli við fast hitastig og skilgreinum

$$\boxed{\Delta S = \frac{Q}{T}}$$

11

Ef ferlið er ekki við fast T

$$\Delta S = S_B - S_A = \int_A^B \frac{dQ}{T}$$

Fyrir hring Carnots fæst

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 = \frac{Q_h}{T_h} - \frac{Q_c}{T_c}$$

en fyrir hring Carnots gildir líka

$$\frac{Q_h}{T_h} = \frac{Q_c}{T_c} \rightarrow \Delta S = 0$$

Almennt fyrir jafngengt hringferli gildir

$$\boxed{\oint dS = \oint \frac{dQ}{T} = 0}$$

12

Second Law of Thermodynamics (Entropy statement)

The entropy of a closed system and the entire universe never decreases. $\Delta S \geq 0$

3. lögmál varmafræðinnar (þarf skammtafræði)

$$\boxed{\lim_{T \to 0} (\Delta S)_T = 0}$$

Safneðlisfræði setur varmafræðinni grunn

$$\boxed{S = k_B \ln \Omega}$$

Ex. 4.8

Óvermin frjáls pensla

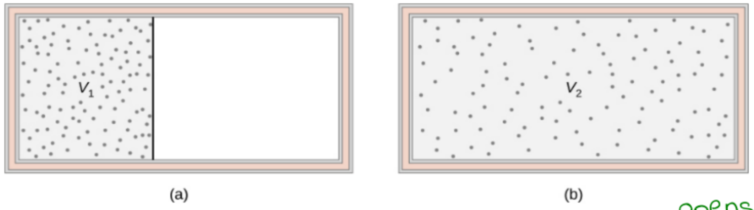

Figure 4.18 The adiabatic free expansion of an ideal gas from volume V_1 to volume V_2.

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kjörgas

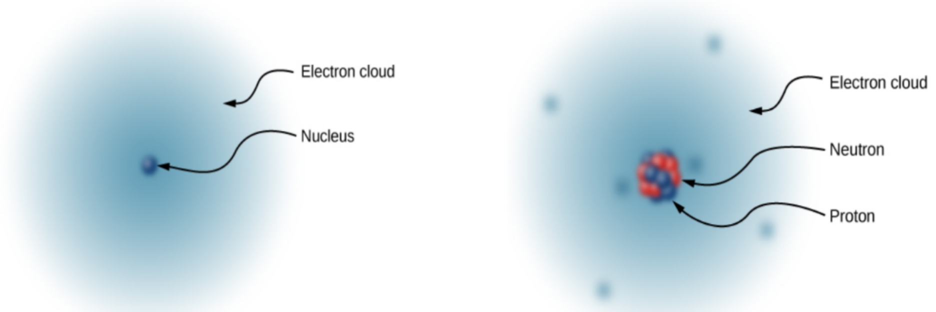
$$\Delta T = 0 \rightarrow \Delta S = \frac{\Delta Q}{T}, \quad \Delta E_{int} = 0$$

$$Q = W = \int_{V_1}^{V_2} p\,dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$\rightarrow \Delta S = \frac{\Delta Q}{T} = nR \ln\left(\frac{V_2}{V_1}\right) \geq 0$$

eingengt ferli

Rafhleðslur og kraftar



H - atóm

C - atóm

Rafeindir með einingarhleðslu $-e$ og róteindir með einingarhleðslu $+e$
Hleðsla varðveitist staðbundið og víðvært (um þær gildir samfelldnijafna)
Einingarhleðslur - lokuð eða opin kerfi - skömmtun hleðslu

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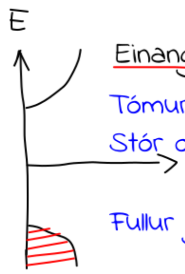
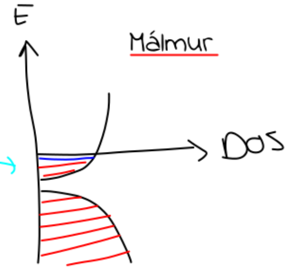
Orkustig - borðar í föstu efni, ástandaþéttleiki (DOS)



Einangrari

Tómur leiðniborði, engin leiðni
Stór orkugeil

Fullur gildisborði



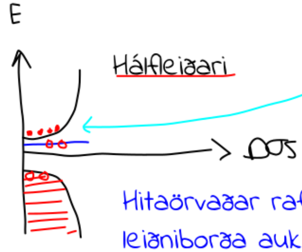
Málmur

Rafeindir í leiðniborða auk tómra ástanda
--> mikil leiðni

Aðeins ein rafeind getur setið í hverju ástandi.
Til að hreyfa sig þurfa þær nálæg tóm ástönd

Í jafnvægi eru engir straumar

Kulgas
hlýgas



Hálfleiðari

Hitaörvaðar rafeindir upp í leiðniborða auk íbótar rafeinda
Stýranleg leiðni

Lögmál Coulombs

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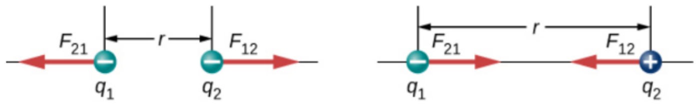
Coulomb's Law

The magnitude of the electric force (or **Coulomb force**) between two electrically charged particles is equal to

$$|\mathbf{F}_{12}| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r_{12}^2} \qquad 5.1$$

The unit vector $\boldsymbol{r}$ has a magnitude of 1 and points along the axis as the charges. If the charges have the same sign, the force is in the same direction as $\boldsymbol{r}$ showing a repelling force. If the charges have different signs, the force is in the opposite direction of $\boldsymbol{r}$ showing an attracting force. (Figure 5.14).



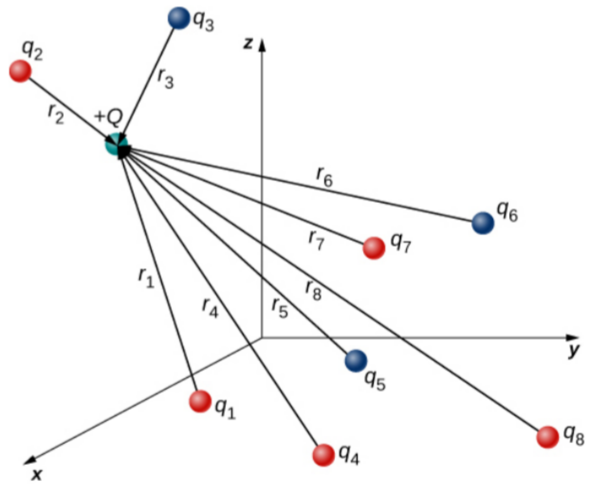
Samskonar lögmál gildir um aðdráttarkraft tveggja massa, en hleðslur geta haft sitt hvort formerkið, ekki massar

Í sígildu tómarúmi er rafsegulfræðin línuleg

$$\vec{\mathbf{F}}(r) = \frac{1}{4\pi\varepsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$$

Principle of superposition

$\bar{F}(r)$ er krafturinn á hleðslu Q í punktinum $\bar{r}$, en $\bar{r}_i$ er vigur frá hleðslu q_i að Q



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Rafsvið - electrical field

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \cdots + \vec{\mathbf{F}}_N$$
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Qq_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{Qq_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{Qq_3}{r_3^2}\hat{\mathbf{r}}_3 + \cdots + \frac{Qq_N}{r_1^2}\hat{\mathbf{r}}_N\right)$$
$$= Q\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{q_3}{r_3^2}\hat{\mathbf{r}}_3 + \cdots + \frac{q_N}{r_1^2}\hat{\mathbf{r}}_N\right)\right].$$

Kraftar N hleðslna á hleðslu Q

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$$\boxed{\vec{\mathbf{F}} = Q\vec{\mathbf{E}}}$$

Skilgreinum rafsvið

$$\boxed{\vec{\mathbf{E}} \equiv \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{q_3}{r_3^2}\hat{\mathbf{r}}_3 + \cdots + \frac{q_N}{r_1^2}\hat{\mathbf{r}}_N\right)}$$

Vigursvið í öllum punktum rúmsins

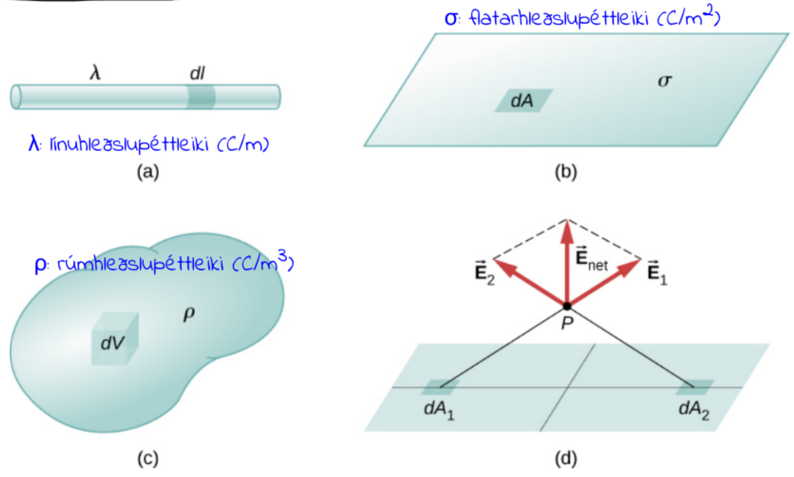
$$\vec{\mathbf{E}}(P) \equiv \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i.$$

Línuleg samantekt eins og fyrir kraftsvið

Direction of the Electric Field

By convention, all electric fields $\vec{\mathbf{E}}$ point away from positive source charges and point toward negative source charges.

Samfelld hleðsla



(a) (b) (c) (d)

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Point charges: $$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\left(\frac{q_i}{r^2}\right)\hat{\mathbf{r}}$$

Line charge: $$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{line}}\left(\frac{\lambda dl}{r^2}\right)\hat{\mathbf{r}}$$

Surface charge: $$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{surface}}\left(\frac{\sigma dA}{r^2}\right)\hat{\mathbf{r}}$$

Volume charge: $$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{\text{volume}}\left(\frac{\rho dV}{r^2}\right)\hat{\mathbf{r}}$$

Einingarvigur frá hleðslufrymi til athuganda

P: Staðsetning athuganda
r: Fjarlægð hleðslufrymis frá athuganda

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Ex. 5.8

Rafsvið beint ofan jafnhlaðinnar skífu

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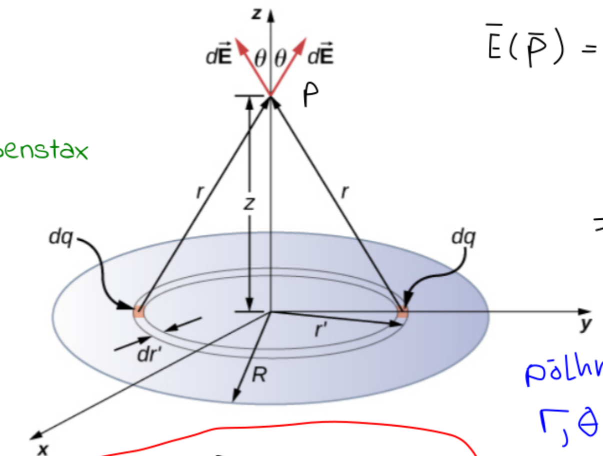

$$\bar{E}(\bar{P}) = \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma dA}{r^2}\hat{r}$$

$$= \frac{1}{4\pi\epsilon_0}\int_S \frac{\sigma dA}{r^2}\cos\theta \cdot \hat{k}$$

pólhnit → sívalningshnit

r, θ, z

$dA = 2\pi r' dr'$

$r^2 = r'^2 + z^2$

$\cos\theta = \frac{z}{\sqrt{r'^2 + z^2}}$

$$\boxed{\bar{E}(P) = \bar{E}(z) = \frac{1}{4\pi\epsilon_0}\int_0^R \frac{\sigma(2\pi r' dr')\, z}{(r'^2 + z^2)^{3/2}}\hat{k}}$$

(9)

$$= \frac{1}{4\pi\epsilon_0}(2\pi\sigma z)\left\{\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right\}\hat{k}$$

$$= \frac{1}{4\pi\epsilon_0}\left\{2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2+z^2}}\right\}\hat{k}$$

Nákvæm lausn

Aðfellulausn fyrir $z \gg R$

$$\xrightarrow[z \gg R]{} \approx \frac{1}{4\pi\epsilon_0}\frac{\sigma\pi R^2}{z^2}\hat{k} = \frac{1}{4\pi\epsilon_0}\frac{Q_T}{z^2}\hat{k}$$

Úr mikilli hæð "lítur" diskurinn út eins og punkthleðsla $Q_T = \sigma\pi R^2 = \sigma A$

(10)

Rafsvið - sviðslínur

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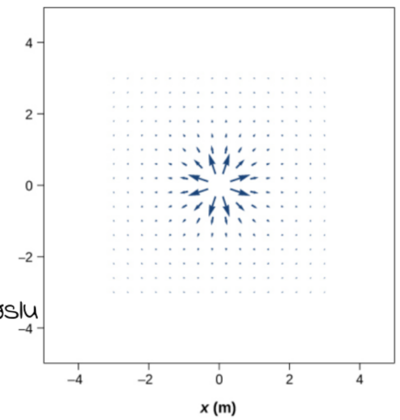
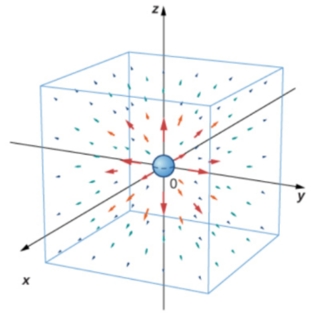
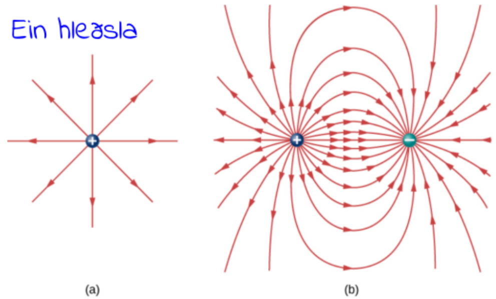




Ein hleðsla

Línufjöldi í réttu hlutfalli við hleðslu
Hefjast alltaf í +hleðslu og enda alltaf í -hleðslu

Ein hleðsla



Tvær jafnstórar hleðslur
-- tvískaut

Sviðslínur skerast aldrei
(þá væri sviðið ekki einkvæmt)
Flæði ...

