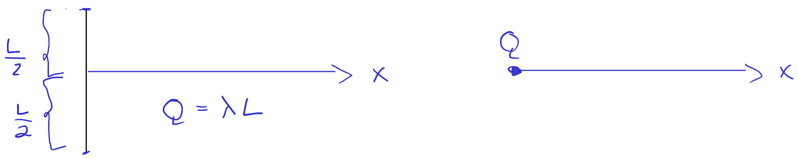
Problem 1: (11-07-54)

Compare the electrical potential from a point charge and a short line charge



Point charge:

$$V_p(x) = \frac{1}{4\pi\epsilon_0}\frac{Q}{x}, \quad V_p(\infty) = 0 \leftarrow \text{boundary condition}$$

line charge:

Use Ex. 7.13 $\rightarrow$ $V_L(x) = \frac{1}{4\pi\epsilon_0}\lambda \ln\left[\frac{L + \sqrt{L^2 + 4x^2}}{-L + \sqrt{L^2 + 4x^2}}\right]$

Same boundary condition, $\lambda L = Q$

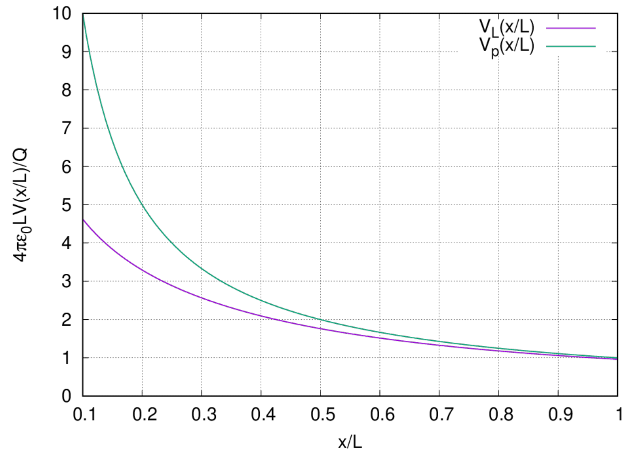
(1)

rewrite

$$V_L(x) = \frac{Q}{4\pi\epsilon_0 L}\ln\left[\frac{\sqrt{1 + 4\left(\frac{x}{L}\right)^2} + 1}{\sqrt{1 + 4\left(\frac{x}{L}\right)^2} - 1}\right] \rightarrow$$

plot $\left(\frac{4\pi\epsilon_0 L V}{Q}\right)$ vs. $\frac{x}{L}$ dimensionless

$$V_p(x) = \frac{Q}{4\pi\epsilon_0 L}\left(\frac{L}{x}\right)$$

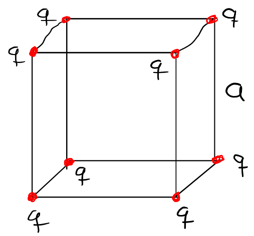


close to the charge the potential of the point charge is stronger. At distance $x/L = 1$ both have the same value, and at infinity they have the same value o

why? Yes, as far away both tried to look like a point charge. This would never be true for an infinite line charge, which does not have any natural length scale

(2)

Problem 2: (11-07-74)



$q = +3\mu C$ Find the energy of
$a = 2\,cm$ the configuration

$$W = \frac{k}{2}\sum_{\substack{i,j \\ i \neq j}}^{N}\frac{q_i q_j}{r_{ij}}$$

sum over pairs, does not matter where the origin of the coordinate system is,

7·8/2 pairs

$$W = kq^2\left\{\frac{3}{a} + \frac{3}{\sqrt{2}\,a} + \frac{1}{\sqrt{3}\,a}\right\}\cdot 28$$

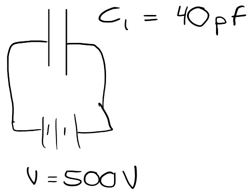
$$= \frac{kq^2}{a}\left\{3 \cdot \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right\}\cdot 28$$

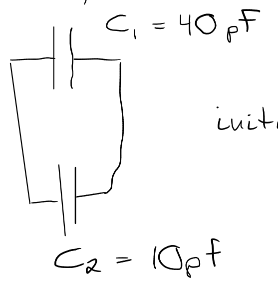
$$\simeq \frac{kq^2}{a}\cdot 160$$

(3)

Problem 3: (11-08-38)

Initially $\rightarrow$ Finally





initially $Q_2 = 0$

a) Find Q_1^i

$$C_1 = \frac{Q_1^i}{V_1^i} \rightarrow Q_1^i = V_1^i C_1$$

b) Find Q_1^f and Q_2^f

c) $V_1^f = V_2^f$: equilibrium, we us this in b)

(4)

$$V_1^f = \frac{Q_1^f}{C_1} \quad \text{and} \quad V_2^f = \frac{Q_2^f}{C_2}$$

$V_1^f = V_2^f$ →

$$\frac{Q_1^f}{C_1} = \frac{Q_2^f}{C_2} \quad (1)$$

and Conservation of charge →

$$Q_1^f + Q_2^f = Q_1^i \quad (2)$$

Two linear equations for the two unknown quantities Q_1^f and Q_2^f

(1) → $Q_2^f = Q_1^f \frac{C_2}{C_1}$ use in (2)

→ $Q_1^f + Q_1^f \frac{C_2}{C_1} = Q_1^i$ → $Q_1^f \left\{1 + \frac{C_2}{C_1}\right\} = Q_1^i$

→ $$Q_1^f = \frac{Q_1^i}{1 + \frac{C_2}{C_1}} \qquad Q_2^f = \frac{Q_1^i \frac{C_2}{C_1}}{1 + \frac{C_2}{C_1}}$$

Problem 4: (11-09-58)

100 W incand. ~ 16 W LED in terms of Light

1 kWhr = \$ 0.10

4 hr per day in one year

→ $P_{LED} \cdot 4 \cdot 365 = 16 \cdot 4 \cdot 365$ = Energy

= 23 360 Whr

= 23.360 kWhr

→ Cost = \$ 2.34