1

I-01-50

Check which equations for volume v and area A are dimensionally consistent

a) $V = \pi r^2 h$, $[V] = L^2 \cdot L = L^3$, OK

b) $A = 2\pi r^2 + 2\pi r h$, $[A] = L^2 + L \cdot L = L^2$, OK

c) $V = 0{,}5\, bh$, if $[b] = L \rightarrow [V] = L \cdot L = L^2$, not OK

d) $V = \pi d^2$, $[V] = L^2$, not OK

e) $V = \pi d^3/6$, $[V] = L^3$, OK

1-01-53

$[s] = L$, $[t] = T$, $v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$

$\rightarrow$

a) $[v] = \frac{L}{T}$

b) $[a] = \frac{L}{T} \cdot \frac{1}{T} = \frac{L}{T^2}$

2

c) $\left[\int v\,dt\right] = \frac{L}{T} T = L$

d) $\left[\int a\,dt\right] = \frac{L}{T^2} \cdot T = \frac{L}{T}$

e) $\left[\frac{da}{dt}\right] = \frac{L}{T^2} \cdot \frac{1}{T} = \frac{L}{T^3}$

f) $E_{kin} = \frac{1}{2} m \left(\frac{dv}{dt}\right)^2 \rightarrow [E_{kin}] = M \frac{L^2}{T^2}$

g) $E_{pot} = \frac{1}{2} m (\omega x)^2 \rightarrow [E_{pot}] = M \frac{1}{T^2} L^2$, as $[\omega] = \frac{1}{T}$

so the different forms of energy we will see later all have the same dimension

3

1-01-64

Estimate the mass of a virus. Lets take C-19, it has close to spherical shape

In https://www.ncbi.nlm.nih.gov/pmc/articles/PMC7224694/ we see that the diameter of the C-19 virus is approximately 100 nm, $d = 100\ nm = 100 \cdot 10^{-9}\ m = 10^{-7} m$

$$V = \frac{4\pi}{3} r^3 = \frac{4\pi}{3} \left(\frac{d}{2}\right)^3 = \frac{4\pi}{3 \cdot 8} d^3 \approx 0.52 \cdot 10^{-21}\ m^3$$
$$\approx 0.5 \cdot 10^6\ nm^3$$

we estimate the virus to have density close to water

$\rho_{H_2O} = 1000\ kg/m^3$ $\qquad m = \rho V \simeq 1000 \frac{kg}{m^3} \cdot 0{,}5 \cdot 10^{-21}\ m^3$
$$\simeq 0{,}5 \cdot 10^{-18}\ kg = 0{,}5\ fg$$

So, we estimate the mass of a C-19 to be 0.5 fg half a femtogram.
C, N, O, all have similar mass, and H is in water and in the virus.
For fun there is a publication estimating the total mass of all C-19 viruses during the pandemic
https://www.pnas.org/doi/10.1073/pnas.2024815118

4

1-02-70

a) If $\bar{A} \times \bar{F} = \bar{B} \times \bar{F}$ is then $\bar{A} = \bar{B}$?

Remember that for two vectors $\bar{G}$ and $\bar{H}$ parallel or antiparallel means that $\bar{G} \times \bar{H} = 0$, $\bar{H} \times \bar{G} = 0$

So, we select $\bar{D}$ parallel to $\bar{F}$ then

$(\bar{A} + \bar{D}) \times \bar{F} = \bar{A} \times \bar{F}$, but $\bar{A} + \bar{D} \neq \bar{A}$ generally

b) what about $\bar{A} \cdot \bar{F} = \bar{B} \cdot \bar{F}$ is then $\bar{A} = \bar{B}$?

Now we select $\bar{D}$ that is perpendicular to $\bar{F}$ $\rightarrow$ $\bar{D} \cdot \bar{F} = 0$

$(\bar{A} + \bar{D}) \cdot \bar{F} = \bar{A} \cdot \bar{F}$, but $\bar{A} + \bar{D} \neq \bar{A}$ generally

c) If $F\bar{A} = \bar{B}F$ is then $\bar{A} = \bar{B}$

F is a scalar $\rightarrow$ $\bar{B}F = F\bar{B}$

$F(\bar{A} - \bar{B}) = 0$, if $F \neq 0$ then $\bar{A} = \bar{B}$

①

1-03-44

Linear motion: $v(0) = 0$, $a = 30\ \mathrm{m/s^2}$ Constant

$x(0) = 0$

Find $x(t)$ at $t = 5$ s

$a = \frac{dv}{dt} \rightarrow dv = a\,dt$

integrate

$$\int_{v_0}^{v(t)} dv' = \int_0^t a(t')\,dt'$$

a Constant

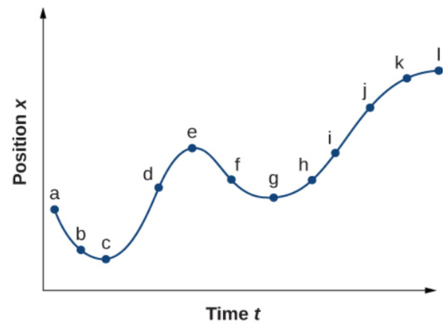
$$\underbrace{v(t) - v(0)}_{=0} = \int_0^t a\,dt' \longrightarrow v(t) = at$$

$$\underbrace{x(t) - x(0)}_{=0} = \int_0^t v(t')\,dt' = a\int_0^t t'\,dt' = a\frac{t^2}{2}$$

$$\rightarrow x(t) = \frac{1}{2}at^2 \qquad \rightarrow x(5) = \frac{30}{2}\frac{\mathrm{m}}{\mathrm{s^2}}\,25\ \mathrm{s^2} = 375\ \mathrm{m}$$

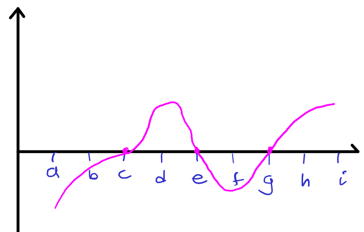
②

1-03-46



a) Sketch the corresponding v(t) graph

$$v(t) = \frac{dx(t)}{dt}$$



very approximately sketchy...

b) Max values for v(t) occur at

t_a, t_d, t_i, t_j

c) when is $v(t) = 0$?