(11)

Tvískaut eins jafnstórar hleðslur misstórar hleðslur

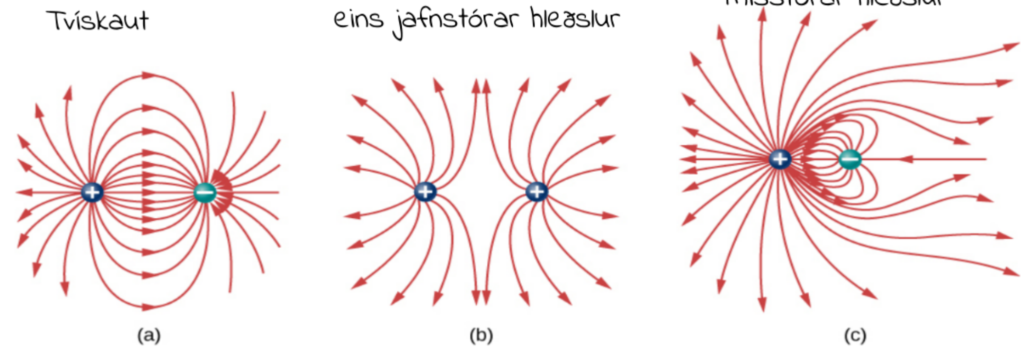


Figure 5.31 Three typical electric field diagrams. (a) A dipole. (b) Two identical charges. (c) Two charges with opposite signs and different magnitudes. Can you tell from the diagram which charge has the larger magnitude?

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(12)

Vægi á tvískaut

$$\begin{aligned}\vec{\tau} &= \left(\frac{\vec{\mathbf{d}}}{2} \times \vec{\mathbf{F}}_+\right) + \left(-\frac{\vec{\mathbf{d}}}{2} \times \vec{\mathbf{F}}_-\right) \\ &= \left[\left(\frac{\vec{\mathbf{d}}}{2}\right) \times \left(+q\vec{\mathbf{E}}\right) + \left(-\frac{\vec{\mathbf{d}}}{2}\right) \times \left(-q\vec{\mathbf{E}}\right)\right] \\ &= q\vec{\mathbf{d}} \times \vec{\mathbf{E}}.\end{aligned}$$

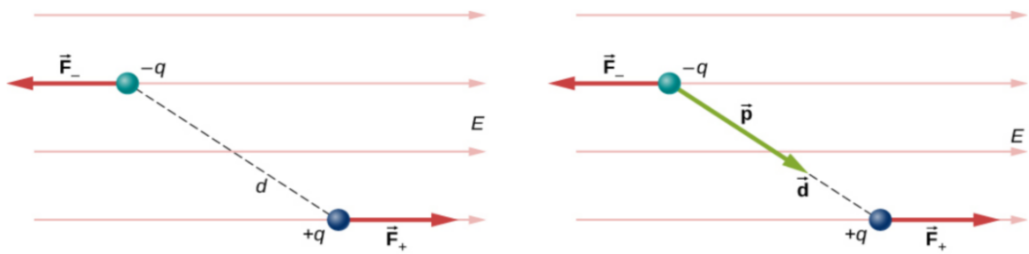

Figure 5.32 A dipole in an external electric field. (a) The net force on the dipole is zero, but the net torque is not. As a result, the dipole rotates, becoming aligned with the external field. (b) The dipole moment is a convenient way to characterize this effect. The $\vec{\mathbf{d}}$ points in the same direction as $\vec{\mathbf{p}}$.

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Tvískaut, stefna og vægi

$$\vec{\mathbf{p}} \equiv q\vec{\mathbf{d}}.$$

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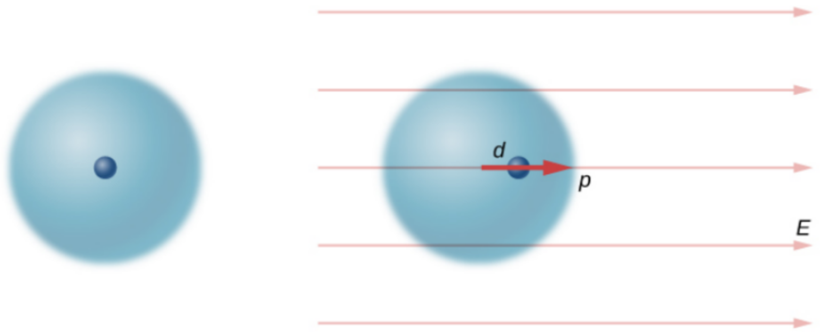
Skautað tvískaut

$$\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}.$$



(a) Neutral atom (b) Induced dipole

A dipole is induced in a neutral atom by an external electric field. The induced dipole moment is aligned with the external

14

Skautun milli atóma vegna flökkts --> veikir aðdráttarkraftar,
víxlverkun van der Waals

$$V(r) \sim \frac{1}{r^6}$$

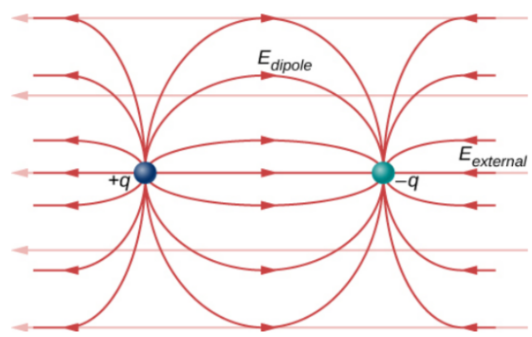


Figure 5.34 The net electric field is the vector sum of the field of the dipole plus the external field.

Recall that we found the electric field of a dipole in Equation 5.7. If we rewrite it in terms of the dipole moment we get:

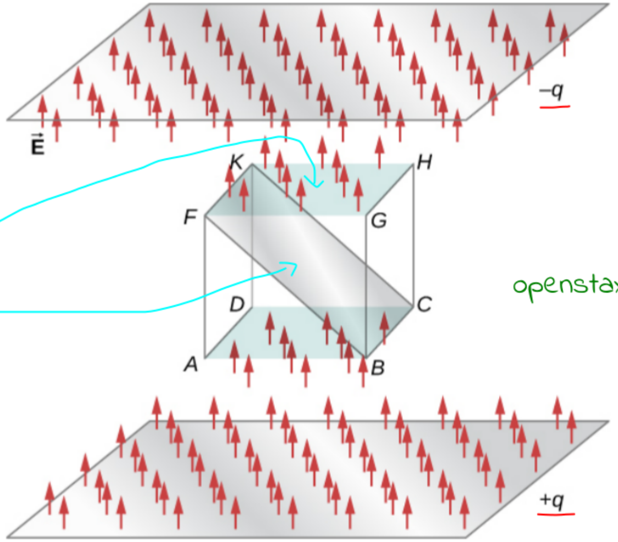
Svið tvískauts

$$\vec{\mathbf{E}}(z) = \frac{-1}{4\pi\varepsilon_0}\frac{\vec{\mathbf{p}}}{z^3}.$$

skammseilið svið

Lögmál Gauß fyrir rafflæði

1



Sama flæði um KHGF og KCBF

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$$\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} \text{ (uniform } \vec{\mathbf{E}}, \text{ flat surface)}$$

2

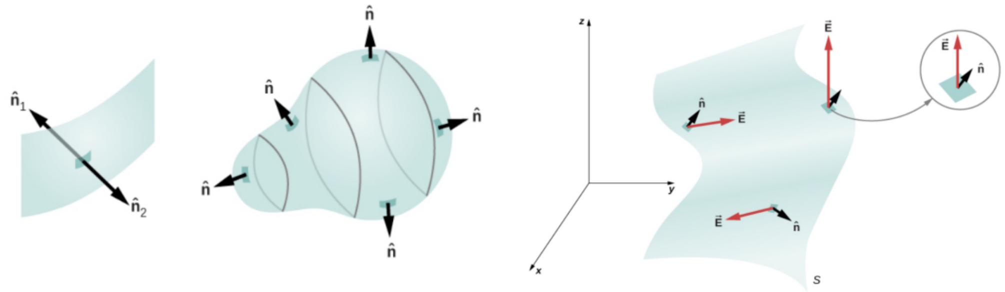
Figure 6.8 A surface is divided into patches to find the flux.

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \underbrace{\hat{\mathbf{n}}\, dA} = \oint_S \vec{\mathbf{E}} \cdot \underbrace{d\vec{\mathbf{A}}} \text{ (closed surface)}$$

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3

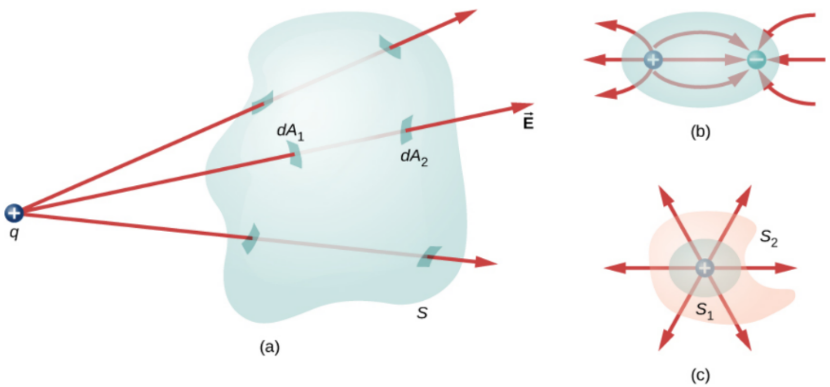

Figure 6.15 Understanding the flux in terms of field lines. (a) The electric flux through a closed surface due to a charge outside that surface is zero. (b) Charges are enclosed, but because the net charge included is zero, the net flux through the closed surface is also zero. (c) The shape and size of the surfaces that enclose a charge does not matter because all surfaces enclosing the same charge have the same flux.

