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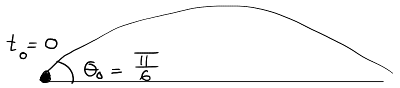
Problem 1: (1-09-34)



$v_0 = 25\ m/s$ No Friction

$M = 0.250\ kg$

find $\bar{p}(t = 0.2\ s)$

$v_{0x} = v_0 \cos\theta_0$

$v_{0y} = v_0 \sin\theta_0$

$v_x(t) = v_{0x}$

$v_y(t) = v_{0y} - gt$

$p_x = M \cdot v_{0x} = M v_0 \cos\theta_0$

$p_y = [v_{0y} - gt] M = M\{v_0 \sin\theta_0 - gt\}$

$$|\bar{p}| = \sqrt{p_x^2 + p_y^2} = M\sqrt{v_0^2\cos^2\theta_0 + \{v_0\sin\theta_0 - gt\}^2}$$

$$= \sqrt{\left[v_0^2 - 2gt\,v_0\sin\theta_0 + (gt)^2\right]}$$

The angle θ depends on time, $\theta_0 = \theta(0)$

②

$$\theta(t) = \arctan\left(\frac{p_y}{p_x}\right) = \arctan\left\{\frac{v_0\sin\theta_0 - gt}{v_0\cos\theta_0}\right\}$$

$$|\bar{p}| = 0.25\sqrt{\left[(25)^2 - 2 \cdot 9.81 \cdot 0.2 \cdot 25\sin\left(\frac{\pi}{6}\right) + (9.81 \cdot 0.2)^2\right]}\ kg\frac{m}{s}$$

$\approx 6.0\ kg\ m/s$

$$\theta(2s) = \arctan\left\{\frac{25 \cdot \sin\left(\frac{\pi}{6}\right) - 9.81 \cdot 0.2}{25 \cdot \cos\left(\frac{\pi}{6}\right)}\right\} \simeq 0.45$$

$\simeq 26°$

So, the angle $\theta(t=0.2\ s)$ is reduced from the initial value, but is still positive. At the top of the track it is 0, and then turns negative after that

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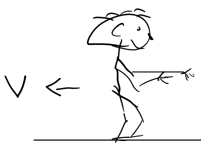
Problem 2: (1-09-50)



$m = 1.0\ kg$

$\circ \longrightarrow v = 6.7\ m/s$

$M = 65\ kg$

I assume V_0 of M to be 0 initially

conservation of momentum

$$\bar{p}_M + \bar{p}_m = 0$$

$$\rightarrow MV + mv = 0 \rightarrow \boxed{V = -\frac{m}{M}v}$$

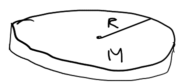
$$V = -\frac{1.0\ m/s}{65\ m/s}\ 6.7\ \frac{m}{s} \simeq -0.10\ \frac{m}{s}$$

You slip on the ice in opposite direction to the ball

④

Problem 3: (1-10-62)

Disk of a sander



$R = 0.10\ m$ $\nu = 15\ \frac{rev}{s}$

$M = 0.7\ kg$ $\omega = 2\pi\nu$

a) when sanding ω decreases by 20% $\rightarrow \omega_1 = \omega_0 \cdot 0.8$

Find $(E_{kin})_1 = \frac{1}{2}I\omega_1^2$, $I = \frac{1}{2}MR^2$ (Ex. 10.5)

$$(E_{kin})_1 = \frac{1}{4}MR^2\left(\omega_0 \cdot 0.8\right)^2 = \frac{1}{4}0.7 \cdot 0.10^2(2\pi \cdot 15 \cdot 0.8)^2$$

$\approx 9.95\ J$

b) How large is the change in the kinetic energy from ω_0 to ω_1 ?

$$\Delta E_{kin} = (E_{kin})_1 - (E_{kin})_0 = \frac{1}{4}MR^2\omega_0^2 - \frac{1}{4}MR^2\omega_1^2$$

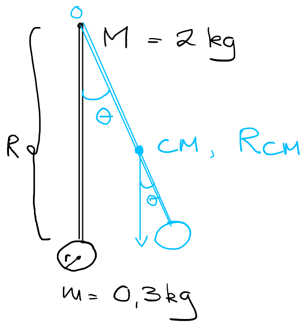
$$= \frac{1}{4}MR^2\omega_0^2\left(1 - 0.8^2\right)$$

$$= (E_{kin})_0 \cdot 0.36$$

--> decreased by 64%

5

Problem 4: (1-10-68)



$R = 1m$, originalli $\theta = \frac{\pi}{6}$, $v_0 = 0$

Find $\omega_1 = \omega(\theta = 0)$

$$I = I_r + I_{sph}$$

$$I_r = \frac{1}{3}MR^2, \quad I_{sph} = \frac{2}{5}mr^2 + m(R+r)^2$$

$$R_{CM} = \frac{\frac{1}{2}MR + (R+r)m}{M+m}$$

I use the energy conservation, as both the torque and the angular acceleration are not constant.