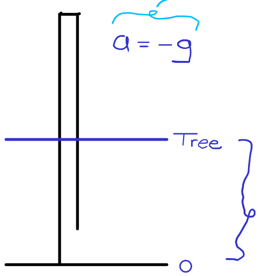
t_c, t_e, t_g, t_l

d) $v(t) < 0$ for t_b, t_f, t_a

③

1-03-74

constant acceleration



$a = -g$

$h = 7.0$ m

$v_0 = v(0) = 15.0\ \mathrm{m/s}$

$x(0) = 0$

want to find Δt, the time interval during which the ball is above the tree branch

$$x(t) = v_0 t - \frac{1}{2}gt^2$$

parabola in t

During the trip of the ball we will hvave twice

$$h = v_0 t - \frac{1}{2}gt^2$$

$$\rightarrow \frac{g}{2}t^2 - v_0 t + h = 0$$

or

$$\boxed{t^2 - \frac{2v_0}{g}t + \frac{2h}{g} = 0}$$

which has two solutions

④

The roots are

$$t = \frac{2v_0}{2g} \pm \frac{1}{2}\sqrt{\left(\frac{2v_0}{g}\right)^2 - 4\frac{2h}{g}}$$

$$= \frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 - \frac{2h}{g}}$$

$$\rightarrow \quad \Delta t = 2\sqrt{\left(\frac{v_0}{g}\right)^2 - \frac{2h}{g}}$$

$$= 2\sqrt{\left(\frac{15}{9.81}\right)^2 \mathrm{s^2} - \frac{2\cdot 7}{9.81}\mathrm{s^2}} \approx 1.91\ \mathrm{s}$$

1-04-44

(5)

Max throw range of a boy is 50 m, assume always the same initial speed and find the max height

$$\underbrace{R = \frac{v_0^2 \sin(2\theta_0)}{g}}_{}$$

max R is for $\theta_0 = 45$ as then $\sin(2\theta_0)$ takes a max value

$$\rightarrow v_0^2 = gR$$

Throw straight up

$$h = v_0 t - \frac{1}{2} g t^2 = \sqrt{gR}\, t - \frac{g}{2} t^2$$

Max height when $\frac{dh}{dt} = 0 \rightarrow \sqrt{gR} - g t_m = 0$

(6)

$$\rightarrow t_m = \frac{\sqrt{gR}}{g} = \sqrt{\frac{R}{g}}$$

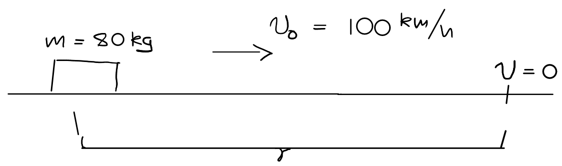
$$h_m = h(t_m) = \sqrt{gR}\sqrt{\frac{R}{g}} - \frac{g}{2}\frac{R}{g}$$

$$= R - \frac{R}{2} = \frac{R}{2} = 25\ m$$

Think, no airresistance, the motion is symmetric in x. The angle is 45 degrees is the answer R/2 then not realistic?

(1)

1-05-36



stops in 45.0 m

Find the force of the seat belt on the passanger.
we approximate by assuming constant acceleration

100 km/h ≈ 27,8 m/s

$$v = v_0 - at \quad \text{or} \quad v^2 = v_0^2 - 2ad$$

$$\rightarrow 0 = v_0^2 - 2ad \rightarrow v_0^2 = 2ad \rightarrow a = \frac{v_0^2}{2d}$$

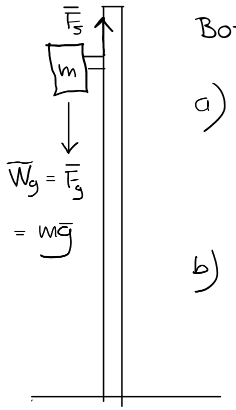
$$F = ma = m\frac{v_0^2}{2d} = \frac{80\,\text{kg}\,(27{,}8)^2\,\text{m}^2/\text{s}^2}{2 \cdot 45{,}0\ \text{m}}$$

$$= 687\ \text{kg}\ \frac{\text{m}}{\text{s}^2} = \underline{687\ \text{N}}$$ in direction ← as $\bar{a}$

(2)

1-05-48

Fireman slides down a pole with acceleration $|\bar{a}| < |\bar{g}|$



Both foces needed are vertical

a) $ma = F_s - mg$, $a < 0$

$$\rightarrow F_s = ma + mg = m(a+g)$$

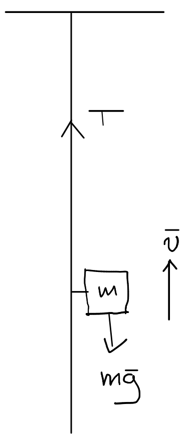
b) $m = 90{,}0$ kg , $a = -5{,}00\ \text{m/s}^2$

$$F_s = m(a+g) = 90{,}0\{-5{,}00 + 9{,}81\}\ \frac{\text{m}}{\text{s}^2}\text{kg}$$

$$= \underline{433\ \text{N}}$$

(3)

1-05-60



a) Find T in the rope if v = constant, a = 0,
mass less rope
$m = 60{,}0$ kg

$$am = T - mg$$

$$\text{if } a = 0 \rightarrow T = mg = 60 \cdot 9{,}81\ \text{N}$$

$$= \underline{589\ \text{N}}$$

b) $am = T - mg$, now $a = 1{,}50\ \text{m/s}^2$

$$\rightarrow T = m(a+g)$$

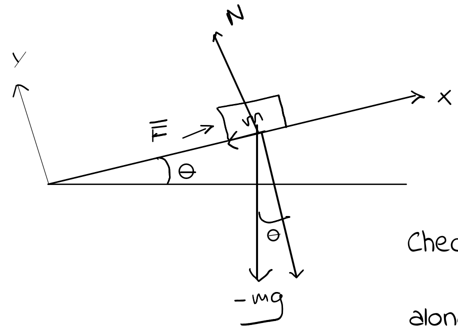
$$= 60 \cdot \{1{,}50 + 9{,}81\}\ \text{N}$$

$$= \underline{679\ \text{N}}$$

(4)

1-05-66

$m = 100$ kg $\theta = 30°$, or $\frac{\pi}{6}$ rad



How large force F do we need to push the crate up the slope with acceleration a?

$a = 2{,}0\ \text{m/s}^2$

Check the forces:

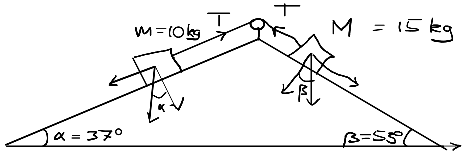
along y-direction: $N - mg\cos\theta = 0$

along x: $F - mg\sin\theta = am \rightarrow F = am + mg\sin\theta$

$$= m\{a + g\sin\theta\}$$

$$\rightarrow F = 100\{2{,}0 + 9{,}81 \cdot \sin(\tfrac{\pi}{6})\}\ \text{kg}\frac{\text{m}}{\text{s}^2} = \underline{691\ \text{N}}$$

1-06-44



(m): $-mg\sin\alpha + T = am$

(M): $Mg\sin\beta - T = aM$

we can add the two equations to eliminate T:

$$-mg\sin\alpha + Mg\sin\beta = am + aM$$

$$\rightarrow \quad a = \frac{g\{M\sin\beta - m\sin\alpha\}}{m+M}$$

$a = 2{,}34\ m/s^2$

notice how the direction of the acceleration depends on m, M, and the angles

To find T we obviously can then subtract the equations

$\{ (m) - (M) \}$ symbolically

$$-mg\sin\alpha - Mg\sin\beta + 2T = a\{m - M\}$$

$$\rightarrow \quad -g\{m\sin\alpha + M\sin\beta\} + 2T = \frac{g\{M\sin\beta - m\sin\alpha\}(m-M)}{m+M}$$