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Gauss's Law

The flux Φ of the electric field $\vec{\mathbf{E}}$ through any closed surface S (a Gaussian surface) is equal to the net charge enclosed (q_{enc}) divided by the permittivity of free space (ε_0):

$$\Phi = \oint_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}}\, dA = \frac{q_{enc}}{\varepsilon_0}. \qquad 6.5$$

4

Kúlusamhverfar hleðslur

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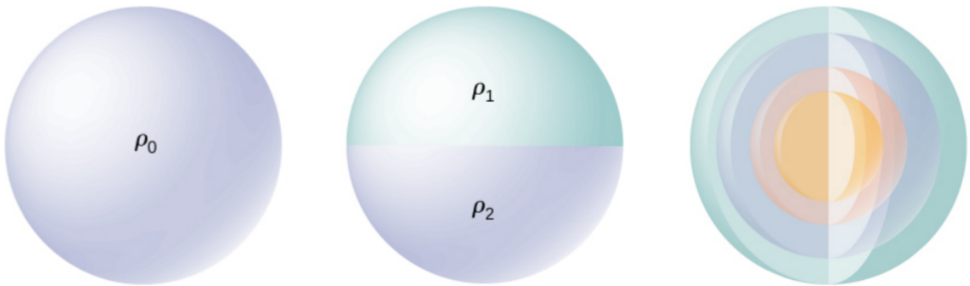


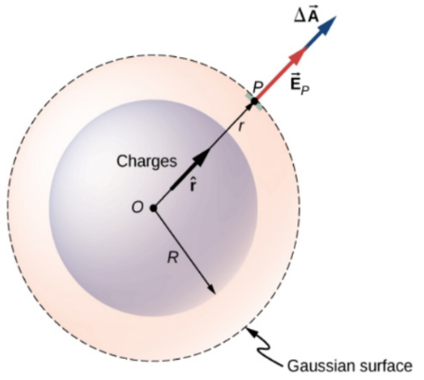
(a) Spherically symmetric (b) Not spherically symmetric (c) Spherically symmetric

Fyrir kúlusamhverfa hleðsludreifingu er hægt að hugsa kúluyfirborð með sömu miðju og hleðsludreifingin. Á hugsaða Gauß-yfirborðinu er rafsviðið jafn sterkt alls staðar og samsíða eða andsamsíða stefnu útþáttar (radial)

--> Getum reiknað E. Lögmál Gauß gildir alltaf, en við þurfum heppilega samhverfu til að nota það til að reikna E

5

Rafsvið innan og utan jafnhlaðinnar kúlu



Byrjum utan kúlu, hleðsludreifing innan hennar (fyrir r > R)

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4\pi R^3}{3}} = \frac{3Q}{4\pi R^3}$$

Q er heildarhleðsla kúlunnar

$$\oint \bar{E} \cdot d\bar{A} = \frac{Q}{\epsilon_0}$$

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E er fasti á Gauß-yfirborðinu

$$4\pi r^2 E = \frac{Q}{\epsilon_0} \rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

6

því fæst fyrir r > R

$$\boxed{\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}} = \frac{\frac{4\pi R^3 \rho}{3}\hat{r}}{4\pi\epsilon_0 r^2} = \frac{R^3\rho}{3\epsilon_0 r^2}\hat{r}$$

sem er sams konar og fyrir punkthleðslu Q í r = 0

Fyrir innan kúlu (þetta er ekki málmkúla, heldur einangrari með jafna hleðslu) r < R. Þá þurfum við að finna hleðsluna innan Gauß-yfirborðsins Q_{enc}

$$Q_{enc} = \frac{4\pi r^3}{3}\rho = V\rho = \frac{4\pi r^3}{3}\frac{Q\,3}{4\pi R^3} = Q\left(\frac{r}{R}\right)^3$$

Höfum

$$\oint \bar{E} \cdot d\bar{A} = \frac{Q_{enc}}{\epsilon_0}$$

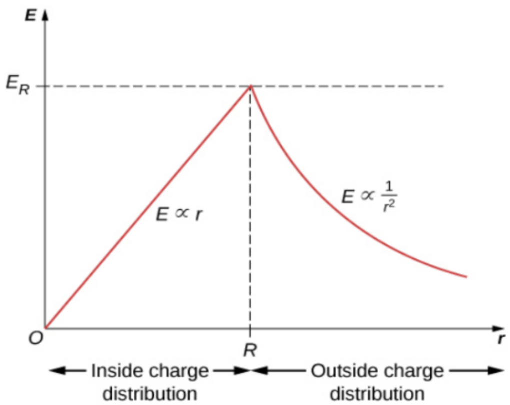
$$\rightarrow 4\pi r^2 E = \frac{Q}{\epsilon_0}\left(\frac{r}{R}\right)^3 \rightarrow E = \frac{Q r}{4\pi\epsilon_0 R^3}$$

7

og því, innan kúlu, fyrir r < R fæst

$$\boxed{\bar{E} = \frac{Q r}{4\pi\epsilon_0 R^3}\hat{r} = \frac{\rho r}{3\epsilon_0}\hat{r}}$$

Innan kúlu vex rafsviðið línulega með r að yfirborðinu og það er samfellt í yfirborðinu (r = R).



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8

Jafnhlaðinn sívalningur

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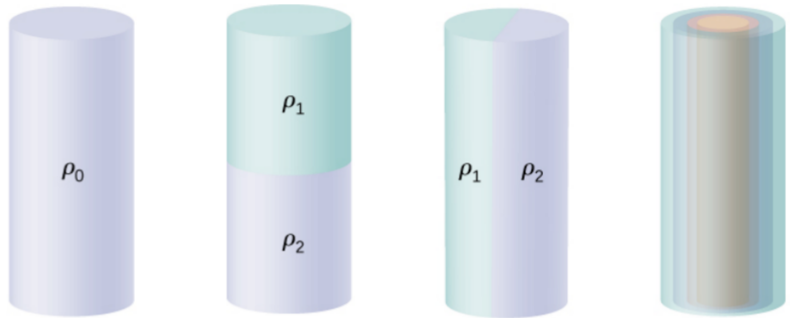


(a) Cylindrically symmetric
(b) Not cylindrically symmetric
(c) Not cylindrically symmetric
(d) Cylindrically symmetric

verðum að hugsa okkur óendanlegan langan sívalning til að uppfylla samhverfuna. Rafsviðið verður þá að vera alls staðar aðeins með útþátt, og við þurfum að huga að hleðslunni

$$Q = \rho V = \rho A L, \quad (L \rightarrow \infty, \text{ en bíðum})$$
$$= \lambda L$$

9

Utan sívalnings, $r > R$

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q}{\epsilon_0}$$

$$\rightarrow \quad E\,(2\pi r L) = \lambda L/\epsilon_0 \quad , \quad L \rightarrow \infty$$

$$\rightarrow \quad E = \frac{\lambda}{2\pi\epsilon_0 r} \quad \rightarrow \quad \boxed{\bar{E} = \frac{\lambda\,\hat{r}}{2\pi\epsilon_0 r}}$$

sem er sama niðurstaða og fæst fyrir örgranna línuhleðslu λ

Innan sívalnings, $r < R$

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q_{enc}}{\epsilon} \quad \rightarrow \quad E\,(2\pi r L) = \frac{\rho\pi r^2 L}{\epsilon_0}$$

$$\rightarrow \quad E = \frac{\rho r}{2\epsilon_0} = \frac{\lambda r}{2A\epsilon_0} = \boxed{\frac{\lambda r}{2\pi R^2\epsilon_0}}$$

10

þannig að $\boxed{\bar{E} = \frac{\lambda r\,\hat{r}}{2\pi\epsilon_0 R^2}}$

Rafsviðið er því samfellt í yfirborði sívalningsins, og vex línulega með r innan hans

Utan kúlunnar er sviðið eins og fyrir punkthleðslu, ekki fyrir sívaninginn, enda er aldrei hægt að komast nógu langt frá honum til að hann líti út sem punkthleðsla!

Óendanleg hlaðin örþunn slétta (ekki málmur)

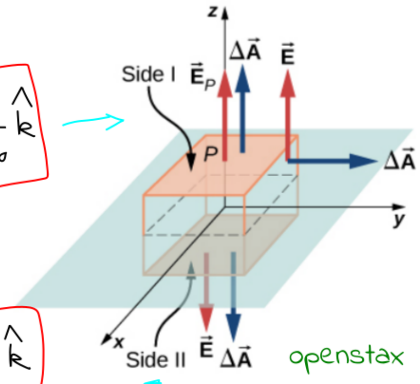
Yfirborðshleðsluþéttleiki σ

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q}{\epsilon_0}$$

$$EA + EA = \frac{\sigma A}{\epsilon_0}$$

$$\rightarrow \quad E = \frac{\sigma}{2\epsilon_0}$$

$$\boxed{\bar{E} = \frac{\sigma}{2\epsilon_0}\hat{k}} \qquad \boxed{\bar{E} = -\frac{\sigma}{2\epsilon_0}\hat{k}}$$



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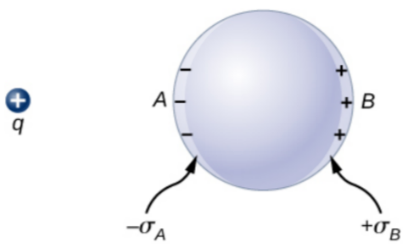
11

Leiðari í rafstöðujafnvægi



Ekkert háð tíma - jafnvægi

Óhlaðinn leiðari í upphafi, ytri hleðsla skautar yfirborðshleðslu

Skautunarhleðslan á yfirborðinu kemur í veg fyrir rafsvið innan leiðarans. Alger skýling

Í jafnvægi er ekkert rafsvið innan leiðara, annars yrðu straumar - ójafnvægi

Í jafnvægi getur aðeins verið yfirborðshleðsla á leiðara, ekki inni í honum

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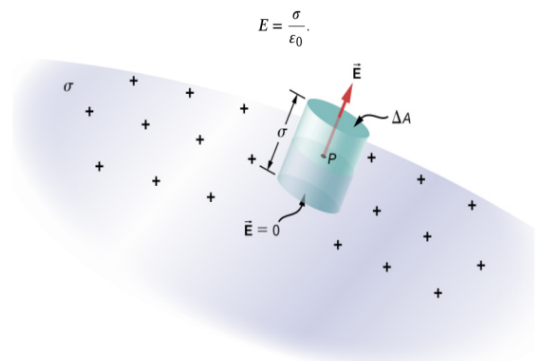
12

Rafsvið við yfirborð leiðara

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q}{\epsilon_0}$$

$$\rightarrow \quad EA = \frac{\sigma A}{\epsilon_0}$$

$$\rightarrow \quad \boxed{\bar{E} = \frac{\sigma\,\hat{n}}{\epsilon_0}}$$



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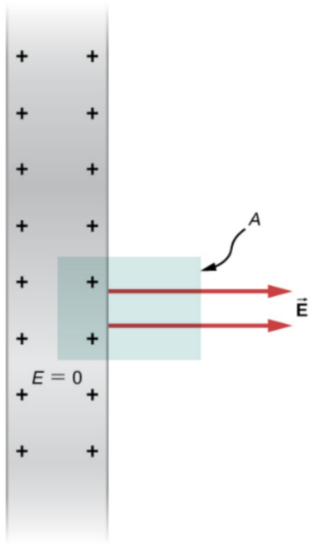
fyrir utan leiðarann ($E = 0$ fyrir innan) þar sem $\hat{n}$ er normalvigur á yfirborð leiðarans

Við yfirborð leiðarans er rafsviðið ósamfellt, ósamfellan er í réttu hlutfalli við yfirborðshleðslu leiðarans á hverjum stað

Hlaðinn sléttur leiðari

13



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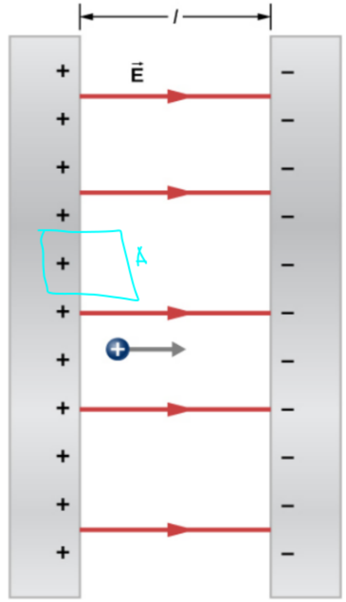
$$\oint \bar{E} \cdot d\bar{A} = \frac{Q}{\epsilon_0}$$

$$EA = \frac{\sigma A}{\epsilon_0}$$

$$\rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$$ þvert á leiðarann

Samsíða hlaðnir sléttir leiðarar með sitthvora hleðslutegundina

14



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$$\oint \bar{E} \cdot d\bar{A} = \frac{Q}{\epsilon_0}$$

$$\rightarrow EA = \frac{\sigma A}{\epsilon_0}$$

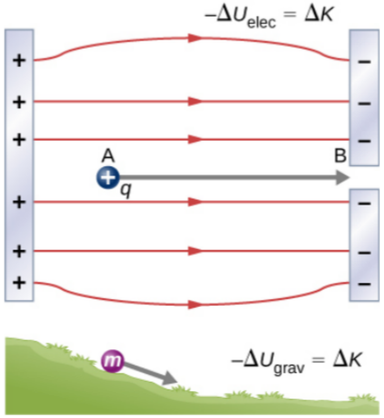
$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