measured from bottom, where it is 0

$$E_{pot}(0) = R_{CM} \cdot \left\{1 - \cos\theta\right\} Mg \longrightarrow \Delta E_{pot} = R_{CM}\left\{1 - \cos\theta\right\} Mg$$

6

$$E_{kin}\left(\theta = \frac{\pi}{6}\right) = 0$$

$$\rightarrow \Delta E_{kin} = \frac{1}{2}\left[I_r + I_{sph}\right]\omega_1^2$$

conservation of the energy

$$\rightarrow \Delta E_{pot} = \Delta E_{kin} \rightarrow R_{CM}\left[1 - \cos\theta\right] Mg = \frac{1}{2}\left\{I_r + I_{sph}\right\}\omega_1^2$$

$$\rightarrow \omega_1 = \sqrt{\frac{2(1-\cos\theta)\, R_{CM}\, Mg}{(I_r + I_{sph})}}$$

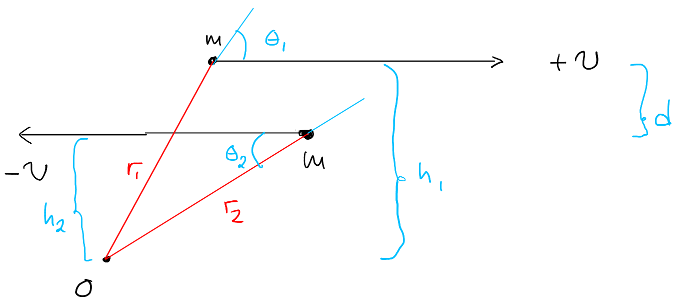
to check, dimension

$$[\omega_1] = \frac{1}{T} = \sqrt{\frac{L\, M\, L}{T^2\, M\, L^2}} = \frac{1}{T} \qquad \text{ok}$$

7

$$\boxed{\omega_1 = \sqrt{\frac{2(1-\cos\theta)\cdot\left(\frac{M}{2}R + m(R+r)\right) Mg}{(M+m)\left\{\frac{MR^2}{3} + \frac{2mr^2}{5} + m(R+r)^2\right\}}} \approx 0.99\ 1/s}$$

Problem 5: (1-11-40)



Remember

$$\bar{L}_1 = \bar{r}_1 \times \bar{p}_1$$

$$L_1 = r_1 p_1 \sin\theta_1 = p_1 \cdot h_1$$

8

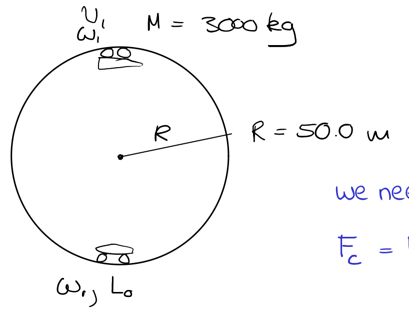
$$\bar{L} = \bar{L}_1 + \bar{L}_2 \qquad \bar{L}_1 \text{ and } \bar{L}_2 \text{ have opposite directions}$$

$$|\bar{L}| = h_1 m v - h_2 m v = m v\left\{h_1 - h_2\right\}$$

$$\text{but, } |h_1 - h_2| = d$$

Thus we will always have the same angularmomentum for the system, independent of the choice we make for the reference point o

Problem 6: (1-11-50)



Find minimum L_o for the roller coaster to stay on the track

we need at least gravity to supply

$$F_c = MR\omega_1^2 = M\frac{v_1^2}{R}$$

So, minimum angular frequency

$$W = Mg = MR\omega_1^2 \longrightarrow \omega_1^2 = \frac{g}{R} \longrightarrow L_1 = R^2M\sqrt{\frac{g}{R}}$$

$$L_1 = R\cdot M\underbrace{(\omega_1 R)}_{v_1} = MR^2\omega_1$$

$$L_o = MR^2\omega_o$$

Energy conservation

$$E_1 = E_0$$

$$\frac{1}{2}M(\omega_1 R)^2 + gM2R = \frac{1}{2}M(\omega_0 R)^2$$

$$\rightarrow (\omega_1 R)^2 + g4R = (\omega_0 R)^2$$

$$\rightarrow (\omega_0 R) = \sqrt{(\omega_1 R)^2 + g4R}$$

$$\rightarrow L_0 = MR^2\omega_0 = MR\sqrt{(\omega_1 R)^2 + 4gR}$$

So,

$$L_0 \geq MR\sqrt{(\omega_1 R)^2 + 4gR} = MR\sqrt{gR + 4gR} = MR\sqrt{5gR}$$

$$L_0 \geq 7.43\cdot 10^6\ \frac{kg\,M^2}{s}$$