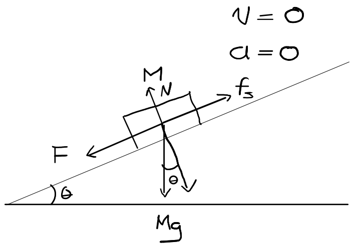
$$\rightarrow \quad T = \underbrace{\frac{g}{2}\{m\sin\alpha + M\sin\beta\}}_{T \text{ for } a = 0} + \underbrace{\frac{g}{2}\{M\sin\beta - m\sin\alpha\}\frac{m-M}{m+M}}_{\text{This term vanishes if } a = 0}$$

or we could have used

$a \rightarrow (m)$: $\quad -mg\sin\alpha + T = \frac{mg}{m+M}\{M\sin\beta - m\sin\alpha\}$

$$\rightarrow \quad T = mg\sin\alpha + \frac{mg}{m+M}\{M\sin\beta - m\sin\alpha\} = 82{,}4\ N$$

1-06-56



$$f_s = \mu_s N = \mu_s Mg\cos\theta$$

$$F = Mg\sin\theta$$

$$a = 0 \quad \rightarrow \quad \vec{f_s} + \vec{F} = 0$$

i.e. $\quad Mg\sin\theta = \mu_s Mg\cos\theta$

$$\rightarrow \sin\theta = \mu_s\cos\theta \quad \rightarrow \quad \tan\theta = \mu_s$$

$$\rightarrow \quad \theta = \arctan\mu_s$$

1-06-74

The Bohr model

$R = 5{,}28 \cdot 10^{-11}\,m$ for e

$v = 2{,}18 \cdot 10^{6}\ m/s$

$m_e = 9{,}11 \cdot 10^{-31}\ kg$

$$F_c = m_e\frac{v^2}{R} = \frac{9{,}11\cdot 10^{-31}(2{,}18\cdot 10^{6})^2}{5{,}28\cdot 10^{-11}} = 8{,}2\cdot 10^{-8}\ N$$

Corresponding acceleration

$$a_c = \frac{v^2}{R} = 9\cdot 10^{22}\ m/s^2\ !,$$ but . . . .

$L = 0$ in QM

1-06-88

Air resistance on a skydiver $f = -bv^2$, $v_T = 60\ m/s$

$M = 50\ kg$, Find b

Equation of motion

$$\underbrace{m\frac{dv}{dt}}_{=0} = Mg - bv^2 \longrightarrow Mg = bv_T^2$$

$$\rightarrow b = \frac{Mg}{v_T^2} = \frac{50\,kg \cdot 9{,}81\ m/s^2}{(60)^2\ \frac{m^2}{s^2}}$$

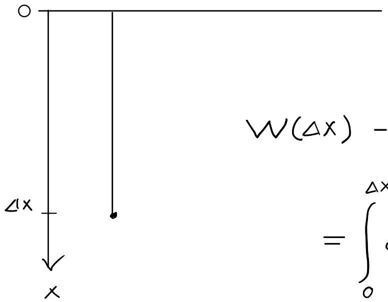
$$= 0{,}136\ kg/m$$

1-07-40

Force of a bungee cord is $\vec{F}(x) = k_1 x + k_2 x^3$, $k_1 = 204 \ \frac{N}{m}$

How much work is needed to strech it $k_2 = -0{,}233 \ \frac{N}{m^3}$

to $\Delta x = 16{,}7$ m



$$dW = \vec{F} \cdot d\vec{r}$$

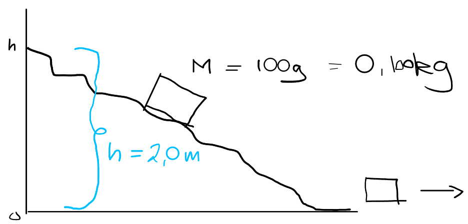
$$W(\Delta x) - W(0) = \int_0^{\Delta x} F(x)\,dx$$

$$= \int_0^{\Delta x} dx \left\{ k_1 x + k_2 x^3 \right\} = \left[k_1 \frac{x^2}{2} + k_2 \frac{x^4}{4} \right]_0^{\Delta x}$$

$$= k_1 \frac{(\Delta x)^2}{2} + k_2 \frac{(\Delta x)^4}{4}$$

$$W(16{,}7) = 204 \cdot \frac{(16{,}7)^2}{2} - 0{,}233 \cdot \frac{(16{,}7)^4}{4} = 2{,}39 \cdot 10^4 \, Nm = \underline{23{,}9 \, kNm}$$

1-07-64



$\to v = 4{,}0 \ m/s$

How much work does the work of friction do?

Total energy is conseved. Note initial with "i" and final with "f"

$$E^i_{pot} = Mgh, \quad E^f_{pot} = 0, \quad \to \Delta E_{pot} = E^f_{pot} - E^i_{pot} = -Mgh$$

$$E^i_{kin} = 0, \quad E^f_{kin} = \frac{1}{2} M v^2, \quad \to \Delta E_{kin} = \frac{1}{2} M v^2$$

If there was no resistance, then

$$\Delta E_{Total} = 0 = \Delta E_{kin} + \Delta E_{pot} = \frac{1}{2} M v^2 - Mgh$$

but we get

$$\Delta E_{Total} = -1{,}16 \ Nm \to \underline{-1{,}16 \ Nm}$$ is the work done by the friction

1-08-26

$$U(x) = -\frac{a}{x} + \frac{b}{x^2}$$

$$F = -\frac{dU(x)}{dx} = -\frac{a}{x^2} + \frac{2b}{x^3}$$

hvaða kraftur gæti þetta verið?

1-08-36 Tarsan jumps onto a vine with v = 9.0 m/s

a) how high can he swing?

$$E^i_k = \frac{1}{2} M v^2$$

the highest he could get is if all the kinetic energy is changed into potential energy

$$E_{pot} = Mgh = \frac{1}{2} M v^2 \to gh = \frac{1}{2} v^2 \to h = \frac{v^2}{2g}$$

$$= \frac{9^2}{2 \cdot 9{,}81} \ m$$

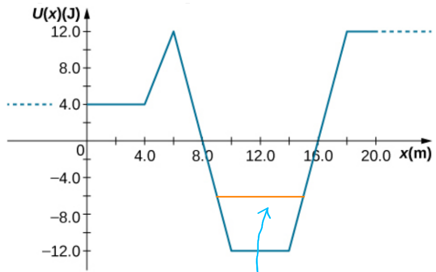
$$= \underline{4{,}13 \ m}$$

Independent of M

b) Does the length of the vine influence h?

Not if $L > h$, otherwise Tarsan could be in trouble.

1-08-50



$$F(x) = -\frac{dU}{dx}$$

a) Find F(x) for some values of x

$$F(2) = 0$$

$$F(5) = -\frac{12-4}{6-4} N = -4 \ N$$

$$F(8) = -\frac{(-12-12)}{10-6} = +6 \ N$$

$$F(12) = 0$$

b) If the total energy of a particle is -6.0 J, find min and max x for the motion of the particle

$\underline{x_{min} = 9 \, m, \quad x_{max} = 15 \, m}$ bound motion

c) If $E_T = 2{,}0 J$, bit more difficult, find the slope in the region $\pm 6 \ J/m$

$$\to x_{min} = \left(8 - \frac{1}{3}\right) m, \quad x_{max} = \left(16 + \frac{1}{3}\right) m$$

5

d) If the total energy is 16 J, what is the velocity of the partice at x = 2, 5, 8, 12?