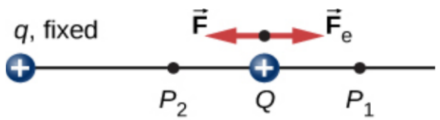
1

Rafstöðumætti - spenna

Hægt er að hraða hleðslu með rafsviði

Skoðum hreyfanlega hleðslu Q nærri fastri hleðslu q





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$\vec F_e$ er rafkraftur q á Q, $\vec F$ er ytri kraftur á Q
vinna $\vec F$ á Q vegna færslu frá P_1 til P_2 er

$$W_{12} = \int_{P_1}^{P_2} \vec F \cdot d\vec l$$

veljum $\vec F = -\vec F_e$

$$\vec F = -\vec F_e = -\frac{kQq}{r^2}\hat r$$

breyting stöðuorku Q er þá

$$\Delta U = -W$$

2

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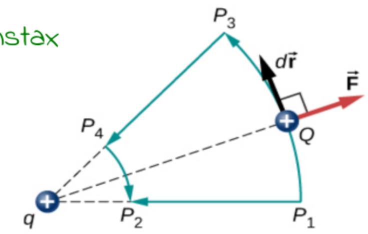

Vinnan fyrir $P_4 \to P_2$ og $P_1 \to P_3$ er 0, því $\vec F \cdot d\vec r = 0$ þar

$W_{34} = W_{12} = -W_{21}$

$$\rightarrow \boxed{\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = 0}$$

$$W_{12} = kqQ\int_{r_1}^{r_2} \frac{1}{r^2}\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}\, dr = kqQ\frac{1}{r_2} - kqQ\frac{1}{r_1}$$

$$\Delta U = -\int_{r_{\text{ref}}}^{r} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$$

$$\boxed{U(r) = k\frac{qQ}{r} - U_{\text{ref}}}$$

oft er hægt að velja U_{ref} í óendanlegri fjarlægð en ekki alltaf (t.d. gengur ekki fyrir línuhleðslu og sívalning)

3

Uppröðun hleðslu kostar vinnu - stöðuorka rafhleðslna

$$W_{12 \ldots N} = \frac{k}{2}\sum_{i}^{N}\sum_{j}^{N}\frac{q_i q_j}{r_{ij}} \text{ for } i \neq j \quad , \quad W_e = \frac{1}{2}\sum_{i=1}^{N} q_i V_i$$

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$$V_i = \frac{U_i}{q_i}$$

$$W_e = \frac{1}{2}\int_V \rho V \, dV' \qquad W_e = \frac{1}{2}\epsilon_0 \int_V E^2 dV'$$

Fyrir samfellda hleðsludreifingu ρ

U_i er rafstöðuorka q_i vegna hinna hleðslanna

4

Rafstöðumætti og mættismunur - spennumunur

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Electric Potential

The electric potential energy per unit charge is

$$V = \frac{U}{q}. \qquad 7.4$$

Electric Potential Difference

The **electric potential difference** between points A and B, $V_B - V_A$, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1 \text{ V} = 1 \text{ J/C}$$

Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta U}{q} \text{ or } \Delta U = q\Delta V. \qquad 7.5$$

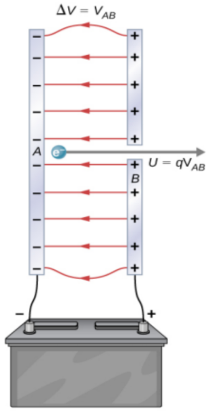
5

Rafeindavolt (eV) - orkueining

Electron-Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron-volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J}.$$



Jónunarorka einu rafeindar vetnisatóms er 13.6 eV

Massaorka rafeindar er 511 keV

$k_B = 8.617 \times 10^{-5}$ eV/K $=$ 0.08617 meV/K

--> $k_B T = 0.26$ meV fyrir $T = 3.0$ K

$k_B T = 25$ meV fyrir $T = 20$ °C

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6

Spenna og rafsvið

$$U_P = -\int_R^P \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}} \qquad U_P = -q\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

$$\boxed{V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}}$$

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Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

$$V = \frac{kq}{r} \text{(point charge)} \qquad 7.8$$

where k is a constant equal to $8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

7

Tvískaut

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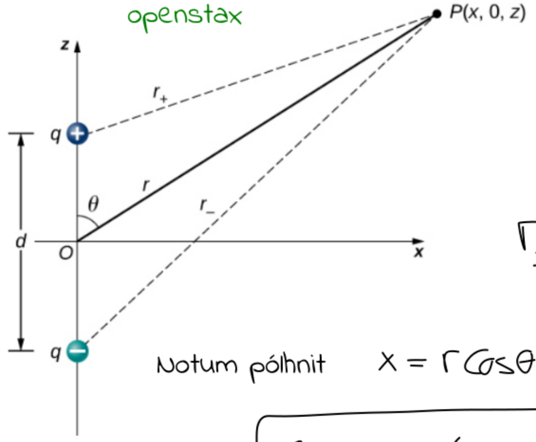


$$V_P = V_+ + V_-$$

$$= k\left[\frac{q}{r_+} - \frac{q}{r_-}\right]$$

$$r_\pm = \sqrt{x^2 + \left(z \mp \frac{d}{2}\right)^2}$$

Notum pólhnit $x = r\cos\theta$, $y = r\sin\theta$

$$r_\pm = \sqrt{r^2\sin^2\theta + \left(r\cos\theta \mp \frac{d}{2}\right)^2}$$

$$= r\sqrt{\sin^2\theta + \left(\cos \mp \frac{d}{2r}\right)^2}$$

8

$$r_\pm = r\sqrt{\underbrace{\sin^2\theta + \cos^2\theta}_{=1} \mp \frac{d}{r}\cos\theta + \left(\frac{d}{2r}\right)^2}$$

$$= r\sqrt{1 \mp \frac{d}{r}\cos\theta + \left(\frac{d}{2r}\right)^2}$$

viljum skoða fjærsviðið þegar $r >> d$

$$\rightarrow \quad r_\pm \simeq r\sqrt{1 \mp \frac{d}{r}\cos\theta}$$

viljum líka nota

$$\frac{1}{\sqrt{1 \mp \alpha}} \simeq 1 \pm \frac{\alpha}{2}$$

ef $\alpha << 1$

$$\rightarrow \quad V_P = k\left\{\frac{q}{r}\left(\cancel{1} + \frac{d\cos\theta}{2r}\right) - \frac{q}{r}\left(\cancel{1} - \frac{d\cos\theta}{2r}\right)\right\}$$

$$= k\,\frac{qd\cos\theta}{r^2}$$

9

Skilgreinum tvískautsvægi $\overline{p} = q\overline{d}$ → $V_p = k\frac{\overline{p}\cdot\hat{r}}{r^2}$

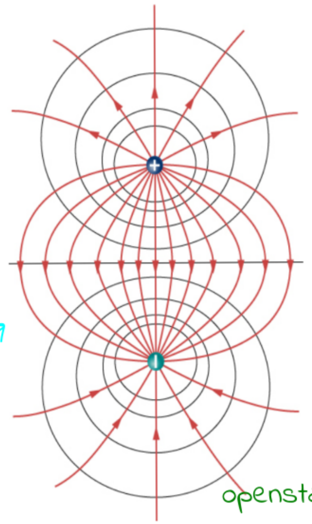
Tvískautið hefur því aðfelluform $V \sim \frac{1}{r^2}$ þegar $r \to \infty$
meðan stök hleðsla hefur $V \sim \frac{1}{r}$

Tvískautið er með rafsvið og rafmætti sem ekki eru stefnusnauð

Tvískaut og hærri skaut koma mikið fyrir í sameindum og hafa mikil áhrif á efnafræði þeirra

Tímaháð tvískaut geta geislað rafsegulbylgjum
tímaháð einskaut getur það ekki

Hér sjást rafsviðslínur og jafnspennufletir sem við komum að rétt bráðum



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10

Rafmætti samfelldrar hleðsludreifingar

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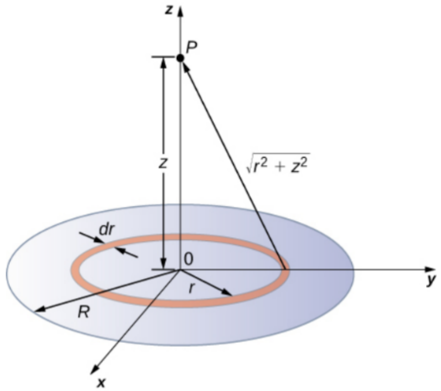
$$V_P = k\int \frac{dq}{r} \qquad dq = \begin{cases} \lambda\, dl & \text{(one dimension)} \\ \sigma\, dA & \text{(two dimensions)} \\ \rho\, dV & \text{(three dimensions)} \end{cases}$$

Skoðum skífu eða disk



$$dV_p = k\frac{dq}{\sqrt{z^2+r^2}}$$

$$dq = \sigma \cdot 2\pi r\,dr$$

$$V_p = k2\pi\sigma\int_0^R \frac{r\,dr}{\sqrt{z^2+r^2}}$$

$$= k2\pi\sigma\left[\sqrt{z^2+R^2} - \sqrt{z^2}\right]$$

11

Rafsvið frá rafmætti

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Relationship between Voltage and Uniform Electric Field

In equation form, the relationship between voltage and uniform electric field is

$$E = -\frac{\Delta V}{\Delta s}$$

where Δs is the distance over which the change in potential ΔV takes place. The minus sign tells us that E points in the direction of decreasing potential. The electric field is said to be the gradient (as in grade or slope) of the electric potential.

$$\boxed{\vec{\mathbf{E}} = -\vec{\nabla} V}$$

$$\vec{\nabla} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$$ í Kartískum hnitum

$$\text{Cylindrical: } \vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

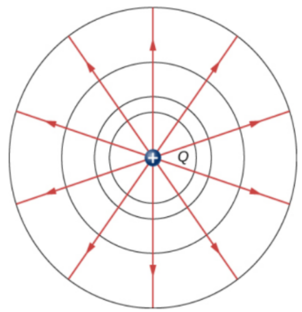
$$\text{Spherical: } \vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}$$

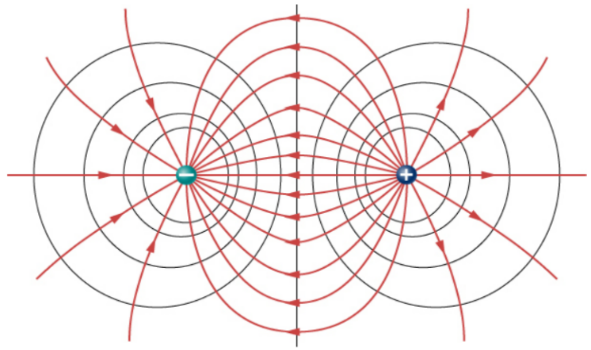
$\overline{E}$ er vigur, V er skalar og $\overline{\nabla}$ er afleiðuvirki sem varpar skalar í vigur

12

Jafnspennufletir og rafsvið

Rafsvið er alltaf hornrétt á jafnspennufleti. Jafnspennuflötur er flötur þar sem V er fasti





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Í jafnvægi er góður leiðari jafnspennuflötur

Þess vegna er hleðsla á hlöðnum leiðara í jafnvægi ekki endilega jafndreifð

$$k\frac{q_1}{R_1} = k\frac{q_2}{R_2} \rightarrow \frac{q_1}{R_1} = \frac{q_2}{R_2}$$

$$q = \sigma(4\pi R^2)$$

$$\sigma_1 R_1 = \sigma_2 R_2$$

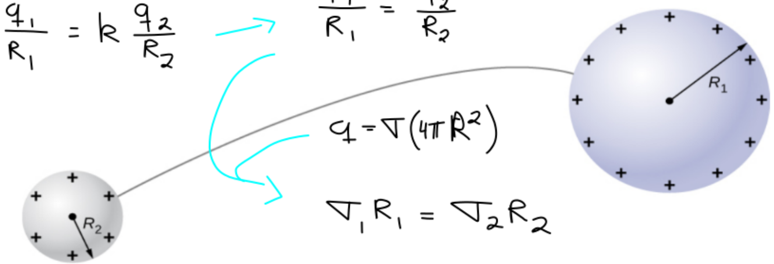
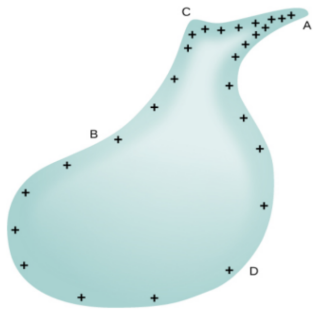


Figure 7.39 Two conducting spheres are connected by a thin conducting wire.

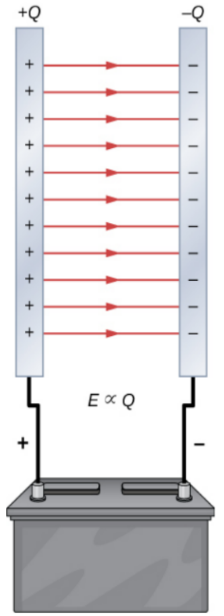


Ef við tengjum R við krappageisla sjáum við að mest hleðslan safnast fyrir þar sem krappageislinn er minnstur

Eldingavarar

1

Rýmd - capacitance



Rafkraftar milli hleðsna á leiðurum halda hleðslunum þar, rýmd er skilgreind sem

$$C = \frac{Q}{V} \qquad \text{Eining} \qquad 1\,\mathrm{F} = \frac{1\,\mathrm{C}}{1\,\mathrm{V}}$$

Almennur eiginleiki leiðara, ský getur líka haft rýmd miðað við jörð...

Þéttar eru mikilvægir í rafrásum, þeir geta einnig geymt rafhleðslu og verkað sem "rafgeymar"

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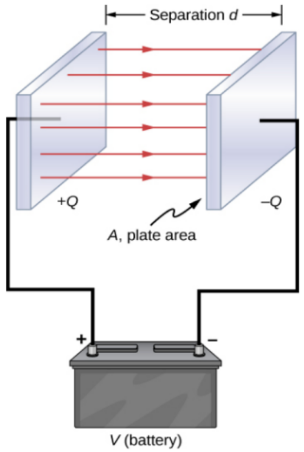
2

Rýmd plötuþéttis



Hleðsluþéttleiki á plötu

$$\sigma = \frac{Q}{A}$$

$$\rightarrow \quad E = \frac{\sigma}{\epsilon_0} \qquad \text{fastur sviðsstyrkur}$$

$$\rightarrow \quad V = Ed = \frac{\sigma d}{\epsilon_0} = \frac{Qd}{\epsilon_0 A}$$

$$\rightarrow \quad \boxed{C = \frac{Q}{V} = \frac{Q\epsilon_0 A}{Qd} = \epsilon_0 \frac{A}{d}}$$

Rýmd einfalds línulegs þéttis er aðeins háð lögun hans (og efninu milli platnanna)

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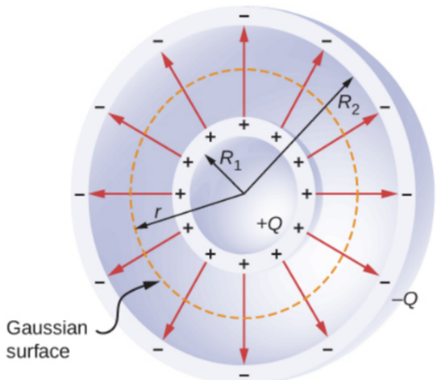
3

Rýmd kúluþéttis



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Milli kúluskeljanna

$$\oint_S \bar{E} \cdot d\bar{A} = \frac{Q}{\epsilon_0} \qquad \text{Lögmál Gauß}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\rightarrow \quad \bar{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

$$V = \int_{R_1}^{R_2} \bar{E} \cdot d\bar{l} = \int_{R_1}^{R_2} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \cdot (\hat{r}\, dr)$$

$$\rightarrow \quad V = \frac{Q}{4\pi\epsilon_0}\int_{R_1}^{R_2}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left\{\frac{1}{R_1} - \frac{1}{R_2}\right\}$$

$$\rightarrow \quad \boxed{C = \frac{Q}{V} = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}}$$

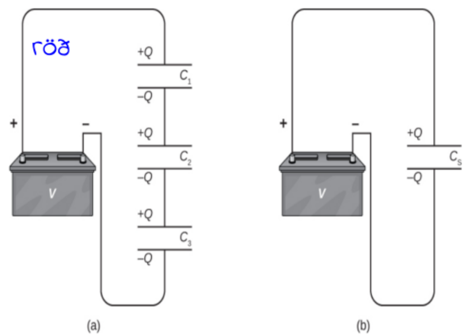
Rýmd einnar kúlu, $R_2 \rightarrow \infty$

$$\boxed{C = 4\pi\epsilon_0 R_1}$$

4

Uppröðun þétta

röð



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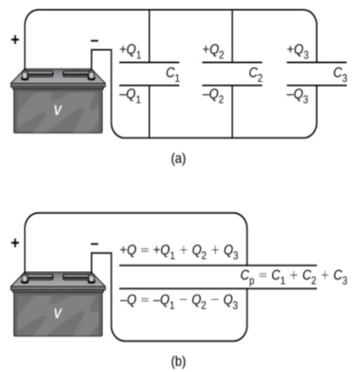
Samsíða



$$V = V_1 + V_2 + V_3$$

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$Q_p = Q_1 + Q_2 + Q_3$$

$$C_p V = C_1 V + C_2 V + C_3 V$$

$$\rightarrow \quad \boxed{C_p = C_1 + C_2 + C_3}$$

Orka í þétti

5

Flutningur á hleðslu dq frá annarri þéttaplötunni yfir á hina krefst vinnu

$$dW = V\,dq = \frac{q}{C}\,dq$$

$$\boxed{W} = \int_0^{W(Q)} dW = \int_0^{Q} \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV = U_C$$

tengist V, Q og C sem tengja má við þéttinn og plötur hans, en hver er orkuþéttleikinn í geilinni milli platnanna

$$u_E = \frac{U_C}{Ad} = \frac{1}{2}\frac{Q^2}{C}\frac{1}{Ad} = \frac{1}{2}\frac{Q^2}{\varepsilon_0 A/d}\frac{1}{Ad} = \frac{1}{2}\frac{1}{\varepsilon_0}\left(\frac{Q}{A}\right)^2 = \frac{\sigma^2}{2\varepsilon_0} = \frac{(E\varepsilon_0)^2}{2\varepsilon_0} = \frac{\varepsilon_0}{2}E^2$$

rafsviðið eða rafmættið í geilinni milli platnanna hefur orkuþéttleika

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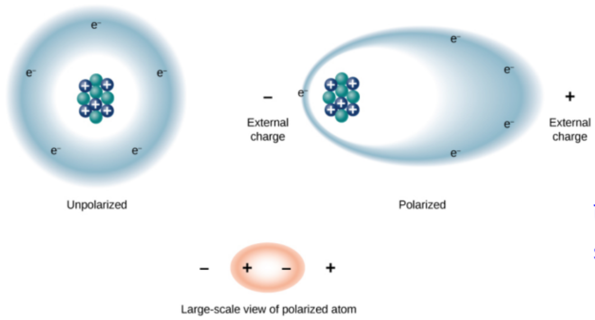
Rafsvarar - dielectric

6



Áhrif ytra rafsvið á atóm -- skautun

induced electric dipole moment
skautað tvískautsvægi

Áhrif ytra rafsviðs á einangrandi efni sem getur skautast

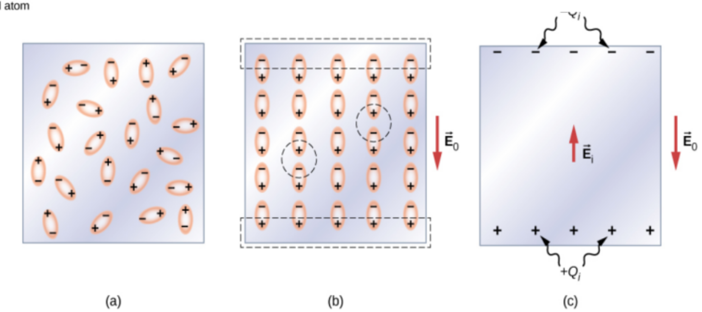


Figure 8.20 A dielectric with polar molecules: (a) In the absence of an external electrical field; (b) in the presence of an external electrical

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Áhrif rafsvara á rýmd

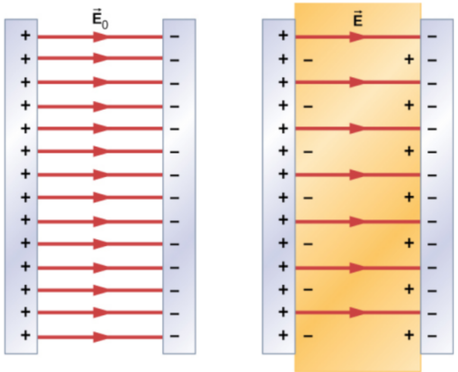
7

Hleðslan Q_0 veldur E_0 inni í þéttinum
Skautun rafsvarans leggur til E_i
Heildarrafsviðið er

$$\bar{E} = \bar{E}_0 + \bar{E}_i$$

Fyrir línulega rafsvara

$$E_0 = \kappa E$$



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skilgreining rafsvörunarfastans κ (grískt kappa), κ > 1

$$\boxed{C = \kappa C_0, \qquad U = \frac{1}{\kappa} U_0}$$

Rýmdin vex með rafsvara, orkan geymd minnkar

Rafsegulfræði í efni er miklu flóknari en rafsegulfræðin fyrir stakar hleðslur í tómarúmi

Straumar - leiðni - viðnám

8

Færumst frá jafnvægi yfir í sístætt ástand (steady state)

Electrical Current

The average electrical current I is the rate at which charge flows,

$$I_{\text{ave}} = \frac{\Delta Q}{\Delta t}, \tag{9.1}$$

where ΔQ is the amount of net charge passing through a given cross-sectional area in time Δt (Figure 9.2). The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \frac{\Delta Q}{\Delta t}$, we see that an ampere is defined as one coulomb of charge passing through a given area per second:

$$1\text{A} \equiv 1\frac{\text{C}}{\text{s}}. \tag{9.2}$$

The instantaneous electrical current, or simply the **electrical current**, is the time derivative of the charge that flows and is found by taking the limit of the average electrical current as $\Delta t \to 0$:

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}. \tag{9.3}$$

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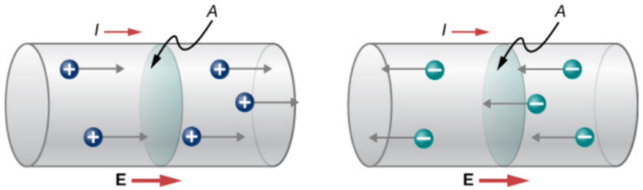
9

Hvað flæðir?



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+ hlaðnar eindir
samkvæmt skilgreiningu
sem er eldri en þekking á
rafeindum

rafeindir?

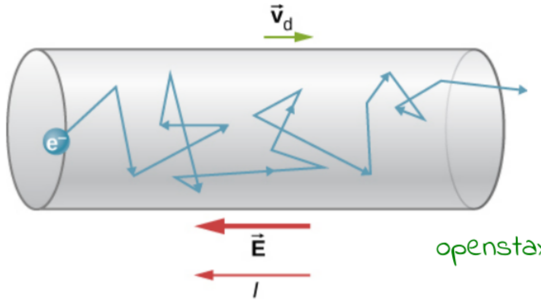
Málið er flóknara. Vissulega flæða rafeindir, en það er einfaldara að skoða flæði "sýndareinda" (quasi-particles) sem geta verið með + eða - hleðslu, eða jafnvel hleðslu sem er brot af e, einingarhleðslunni

Sýndareindirnar koma fram í tilraunum og reikningum, sem veikt víxlverkandi einingar....