$E_T = 16\,J$, $E_{pot} = U(x)$, $E_k = \frac{1}{2} m v^2$

$$E_T = \frac{1}{2} m v^2 + U(x) \rightarrow \frac{1}{2} m v^2 = E_T - U(x)$$

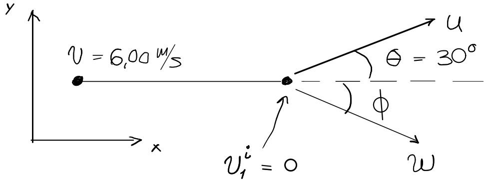
$$\rightarrow v(x) = \sqrt{\frac{2}{m}\left\{E_T - U(x)\right\}}$$

$m = 0{,}50$ kg

$$v(2) = \sqrt{\frac{2}{0{,}50}\left\{16 - 4\right\}}\ m/s = 6{,}9\ m/s$$

1-09-36 ①

Identical pucks on a hocky air table $m_1 = m_2 = m$ Elastic collision, $\Delta E_T = 0$



$\Delta \bar{p} = 0$, no external force

Find w and ϕ

Conservation of momentum

(y): $\quad mu\sin\theta + mw\sin\phi = 0 \qquad ①$

(x): $\quad mv + 0 = mu\cos\theta + mw\cos\phi \qquad ②$

$\theta = \frac{\pi}{6} \rightarrow \sin\theta = \frac{1}{2}, \quad \cos\theta = \frac{\sqrt{3}}{2}$

Conservation of energy

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mu^2 + \frac{1}{2}mw^2 \qquad ③$$

②

3 equations, we need to find ϕ and w (and u)

Simplify the equations

①: $\quad \frac{u}{2} + w\sin\phi = 0$

②: $\quad v = \frac{\sqrt{3}}{2}u + w\cos\phi$

③: $\quad v^2 = u^2 + w^2$

This is not a system of coupled linear equations, but many routs can be taken to find a solution. I want to start to find the angle ϕ

① $\rightarrow \quad u = -2w\sin\phi \quad \rightarrow$ ②

$\rightarrow \quad v = -\sqrt{3}\,w\sin\phi + w\cos\phi = \underline{w\{\cos\phi - \sqrt{3}\sin\phi\}}$

① $\rightarrow$ ③ $\rightarrow \quad v^2 = 4\sin^2\phi \cdot w^2 + w^2 = \underline{w^2\{1 + 4\sin^2\phi\}}$

These 2 equation I combine to get an equation only for the angle ϕ

③

$$\{\cos\phi - \sqrt{3}\sin\phi\}^2 = 1 + 4\sin^2\phi$$

$\rightarrow \quad \cos^2\phi + 3\sin^2\phi - 2\sqrt{3}\cos\phi\cdot\sin\phi = 1 + 4\sin^2\phi$

use the fact that $\cos^2\phi + \sin^2\phi = 1$

$\rightarrow \quad \sin^2\phi = -\sqrt{3}\sin\phi\cdot\cos\phi \quad \rightarrow \quad \sin\phi = -\sqrt{3}\cos\phi$

$\rightarrow \quad \phi = \arctan(-\sqrt{3}) \quad \rightarrow \quad \underline{\underline{\phi = -60^\circ}}$

This we use in ① and ②

$$\rightarrow \boxed{\begin{array}{l} \frac{u}{2} - \frac{\sqrt{3}}{2}w = 0 \\ \frac{\sqrt{3}}{2}u + \frac{w}{2} = v \end{array}}, \quad v = 6{,}00\ m/s$$

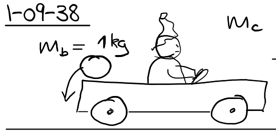
The solution of this set of linear equations gives

$u = (\sqrt{3})^3 \simeq 5{,}20\ m/s$

$\underline{\underline{w = 3\ m/s}}$

④

1-09-38



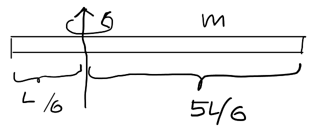
$m_c = 12\ kg$

$\rightarrow \quad v = 3{,}3\ m/s$

$M_w = 4{,}0\ kg$

The child releases a ball at $t = 0$ with no velocity relative to the wagon. The momentum of the total system before $t = 0$ is the same as the momentum immediately after the release. There will be no change in the velocity of the wagon and the child.

1-10-66



Find the momentum of inertia of the system with direct integration

$I = \int r^2 dm \qquad dm = \left(\frac{m}{L}\right)dx$

$$\rightarrow \quad I = \int_0^{L/6} x^2 dm + \int_0^{5L/6} x^2 dm = \frac{m}{L}\left\{\int_0^{L/6} x^2dx + \int_0^{5L/6} x^2dx\right\}$$

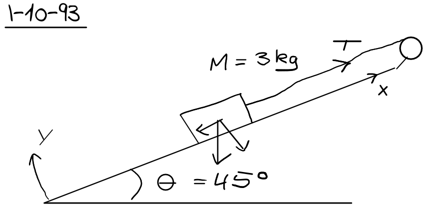
5

$$I = \frac{m}{L}\left[\frac{x^3}{3}\Big|_0^{L/6} + \frac{x^3}{3}\Big|_0^{5L/6} \right] = \frac{m}{L}\left\{ \frac{L^3}{3\cdot 6^3} + \frac{L^3\cdot 5^3}{3\cdot 6^3} \right\}$$

$$= mL^2\left\{ \frac{1}{3\cdot 6^3} + \frac{5^3}{3\cdot 6^3} \right\} = mL^2\left\{ \frac{1+125}{648} \right\}$$

$$= mL^2\left\{ \frac{63}{324} \right\} = m\frac{L^2}{12}\left\{ \frac{63}{27} \right\} = \underline{m\frac{L^2}{12}\cdot\frac{7}{3}}$$

1-10-93



For M:

x: $-Mg\sin\theta + \mu N + T = aM$ (1)

y: $-Mg\cos\theta + N = 0$ (2)

6

For m:

$\tau_T = I\alpha$, (3) $\quad a = R\alpha$, (4) $\quad I = \frac{1}{2}mR^2$, (6) $\quad \tau_T = R\cdot T$ (5)

(2) -> (1) : $\rightarrow$ $-Mg\sin\theta + \mu Mg\cos\theta + T = aM$ (*)

(3-6) $\longrightarrow$ $-RT = I\frac{a}{R} = \frac{1}{2}mR^2\frac{a}{R} = \frac{m}{2}Ra$

$\rightarrow$ $T = -\frac{m}{2}a$

The last equation can be used in (*) :

$$-Mg\sin\theta + \mu Mg\cos\theta - \frac{m}{2}a = aM$$

$$\rightarrow -Mg\left\{\sin\theta - \mu\cos\theta\right\} = a\left\{M + \frac{m}{2}\right\}$$

$$\rightarrow a = -g\left\{\frac{M}{M+\frac{m}{2}}\right\}\left\{\sin\theta - \mu\cos\theta\right\} = \underline{-3{,}6\ \ m/s^2}$$