10

Rekhraði - drift velocity

Rafeindagas í leiðara
mikill hraði - tíðir árekstrar
(rafeindir - hljóðeindir - óreglur
í kristalli) --> lítill rekhraði



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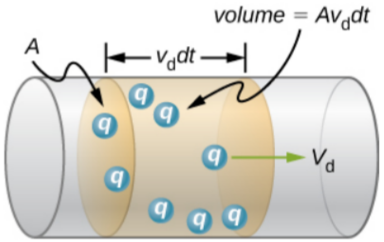


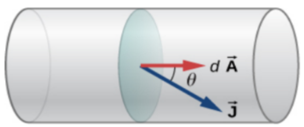
$$\boxed{I = \frac{dQ}{dt} = qnAv_d}$$

n: eindaþéttleiki

$$\boxed{v_d = \frac{I}{nqA}}$$

11

Straumþéttleiki J



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$$I = \int_A \bar{J} \cdot d\bar{A}$$

$$J = \frac{I}{A} = \frac{n|q|Av_d}{A} = n|q|v_d$$

$$\boxed{\bar{J} = nq\bar{v}_d}$$

Eðlisleiðni - conductivity

Fyrir línulega svörun við ytra rafsviði gildir

$$\boxed{\bar{J} = \sigma \bar{E}}$$

eining σ er A/(Vm)

Í smásæjum líkönum er eðlisleiðni eða leiðni reiknuð, en oft er eðlisviðnám eða viðnám mælt

$$\boxed{\bar{E} = \rho \bar{J}} \qquad \rho = \frac{1}{\sigma}$$

eining ρ er $\Omega m = \frac{V}{A} m$

12

Viðnám - leiðni (resistance - conductance)

Resistance

The ratio of the voltage to the current is defined as the **resistance** *R*:

$$R \equiv \frac{V}{I}. \qquad 9.8$$

Lögmál Ohms

$$\boxed{V = RI}$$

The resistance of a cylindrical segment of a conductor is equal to the resistivity of the material times the length divided by the area:

$$R \equiv \frac{V}{I} = \rho \frac{L}{A}. \qquad 9.9$$

$$\boxed{I = GV}$$

G: leiðni

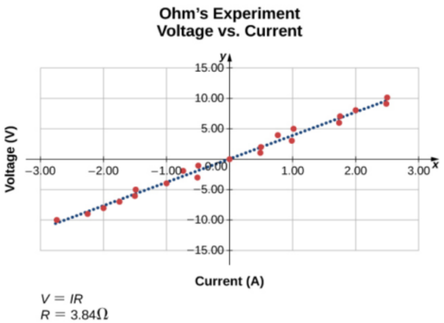
Ef áhrif T eru línuleg fæst

$$R = R_0\{1 + \alpha \Delta T\}$$

Landauer: Allar reikniaðgerðir í tölvu kosta orku

13

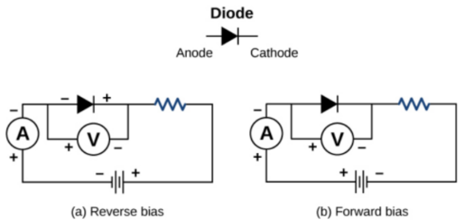
Línuleg eða ólínuleg leiðni

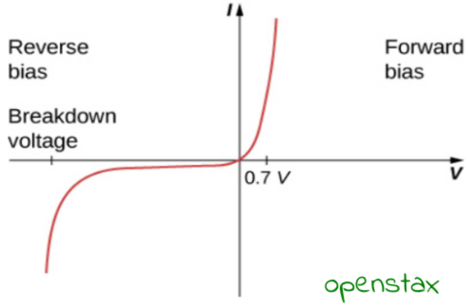


$$\boxed{V = IR}$$

Lögmál Ohms

Tvistur - diode





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14

Rafafl - raforka

opin kerfi með orkutapi

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Electric Power

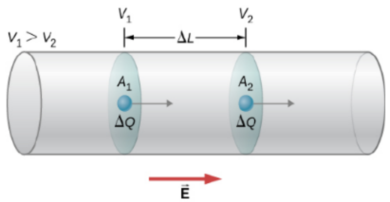
The electric power gained or lost by any device has the form

$$P = IV. \qquad 9.12$$

The power dissipated by a resistor has the form

$$P = I^2 R = \frac{V^2}{R}. \qquad 9.13$$

$\vec{F} = \Delta q \vec{E}$



$$E = -\frac{(V_2 - V_1)}{\Delta L} = \frac{V}{\Delta L}$$

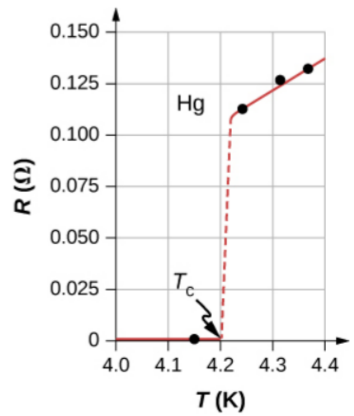
$$W = F\Delta L = (\Delta Q E)\Delta L = \left(\Delta Q \frac{V}{\Delta L}\right)\Delta L$$

$$= \Delta Q V = \Delta U$$

$$P = \frac{\Delta U}{\Delta t} = -\frac{\Delta Q V}{\Delta t} = IV$$

15

Ofurleiðni



Til eru efni sem missa allt viðnám undir vissu hitastigi

Suma má skilja með BCS-líkaninu sem segir að rafeindir parist í Cooper-pör í skriðþungarúminu vegna áhrifa hljóðeinda og þéttist niður í lægstu orkuástöndin (bóseindir)

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16

Segulflæðiskömmtun

$$\Phi_B = \frac{hc}{2e}$$

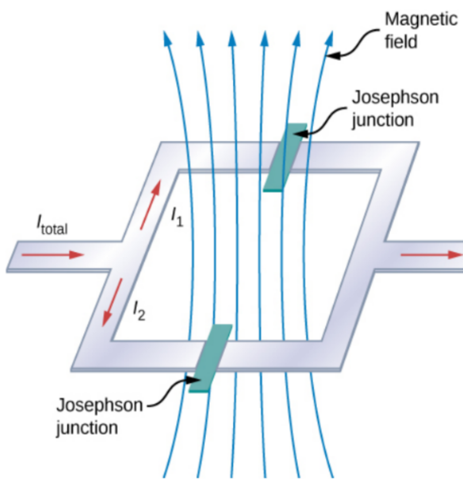

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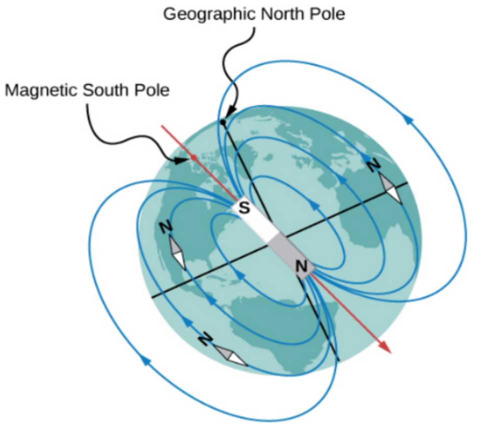
Figure 9.29 The SQUID (superconducting quantum interference device) uses a superconducting current loop and two Josephson junctions to detect magnetic fields as low as 10^{-14} T (Earth's magnet field is on the order of 0.3×10^{-5} T).

Leiðni er líka skömmtuð $G_0 = \frac{2e}{h}$

1

Segulsvið, svið og kraftar



Segulkraftar hafa verið þekktir mjög lengi

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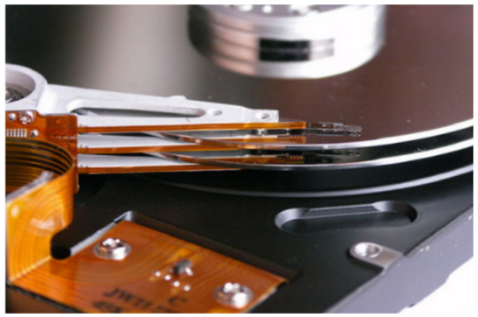

Seguleiginleikar efnis eru mjög mikilvægir í tækni og grunnrannsóknum og í jarðvísindum

2

Kraftar segulsviðs á hleðslur

Á hleðslu kyrra í segulsviði verkar enginn kraftur, en á hleðslu sem hreyfist með hraðanum $\bar{v}$ í segulsviði verkar kraftur Lorentz

$$\boxed{\bar{F} = q\,\bar{v} \times \bar{B}}$$

Styrkur segulsviðs er mældur með einingunni Tesla: $T = N/(Am)$
Af sögulegum ástæðum er líka til einingin gauss: $G = 10^{-4}T$
Svið jarðar 0.5 G. Sterkir fastir seglar eru upp að 2T.
Ofurleiðandi rafseglar eru að 20T
Samsettir ofurleiðandi og venjulegir rafseglar hafa náð að 36-40T
Púlsaðir seglar hafa náð upp að 750T í mjög skamman tíma.

Sterkt segulsvið er mikið notað við rannsóknir á smáum rafeindakerfum vegna þess að það hefur mikil áhrif á rafeindauppbyggingu kerfanna

3

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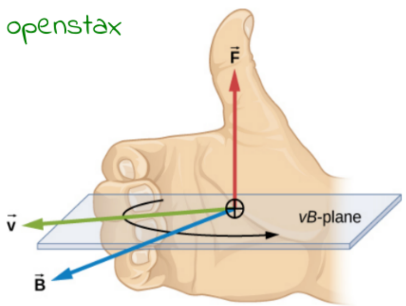
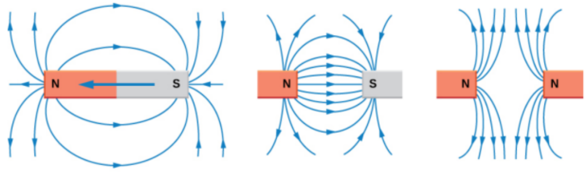


Hægri-handar-reglan

Sviðslínur segulsviðs - fastir seglar



Magnetic field lines of a bar magnet — Magnetic field lines between unlike poles — Magnetic field lines between like poles

Sviðslínur skerast aldrei (einkvæmni sviðs)

Við hugsum okkur sviðslínur út úr N-skauti til S-skauts, en líka innan seguls --> Sviðslínur eru lokaðir ferlar.

Allir seglar hafa bæði skautin -- ekki er til segulеinskaut (engin segulhleðsla)
Lægstu skautin sem finnast eru segultvískaut

4

Brautir hlaðinna einda í segulsviði

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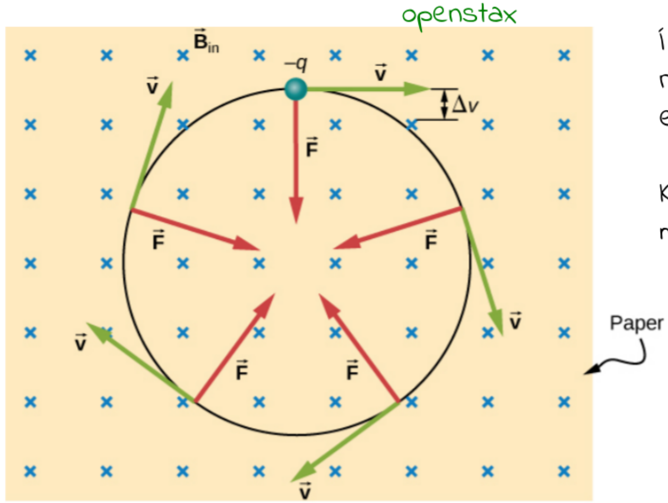


Í föstu segulsviði fara rafeindir og aðrar hlaðnar eindir á hringhreyfingu

Kraftur Lorentz leggur til miðsóknarkraftinn

$$qvB = \frac{mv^2}{r}$$

$$\rightarrow \boxed{r = \frac{mv}{qB}}$$

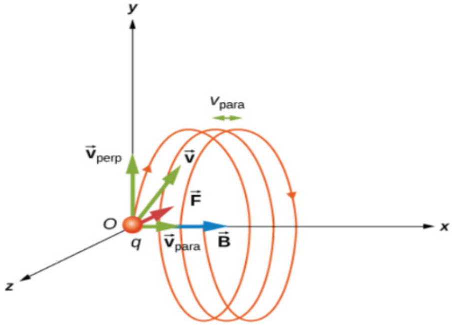
r: geisli hringhraðalshermunnar og lota hennar er

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}}$$

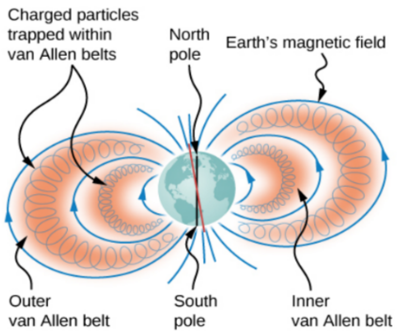
cyclotron resonance

5

Ef eindin hefur hraðaþátt samsíða B verður hreyfingin gormlaga



Gormlaga hreyfingin er ástæða norður- og suðurljósa þegar eindir sem festast í segulsviði jarðar nálgast segulskautin og efri lög andrúmsloftsins



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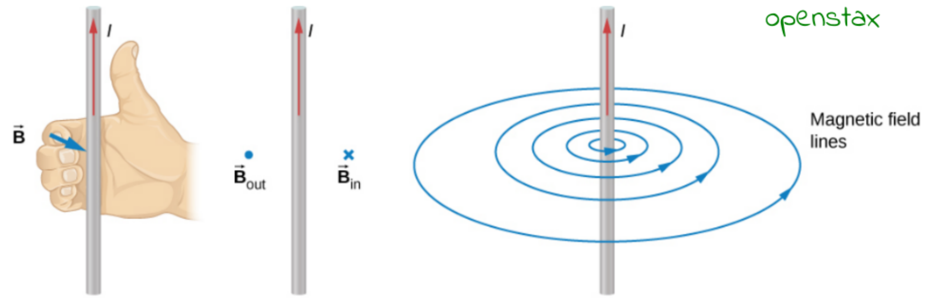
6