7

1-11-30

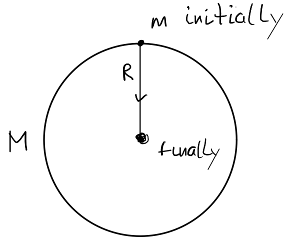
Ball with M = 40.0 kg rolls on a horizontal plane surface with velocity v = 6.0 m/s
How much work is needed to stop it

$I = \frac{2}{5}MR^2$ when turning around an axis through the center

$$E_k = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2, \qquad \omega R = v$$

$$E_k = \frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 = \underline{\frac{7}{10}Mv^2}$$

1-11-52



$M = 2{,}0$ kg

$R = 0{,}60$ m

$m = 0{,}05$ kg

$\omega_i = 2\cdot 2\pi\ s^{-1}$

Find ω_f

after m has slid to the center

8

we assume the sliding of m does not change the angular momentum of the system --> L is conserved:

$$I_i\omega_i = I_f\omega_f$$

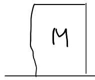
$$\frac{1}{2}R^2\left\{M + m\right\}\omega_i = \frac{1}{2}MR^2\omega_f$$

$$\rightarrow \left[M+m\right]\omega_i = M\omega_f$$

$$\rightarrow \omega_f = \frac{M+m}{M}\omega_i = (1{,}025)\,\omega_i = \underline{\underline{12{,}88\ Hz}}$$

(1)

1-14-70



$M = 80\,kg$, $\rho = 955\,kg/m^3$

a) Find V

$$M = V\rho \;\rightarrow\; V = \frac{M}{\rho} = \frac{80\,kg}{955\,kg/m^3} \approx 0{,}0838\,m^3$$

b) Find F_B due to air

$\rho_{air} = 1{,}29\,kg/m^3$ the weight of the air the man displaces is $gV\rho_{air}$

$$\rightarrow F_B = W_{air} = gV\rho_{air} = 9{,}81\,m/s^2 \cdot 0{,}0838\,m^3 \cdot 1{,}29\,\frac{kg}{m^3} = 1{,}06\,N$$

c) $$\frac{F_B}{W_M} = \frac{gV\rho_{air}}{gM} = \frac{gV\rho_{air}}{gV\rho} = \frac{\rho_{air}}{\rho} = \frac{1{,}29}{955} = 1{,}35\cdot 10^{-3}$$

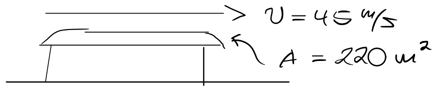
(2)

1-14-86

Wind blows over a house with roof area $A = 220\,m^2$, with speed $v = 45\,m/s$

$P_o = 8{,}89\cdot 10^4\,N/m^2$

we use Bernoullis equation and compare to wind still



$$p + \tfrac{1}{2}\rho v^2 = Const. \;\rightarrow\; P_o + 0 = p + \tfrac{1}{2}\rho v^2$$

where p is the pressure on the roof during the wind is blowing

$$\rightarrow p = P_o - \tfrac{1}{2}\rho v^2 = 8{,}89\cdot 10^4\,\frac{N}{m^2} - \tfrac{1}{2}\,1{,}14\,\frac{kg}{m^3}\cdot(45\,m/s)^2 = 8{,}775\cdot 10^4\,\frac{N}{m^2}$$

Like an upward force on the roof

$$F = (P_o - p)\cdot A = \frac{\rho v^2}{2}A = 254\,kN$$

(3)

1-14-96

Laminar flow through a pipe with fixed cross section, use Eq. (14.19)

$$Q = \frac{(p_2 - p_1)\pi r^4}{8\eta l}$$

Q is the flow rate. we have to find a) how much the flow decreases if the pipe is made narrower $\frac{\Delta r}{r} = -0{,}0500$ and b) how much it is increased if $\frac{\Delta r}{r} = +0{,}0500$

If the variation of the radius r is very small, we could have used a linear approximation built on the derivative

$$\Delta Q = \frac{(p_1 - p_2)}{8\eta l} 4\pi r^3 \Delta r = \frac{(p_1 - p_2)}{8\eta l}\pi r^4 \left(\frac{4\Delta r}{r}\right)$$

$$= Q\left(\frac{4\Delta r}{r}\right)$$

but the variation is not small here, and the authors of the book want a better estimate

(4)

$$\Delta Q = \frac{(p_1 - p_2)}{8\eta l}\pi(r + \Delta r)^4 = \frac{(p_1 - p_2)}{8\eta l}\pi r^4 \left\{1 + \frac{\Delta r}{r}\right\}^4$$

$$= Q\left\{1 + \frac{\Delta r}{r}\right\}^4$$

$$\rightarrow \frac{\Delta Q}{Q} = \left\{1 + \frac{\Delta r}{r}\right\}^4$$

if $\frac{\Delta r}{r} = -0{,}0500 \rightarrow \frac{\Delta Q}{Q} = 0{,}815$

$\sim$ 19% decrease

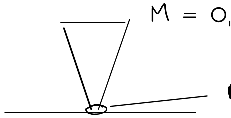
and if $\frac{\Delta r}{r} = +0{,}0500 \rightarrow \frac{\Delta Q}{Q} = 1{,}216$

$\sim$ 22% increase

Beyond a linear approximation, the results are not symmetric!

5

1-14-104



$M = 0{,}001\ \text{kg}$

$r = 0{,}200\ \text{mm} = 0{,}0002\ \text{m}$

$$P = \frac{gM}{\pi r^2} = \frac{9{,}81\ \text{m/s}^2\ \ 0{,}001\ \text{kg}}{\pi\,(0{,}0002)^2\ \text{m}^2} = 7{,}8 \cdot 10^4\ \frac{\text{N}}{\text{m}^2}$$

$$= 7{,}8 \cdot 10^4\ \text{Pa}$$

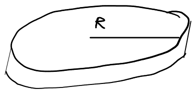
which is a huge pressure only applied to a small area probably causing wear and tear. It is probably not fair to compare it to the standard air pressure at sea level $1.013 \cdot 10^5\ \text{Pa}$

that is homogeneous to the surface of the record, and does not scratch it like the needle

1

11-01-82

Estimate the energy released by a small thunder shower due to the condensation of the evapoured steam into liquid water



$h = 1{,}0\,cm$ $\qquad V = \pi R^2 h, \quad M = V\rho = \pi R^2 h \rho$

$R = 10^3\,m \qquad L_v^{H_2O} \sim 2256\ kJ/kg$

the energy released is

$$E = M L_v^{H_2O} = \pi R^2 \underbrace{h\rho} L_v^{H_2O}$$

$$\approx \pi (10^3)^2 \cdot 0{,}01 \cdot 1000 \cdot 2256 = 7 \cdot 10^{10}\ kJ = \underline{70\,TJ}$$

Compare to an earthquake of magnitude 6.0 Richter releases 63 TJ

2

11-01-100

A home owner adds $\Delta d = 8.0$ cm to the insulation layer of the attic with $d = 15$ cm How much does this improve the insulation of the house

Fiber glass: $\quad k = 0{,}042 \dfrac{W}{(m \cdot {}^\circ C)}$

we have for the power dissipating from the house

$$P = P_{sides} + \frac{kA(T_h - T_c)}{(d + \Delta d)}$$

At the moment we do not worry about P_{sides}, but we know it is also proportional to $(T_h - T_c)$, $\quad P_{sides} = \beta (T_h - T_c)$

we notice that $\Delta d / d$ is by no means small!

3

$$P = P_{sides} + \frac{kA(T_h - T_c)}{d\left(1 + \frac{\Delta d}{d}\right)}$$

$$= P_{sides} + \frac{kA(T_h - T_c)}{d}\left\{1 + \frac{\Delta d}{d} + \left(\frac{\Delta d}{d}\right)^2 - \left(\frac{\Delta d}{d}\right)^3 + \left(\frac{\Delta d}{d}\right)^4 + \dots\right\}$$

$$\to P - P_0 = \frac{kA(T_h - T_c)}{d} \sum_{k=1}^{\infty} \left(-\frac{\Delta d}{d}\right)^k$$

where P_0 is the original power dissipation of the house

$$\to P - P_0 = \Delta P = \frac{kA(T_h - T_c)}{d} \sum_{k=1}^{\infty} \left(-\frac{\Delta d}{d}\right)^k$$

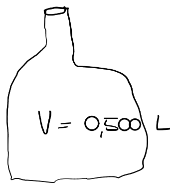
$$= \frac{kA(T_h - T_c)}{d}\left\{\frac{1}{1 + \frac{\Delta d}{d}} - 1\right\}$$

$$= -\frac{kA(T_h - T_c)}{d} \cdot 0{,}35$$

This not a small reduction and without going to further calculations we know that the area of roof is the largest surface of this house

4

11-02-30



$T_c = 25\,°C$

$T_H = 80\,°C$

$pV = N k_B T$, $\quad N = N_A n \quad \Rightarrow \quad pV = nRT$

a) Find n at T_H, open bottle

$$n = \frac{pV}{RT} = \frac{1\,atm \cdot 0{,}5\,L}{0{,}0821 \frac{L \cdot atm}{mol \cdot K} \cdot (273 + 80)}$$

$$= \underline{0{,}0173\ mol}$$

b) Find p_c if the bottle is closed at T_H

$$\frac{p_c V}{T_c} = \frac{p_H V}{T_H} \longrightarrow p_c = p_H \frac{T_c}{T_H}$$

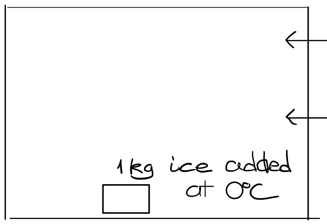
$$= 1{,}0\,atm \frac{(273 + 25)\,K}{(273 + 80)\,K}$$

$$= \underline{0{,}844\ atm}$$

11-02-62

5

Sealed room, initially at 24°C



$T_0 = 24°C$, $P_0 = 9{,}83 \cdot 10^4$ Pa

$V = 24\ m^3$

we want to find the final T_f

Closed system, the energy is conserved

The melting heat of ice $\quad L_f^{H_2O} = 334 \frac{kJ}{kg}$

Heat capacity of air: $\quad C_V^{air} = \frac{d}{2} R, \quad d = \underset{\text{translations}}{3} + \underset{\text{rotation}}{1} + \underset{\text{vibrations}}{1} = 5$

$= 2{,}5 R$

The energy of the air wil be lowered

$\Delta Q^{air} = n C_V^{air} \Delta T, \quad \Delta T = T_i - T_f$

6

The water absorbs energy

$m_{H_2O} \cdot L_f^{H_2O} + \Delta Q^{H_2O}, \qquad \Delta Q^{H_2O} = n^{H_2O} \cdot (T_f - T_i), \quad T_i = 273\ K$

Conservation of energy

$$n^{air} \cdot C_V^{air} \cdot (T_0 - T_f) = m_{H_2O} L_f^{H_2O} + n^{H_2O} C_V^{H_2O} \cdot (T_f - T_i)$$

$1\,kg\ H_2O \rightarrow n^{H_2O} = \frac{1000\,g}{18\,g} = 55{,}6\ mol$

$24\ m^3\ air \rightarrow n^{air} = \frac{pV}{RT} = \frac{0{,}97\ atm \cdot 24 \cdot 10^3\ L}{0{,}0821 \frac{L \cdot atm}{K} (273 + 24)}$

$= 955\ mol$

$1\ atm = 1.013 \cdot 10^5\ Pa$

$P_0 = 9{,}83 \cdot 10^4\ Pa = \frac{9{,}83 \cdot 10^4}{1.013 \cdot 10^5}\ atm = 0{,}97\ atm$

7

$$T_f = \frac{n^{air} \cdot C_V^{air} T_i - m_{H_2O} L_f^{H_2O} + C_V^{H_2O} T_i}{n^{air} C_V^{air} + n^{H_2O} C_V^{H_2O}}$$

$$= \frac{955 \cdot (2{,}50 \cdot 8{,}31)(273+24) - (1 \cdot 334 \cdot 10^3) + 4179 \cdot 273}{955 \cdot 2{,}50 \cdot 8{,}31 + 1 \cdot 4179}$$

$= 279\ K = \underline{6°C}$

Here I do not use
but in stead the heat capacity of
water with respect to mass, I just
have to make sure to use the
same energy units

11-03-40

Ideal gas quasi-static expands isothermally: $p, V \to 4V$

How much heat is needed

$pV = nRT, \quad dE_{int} = dQ - dW$

$E_{int} = E_{int}(T), \quad \Delta T = 0 \to \Delta E_{int} = 0$

$$dQ = dW = p\,dV = \frac{nRT}{V} \to \Delta Q = \int_V^{4V} \frac{nRT}{V'}\,dV'$$

$$\Delta Q = nRT \left[\ln V' \Big|_V^{4V} \right] = nRT \left[\ln(4V) - \ln(V) \right]$$

$$= nRT \ln(4) > 0$$

11-03-72

Ideal diatomic gas at $T_i = 80\,K$ compressed adiabatically $V \to \frac{V}{3}$

Find T_f

$pV = nRT, \quad dE_{int} = dQ - dW, \quad dW = p\,dV$

Adiabatic (óvermið) $\to dQ = 0, \; dW \neq 0 \to dE_{int}(T) \neq 0 \to \Delta T \neq 0$

In section 3.6 the authors of the book derive (3.14):

$$T V^{\gamma-1} = \text{Const.} \to T_i V^{\gamma-1} = T_f \left(\frac{V}{3}\right)^{\gamma-1}$$

$$\to T_f = T_i \left[\frac{3V}{V}\right]^{\gamma-1} = T_i \, 3^{\gamma-1}$$

$$\gamma - 1 = \frac{C_p}{C_v} - 1 = \frac{C_v + R - C_v}{C_v} = \frac{R}{C_v}$$

Diatomic (tvíatóma) kjörgas

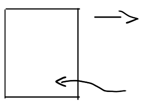
$$C_v = \frac{d}{2}R = \frac{5}{2}R, \quad \text{as } d = \underbrace{3}_{\text{translation 3D}} + \underbrace{1}_{\text{rotation}} + \underbrace{1}_{\text{Vibrations}}$$

$$\to \gamma - 1 = \frac{2R}{5R} = \frac{2}{5}$$

$$\to T_f = T_i (3)^{2/5} \approx 124\ K$$

11-04-66

Carnot refrigerator



$\to T_H = 25°C, \to Q_H$

Freezes $1{,}5\,g$ H_2O /s, $T_c = 0°C$

How much work/s or power is needed?