Kraftur á hleðslur í leiðara -- segulsvið leiðara



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Straumur í leiðara veldur segulsviði í og um leiðarann. Lærum að reikna síðar, en núna viljum við skilja hvernig krafturinn á hleðslur í segulsviði leiðir til krafts á leiðarann sjálfan. Um strauminn gildir

$$I = neAv_d$$

7

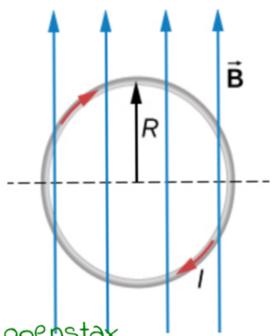
$$dF = \underbrace{\{nA \cdot dl\}e}_{q}\, \bar{v}_d \times \bar{B} = neAv_d\, \bar{dl} \times \bar{B}$$

$$= I\, d\bar{l} \times \bar{B}$$

Fyrir beinan vír fæst þá

$$\bar{F} = I\, \bar{l} \times \bar{B}$$

Fyrir hring



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$$dF = IB\sin\theta\, dl \quad , \quad dl = R\,d\theta$$

$$= IBR\sin\theta\, d\theta$$

$$F = \int_0^{2\pi} IBR\sin\theta = 0$$

Enginn heildarkraftur á hringinn "láréttan" í segulsviðinu

8

.. en á lykkjuna getur verkað vægi

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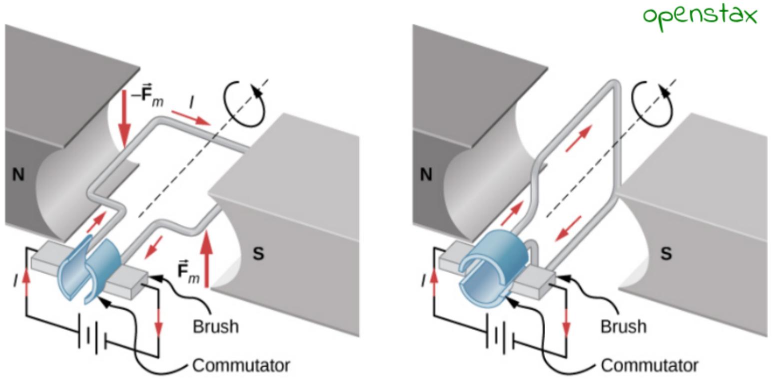
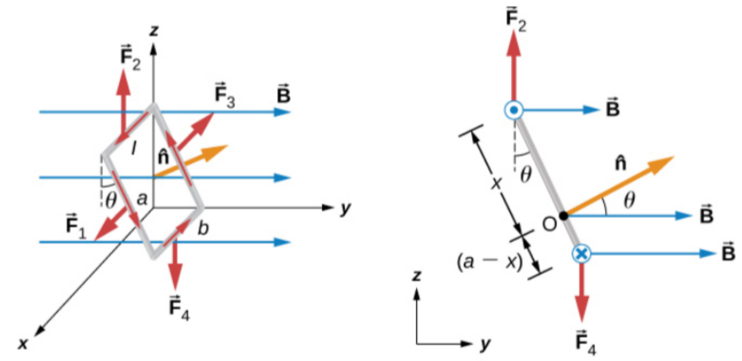


sem er notað til að snúa snúði rafvélar í ytra segulsviði

9



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Heildarkrafturinn á lykkjuna (þó hún sé ekki hringur) er 0, en vægið

$$\bar\tau = \bar\tau_1 + \bar\tau_2 + \bar\tau_3 + \bar\tau_4 \quad = \bar\tau_2 + \bar\tau_4$$

$$= F_2 x \sin\theta\, \hat i \; - F_4 (a-x) \sin\theta\, \hat i$$

$$= -IbBx\sin\theta\, \hat i \; - IbB(a-x)\sin\theta\, \hat i \; = -IAB\sin\theta\, \hat i$$

10

ef $\quad \bar\mu = IA\hat n N$

þvervigur flatar

Flötur lykkju

Fjöldi vafninga

þá fæst

$$\boxed{\bar\tau = \bar\mu \times \bar B}$$

vægi B á tvískautið

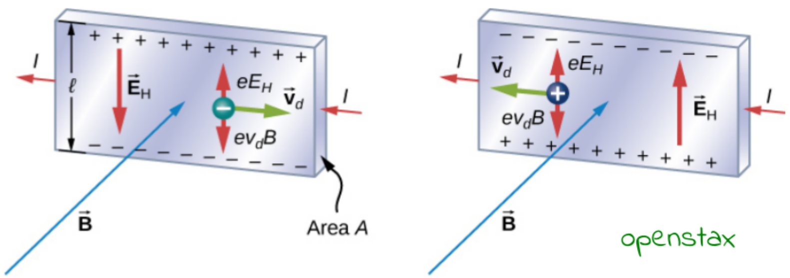
og enn fremur er stöðuorka tvískautsins

$$\boxed{U = -\bar\mu \cdot \bar B}$$

Straumlykkja hefur tvískautsvægi, en eigum eftir að sjá að lykkjan býr til segulsvið (í réttu hlutfalli við I), tvískautssvið

11

Hrif Halls



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Jafnvægi þegar

$$eE = ev_d B \quad \longrightarrow \quad v_d = \frac{E}{B}$$

en $\quad I = nev_d A = ne\left(\frac{E}{B}\right)A \;, \quad E = \frac{V}{L}$

$$\longrightarrow \; V = \frac{IBL}{neA} \;, \quad V = BLv_d$$

spenna Halls

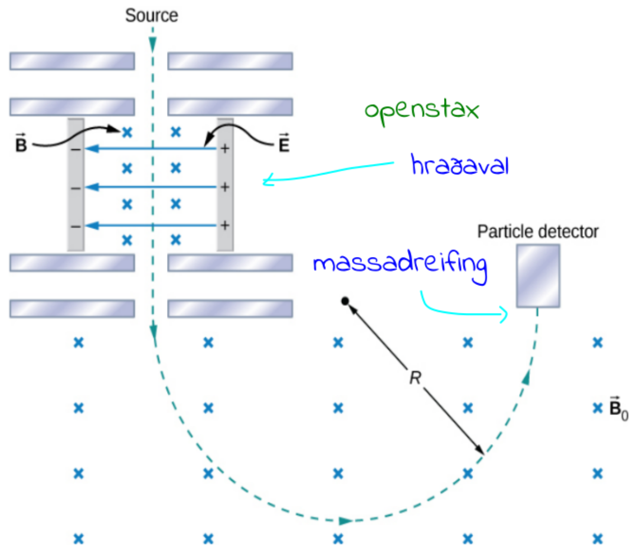
Mæling á V getur ákveðið n, e (+e eða -e) og aðferð til að mæla B

12

Skammtahrif Halls

Heiltöluskömmtun 1980
Brottöluskömmtun 1983
↓
Verðlaun Nobels
1985 -- 1998
+
Skammtapunktar --
leiðni um þrengingar
↓
Talning rafeinda
Skömmtun leiðni
↓
Straumstaðall
Massastaðall

Massagreinir



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hraðaval

massadreifing

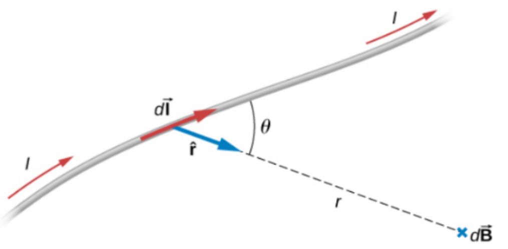
1

Uppsprettur segulsviðs

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}.$$

$$\mu_0 = 4\pi \cdot 10^{-7} \ \frac{Tm}{A}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I dl \sin\theta}{r^2}$$



A current element $Id\vec{\mathbf{l}}$ produces a magnetic field at point P given by the Biot-Savart law.

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Biot-Savart law

The magnetic field $\vec{\mathbf{B}}$ due to an element $d\vec{\mathbf{l}}$ of a current-carrying wire is given by

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\int_{\text{wire}} \frac{I\, d\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}. \qquad 12.4$$

2

Segulsvið þunns beins leiðara

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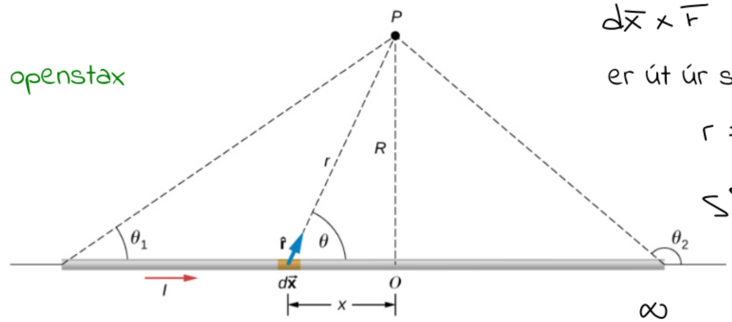


$d\vec{x} \times \vec{r}$

er út úr síðunni fyrir P

$$r = \sqrt{x^2 + R^2}$$

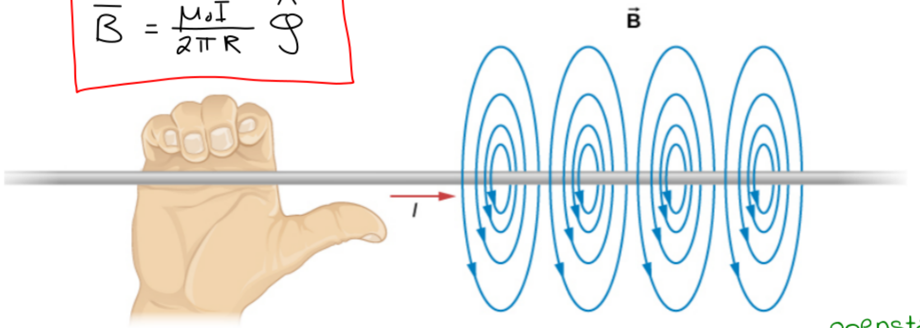
$$\sin\theta = \frac{R}{\sqrt{x^2+R^2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{\sin\theta\, dx}{r^2} = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{R\,dx}{\{x^2+R^2\}^{3/2}}$$

$$= \frac{\mu_0 I}{2\pi}\int_0^{\infty}\frac{R\,dx}{[x^2+R^2]^{3/2}} = \frac{\mu_0 I}{2\pi R}\left\{\frac{x}{(x^2+R^2)^{1/2}}\Big|_0^{\infty}\right\} = \frac{\mu_0 I}{2\pi R}$$

3

$$\boxed{\bar{B} = \frac{\mu_0 I}{2\pi R}\,\hat{\varphi}}$$



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Segulkrafturinn milli tveggja samhliða leiðara

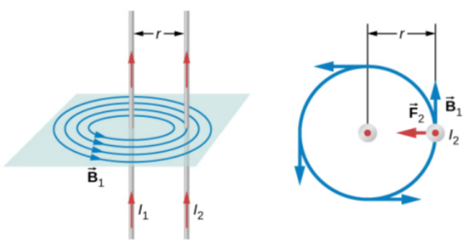
Á leiðara 2 verkar

$$F_2 = I_2 l B_1 = \frac{\mu_0 l I_1 I_2}{2\pi r}$$

$$\boxed{\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

Tengsl við orku

$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$



4

Segulsviðið á samhverfuás lykkju

$$dB = \frac{\mu_0}{4\pi}\frac{I dl}{r^2}\sin\left(\frac{\pi}{2}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{I dl}{y^2+R^2}$$

$$\bar{B} = \hat{j}\oint dB\cos\theta$$

$$= \hat{j}\,\frac{\mu_0 I}{4\pi}\oint\frac{\cos\theta\, dl}{y^2+R^2}$$

$$\cos\theta = \frac{R}{\sqrt{y^2+R^2}}$$

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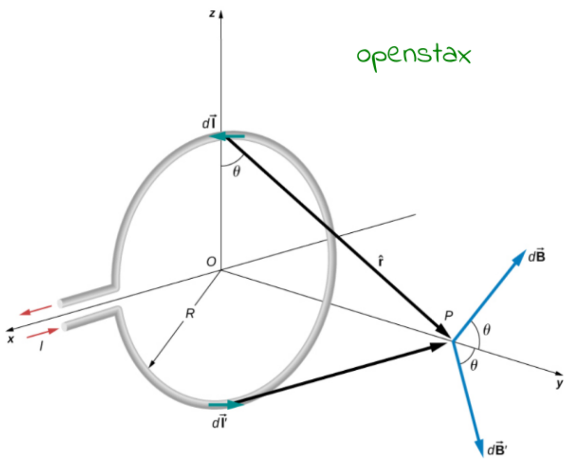

Figure 12.11 Determining the magnetic field at point P along the axis of a current-carrying loop of wire.

$$\bar{B} = \hat{j}\,\frac{\mu_0 I R}{4\pi (y^2+R^2)^{3/2}}\oint dl = \frac{\mu_0 I R^2}{2(y^2+R^2)^{3/2}}\,\hat{j}$$