Latent heat for ice: $334\ kJ/kg = 334\ J/g$

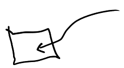
$$\to Q_c = L_f \cdot m_{ice}, \quad Q_c = K_R W, \quad K_R = \frac{T_c}{T_H - T_c}$$

$$\to W = Q_c \frac{1}{K_R} = L_f \cdot m_{ice} \frac{1}{K_R}$$

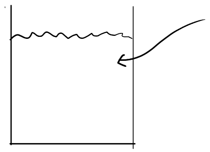
$$P = W/s = L_f \cdot \left(\frac{m_{ice}}{s}\right) \frac{1}{K_R} = 46\ J/s$$

Notice that much higher Q_H is pumped into the environment

11-04-76



$20\,g$ ice, $T_c = 0°C$



$300\,g$ H_2O, $T_H = 60°C$

Estimate the total change in entropy as the mixture comes to equilibrium at T_e

We look at this in steps, the entropy change due to:

1. Melting of the ice
2. Heat transfer from the cold water (coming from the ice) to the hot water
3. mixing of the two bodies of water

The melting is at a fixed temperature

$$\to \Delta S_{melting} = \frac{Q^{ice}}{T} = \frac{m^{ice} \cdot L_f^{H_2O}}{T_c} = \frac{20\,g \cdot 335\ J/g}{273\ K}$$

...but, this lowers the temperature of the hot water to

$$T_H \to T_H'$$

5

Isobaric heat transfer, see Ex. 4.9 in the book

$$\Delta S_c = C \cdot m_c \ln\left\{\frac{T_e}{T_c}\right\} > 0$$

$$\Delta S_H = C\, m_H \ln\left\{\frac{T_e}{T_H'}\right\} < 0$$

$$\Delta S_{T_H \to T_H'} = C\, m_H \cdot \ln\left\{\frac{T_H'}{T_H}\right\} < 0$$

The book does not cover mixing entropy, but wikipedia gives

$$\Delta S_{mix} = -nR\left\{\frac{V_H}{V_H+V_c}\ln\left(\frac{V_H}{V_H+V_c}\right) + \frac{V_c}{V_H-V_c}\ln\left(\frac{V_c}{V_H+V_c}\right)\right\}$$

But, what about T_H'

$$C m_H T_H' = C m_H T_H - L_f m_c$$

$$\rightarrow T_H' = T_H - \frac{L_f m_c}{C m_H} = (273+60) - \frac{335 \cdot 20}{4{,}19 \cdot 300} = \underline{328\,K}$$

55°C

6

I stop here, I am not sure the authors of the book had all these details in mind, but we have to have in mind, that we do not know how the ice melts in the water. The details are not accessible, and this is just an estimate of the total entropy changes. It is positive, and we could calculate it using the initial information given.

11-05-86

①

A thin conducting square with $A = LL$ $\quad Q_T = -10.0\,\mu C$, $\quad L = 2.0\,m$

we have to find E above the square at distance $z = 1$ or 2 cm $<< L$, thus we use Ex. 5.8 in the book, ignore edges and realize the field is perpendicular to the square (away from the edges). For the disk in Ex. 5.8

$$\bar{E}(z) = \frac{2\pi\sigma}{4\pi\epsilon_0}\left\{1 - \frac{z}{\sqrt{R^2+z^2}}\right\}\hat{k}$$

above the center of the disk in the xy-plane

we look at

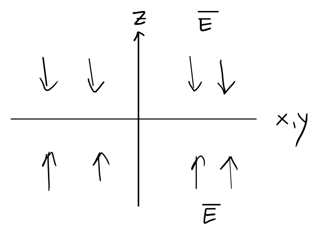
$$1 - \frac{z}{R\sqrt{1+\left(\frac{z}{R}\right)^2}} = 1 - \left(\frac{z}{R}\right)\frac{1}{\sqrt{1+\left(\frac{z}{R}\right)^2}} = f\left(\frac{z}{R}\right)$$

$$\lim_{x\to 0} f(x) = 1 \quad \longrightarrow \quad \bar{E}(z) \simeq \frac{\sigma}{2\epsilon_0}\hat{k} \quad \text{if } z << L$$

for the square in the xy-plane. It is a conductor, the charge is distributed on both sides

$$\longrightarrow \quad \sigma = \frac{Q_T}{2L^2}$$

②



as $\sigma < 0$

In next chapter we see this for an infinite conducting plate with surface charge

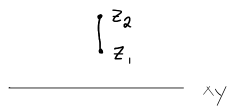
For $z > 0$: $\quad \bar{E} = -\frac{|\sigma|}{2\epsilon_0}\hat{k} = -\frac{|Q_T|}{2\epsilon_0 L^2}\hat{k}$

b) $\quad \bar{F} = -e\frac{\sigma}{2\epsilon_0}\hat{k} = |eQ_T|\frac{\hat{k}}{2\epsilon_0 L^2}$ $\quad$ repulsive for on the electron $z > 0$

c) Same answers as before as the electric field does not change with distance

d) The work by the force, when $-e$ is moved from $z_1 \longrightarrow z_2$

it is positive due to the repulsive force



$$\Delta W = \int_{z_1}^{z_2} \bar{F}\cdot d\bar{r} = \int_{z_1}^{z_2} dz\ \bar{F}\cdot\hat{k} = \bar{F}\cdot\hat{k}\,(z_2 - z_1)$$

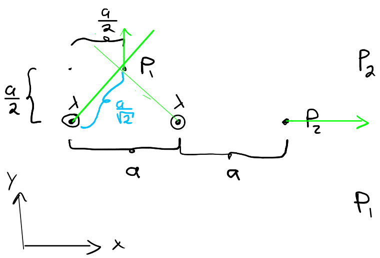
$$= -e\frac{\sigma}{2\epsilon_0}(z_2-z_1)$$

$$= \underline{|eQ_T|\frac{1}{2\epsilon_0 L^2}(z_2-z_1) \ > 0}$$

11-05-96

③

Infinite line charges, find the field at P_1 and P_2



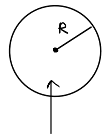
$$P_2: \quad \bar{E} = \frac{\lambda}{2\pi\epsilon_0}\left\{\frac{1}{2a} + \frac{1}{a}\right\}\hat{i} = \underline{\frac{\lambda\hat{i}}{2\pi\epsilon_0}\,\frac{3}{2a}}$$

$$P_1: \quad \bar{E} = \frac{\lambda}{2\pi\epsilon_0}\left[\frac{\sqrt{2}}{a}\cdot 2\right]\cdot\underbrace{\cos\left(\frac{\pi}{4}\right)}_{1/\sqrt{2}}\hat{j}$$

$$= \frac{\lambda}{2\pi\epsilon_0}\,\frac{2}{a}\hat{j} = \underline{\frac{\lambda}{\pi\epsilon_0 a}\hat{j}}$$

11-06-54

Infinite cylinder with space charge density ρ



We use Gauß to find E inside and outside the cylinder (it can not be conducting if it has a charge distribution of this kind)

$$Q_T = 2\pi L\int_0^R r\,dr\cdot\alpha r = 2\pi L\alpha\frac{R^3}{3}$$

$$\underline{= 2\pi L\,\alpha R^2/3}$$

$\rho = \alpha r$, $\quad \alpha = $ Const.