5

því $\oint dl = 2\pi R$

Notum $\bar{\mu} = I A \hat{n} = I \pi R^2 \hat{j}$ hér

Í miðju lykkjunnar, y = 0 $\boxed{\bar{B} = \frac{\mu_0 I}{2R} \hat{j}}$

og langt frá lykkjunni, y >> R, fæst

$$\boxed{\bar{B} = \frac{\mu_0 \bar{\mu}}{2\pi R^3}}$$

sem er svið segultvískauts

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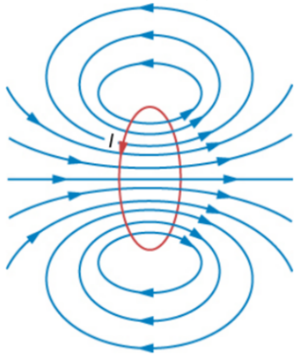

6

Lögmál Amperes

Segulsviðið B er ekki geymið vigursvið

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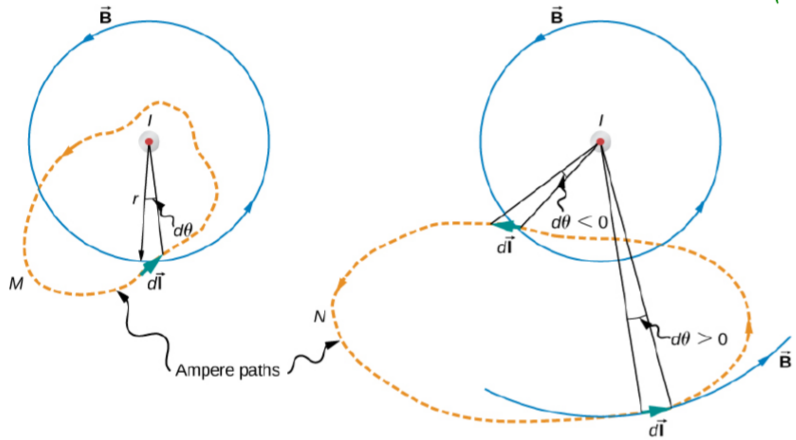


Hægt er að sýna að $\oint_N \bar{B} \cdot d\bar{l} = 0$, en $\oint_M \bar{B} \cdot d\bar{l} = \mu_0 I$

7

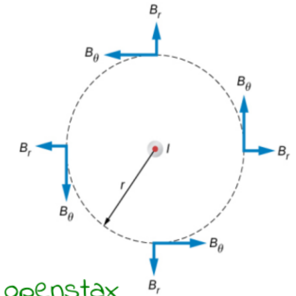
Ampère's law

Over an arbitrary closed path,

$$\boxed{\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0 I} \qquad 12.23$$

where I is the total current passing through any open surface S whose perimeter is the path of integration. Only currents inside the path of integration need be considered.

Beinn langur vír



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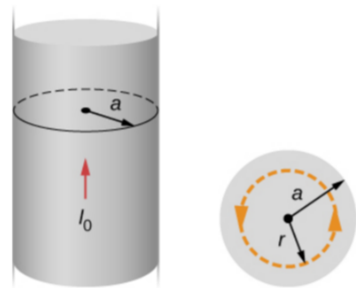
$$\oint \bar{B} \cdot d\bar{l} = \oint \bar{B}_\theta \cdot d\bar{l}$$

$$= 2\pi r B = \mu_0 I$$

$$\rightarrow \boxed{\bar{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}}$$

8

Ex. 12.7, þykkur leiðari með fast straumþykkni

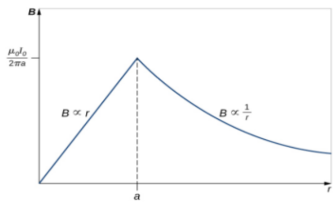


Utan vírs $\oint \bar{B} \cdot d\bar{l} = \oint B\, dl$

$$= B 2\pi r = \mu_0 I$$

$$\rightarrow \bar{B} = \frac{\mu_0 I_0}{2\pi r} \hat{\theta}, \quad r > a$$

Innan vírs $I_{enc} = \frac{r^2}{a^2} I_0 \rightarrow \bar{B} = \frac{\mu_0 I_0}{2\pi} \frac{r}{a^2} \hat{\theta} \quad r \leq a$



Variation of the magnetic field produced by a current I_0 in a long, straight wire of radius a.

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9

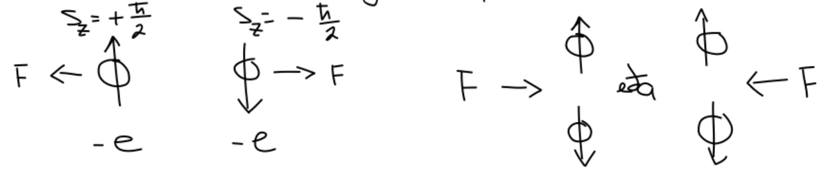
Seguleiginleikar efnis

Allt efni er veikt andseglandi (diamagnetic), líka við!
Sígild eðlisfræði nægir ekki til að skýra seguleiginleika efnis (heldur ekki andseglun)

Rafeindir bera segultvískautsvægi í hlutfalli við spuna þeirra og hverfiþunga á hvelum atóma. Atóm geta því haft segulvægi, sérstaklega um mitt lotukerfið vegna skiptakrafts rafeinda í efri hvelum

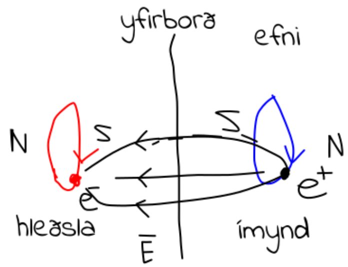
Seglun er til í mörgum flokkum, við minnumst á andseglun (diamagnetism), meðseglun (paramagnetism), járnseglun (ferromagnetism) og andjárnseglun (antiferromagnetism)

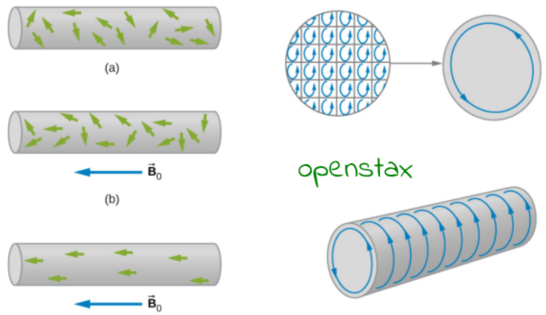
Fráhrindikraftur Coulombs og skiptakraftur - aðdráttarkraftur



10

Andseglun, einfölduð sígild skýring





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Meðseglun

Ytrasvið raðar upp tvískautum en of veikur skiptakraftur nær ekki að viðhalda uppröðun eftir að ytra sviðið hverfur

Heildarsvið B, ytrasvið B_0, innra svið B_m - svörun við B_0

$$\bar{B} = \bar{B}_0 + \bar{B}_M$$

$$\bar{B} = \chi \bar{B}_0$$

segulviðtak

$$\bar{B} = (1+\chi)\bar{B}_0$$

11

fyrir einfalda línulega andseglun eða meðseglun fæst að

$$\mu = (1+\chi)\mu_0$$

χ getur haft annaðhvort formerkið og í flóknari efnum er viðtakið ekki fasti, en flókið fall af B og T

Paramagnetic Materials	χ	Diamagnetic Materials	χ
Aluminum	2.2×10^{-5}	Bismuth	-1.7×10^{-5}
Calcium	1.4×10^{-5}	Carbon (diamond)	-2.2×10^{-5}
Chromium	3.1×10^{-4}	Copper	-9.7×10^{-6}
Magnesium	1.2×10^{-5}	Lead	-1.8×10^{-5}
Oxygen gas (1 atm)	1.8×10^{-6}	Mercury	-2.8×10^{-5}
Oxygen liquid (90 K)	3.5×10^{-3}	Hydrogen gas (1 atm)	-2.2×10^{-9}
Tungsten	6.8×10^{-5}	Nitrogen gas (1 atm)	-6.7×10^{-9}
Air (1 atm)	3.6×10^{-7}	Water	-9.1×10^{-6}

Table 12.2 Magnetic Susceptibilities *Note: Unless otherwise specified, values given are for room temperature.

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12

Járnseglun

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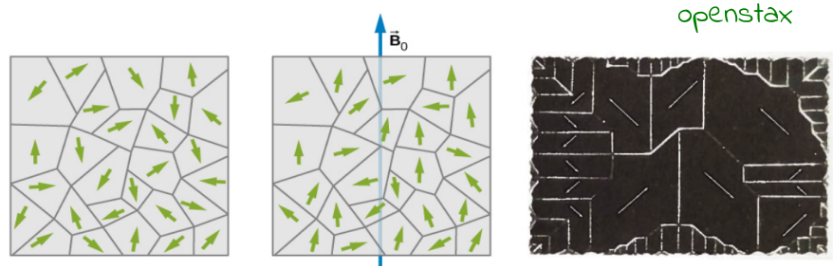
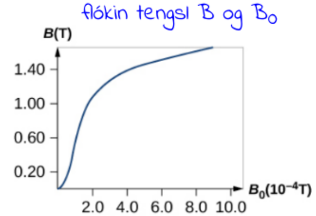


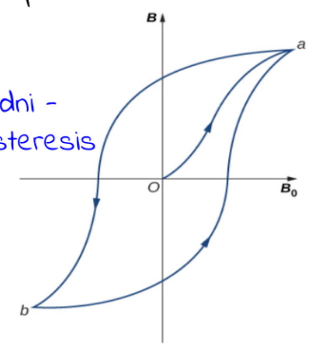
Sterkur skiptakraftur raðar segultvískautum (spunum..) í óðul (domains)

flókin tengsl B og B_0



The magnetic field B in annealed iron as a function of the applied field B_0.

Heldni - hysteresis



13

Samanburður segulsvið spólu og síseguls

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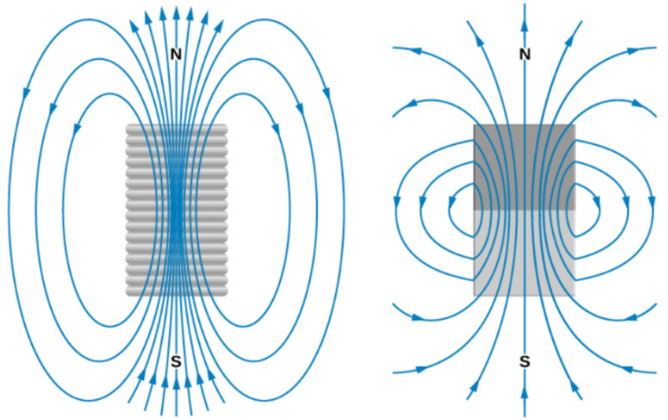

Figure 12.27 Comparison of the magnetic fields of a finite solenoid and a bar magnet.

Segulsviðslínurnar enda og byrja hvergi -- segultvískaut

14

Spurningar - samanburður

Höfum lögmál Coulombs

$$|\bar{F}_e| = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

og fyrir þyngdarkraftinn

$$|\bar{F}_g| = G\frac{|m_1 m_2|}{r^2}$$

Sama lögmál, um báða kraftana gildir lögmál Gauß --->

Hvar er segulþáttur þyngdarsviðsins?

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Til gamans: Hve langt nær samanburður á þyngdarfræði og rafsegulfræði?

Maxwell

$$\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0}$$

$$\bar{\nabla}\cdot\bar{B} = 0$$

$$\bar{\nabla}\times\bar{E} = -\frac{\partial}{\partial t}\bar{B}$$

$$\bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial}{\partial t}\bar{D}$$

$$\bar{D} = \epsilon\bar{E}$$

$$\bar{H} = \frac{1}{\mu}\bar{B}$$

$$\bar{F}_e = q(\bar{E} + \bar{v}\times\bar{B})$$

Almennu sviðsjöfnur Einsteins -> gerðar línulegar

$$\bar{F}_g = m(\bar{g} + 4\bar{v}\times\bar{b})$$

$$\bar{\nabla}\cdot\bar{g} = -4\pi G\rho$$

$$\bar{\nabla}\cdot\bar{b} = 0$$

$$\bar{\nabla}\times\bar{g} = -\frac{\partial}{\partial t}\bar{b}$$

$$\bar{\nabla}\times\bar{b} = -\frac{4\pi G}{c^2}\bar{J}_g + \frac{1}{c^2}\frac{\partial}{\partial t}\bar{g}$$

segulhluti þyngdarsviðs

GPS

þyngdarbylgjur?

En viðhöldum okkur við stöðufræðina