④

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q_{enc}}{\epsilon_0}$$

surface integral, we select a concentric cylinder surface at which E must be constant due to the cylindrical symmetry

$\underline{r > R}$: all the charge is within the Gauß surface

$$L\cdot 2\pi r E_r = \frac{2\pi L\alpha R^3}{3\epsilon_0} \longrightarrow E_r = \frac{\alpha R^3}{3\epsilon_0 r} \longrightarrow \underline{\bar{E} = \frac{\alpha R^3}{3\epsilon_0 r}\hat{r}}$$

$\underline{r < R}$:

$$Q_{enc} = \frac{2\pi}{3}L\alpha r^3$$

$$E_r = \frac{\alpha r^2}{3\epsilon_0} \longrightarrow \underline{\bar{E} = \frac{\alpha r^2}{3\epsilon_0}\hat{r}}$$

In $r = R$ $E(r)$ is continuous, but the derivative is not due to the step in the distribution at the surface

11-06-66

At the surface of a spherical conductor the field is $\overline{E} = \frac{kq}{r^2}\hat{r}$

In equilibrium the field close to a surface of a conductor is perpendicular to it

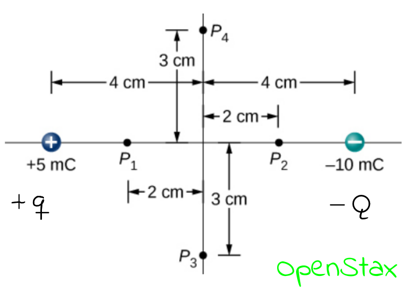
and $E = \frac{\sigma}{\epsilon_0}$

Can we get these facts to be in agreement?

$$q = 4\pi r^2 \sigma$$

$$\hookrightarrow E = \frac{kq}{r^2} = \underbrace{\left(\frac{1}{4\pi\epsilon_0}\right)}_{k} \frac{(4\pi r^2 \sigma)}{r^2} = \frac{\sigma}{\epsilon_0}$$

11-07-52



OpenStax

$q = 5\,mC$

$Q = 10\,mC$

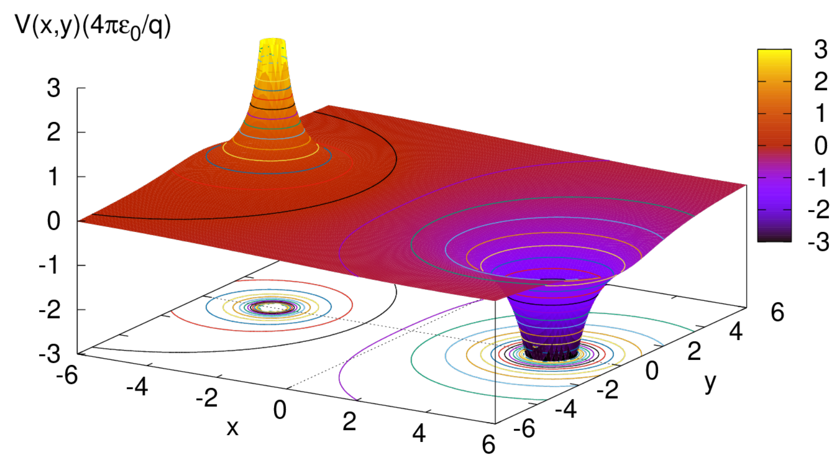
we need to find the electric potential $V(x,y)$

$$V(x,y) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{4}\frac{q_i}{|\vec{x}_i - \vec{x}|} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{4}\frac{q_i}{\sqrt{(x_i - x)^2 + (y_i - y)^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{(-4-x)^2 + (0-y)^2}} - \frac{Q}{\sqrt{(4-x)^2 + (0-y)^2}}\right]$$

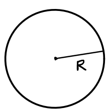
$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{\sqrt{(-4-x)^2 + y^2}} - \frac{2}{\sqrt{(4-x)^2 + y^2}}\right]$$

P_1 : $V(-2,0) = \frac{q}{4\pi\epsilon_0}\, 0{,}1667$ see figure



11-07-62

Long aluminum cylinder (conducting)



L, line charge λ

$\to Q_T = \lambda L$

a) Find $\vec{E}$ inside and outside,

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$r > R$:

$$L \cdot 2\pi r\, E_r = \frac{\lambda L}{\epsilon_0}$$

$$\to E_r = \frac{\lambda}{2\pi\epsilon_0 r} \to \vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}, \quad r > R$$

$r < R$: $Q_{enc} = 0 \to \vec{E} = 0$ inside the cylinder

b) We use here $\vec{E} = -\vec{\nabla} V$, we only need $(\vec{\nabla})_r$ due to the cylinder symmetry

$$(\vec{\nabla})_r = \frac{\partial}{\partial r}$$

$r < R$: $E_r = -\frac{\partial}{\partial r}V(r)$, $E_r = 0$

$\to V(r) = V_i$: Const.

$r > R$:

$$V(r) = -\frac{\lambda}{2\pi\epsilon_0}\ln(r) + V_0$$

Const.

$V(r)$ must be continuous in $r = R$

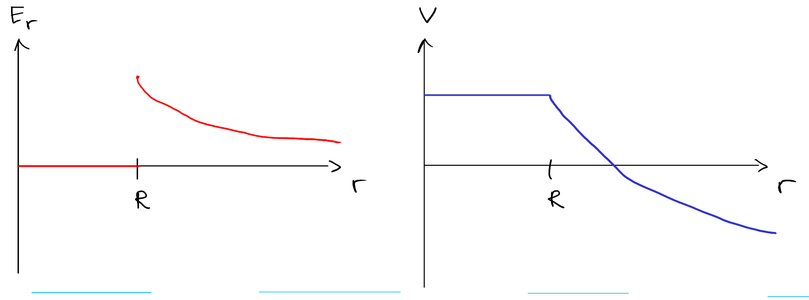
$$\to V(R) = V_i \to -\frac{\lambda}{2\pi\epsilon_0}\ln(R) + V_0 = V_i$$

$$\to V_0 = \frac{\lambda}{2\pi\epsilon_0}\ln(R) + V_i$$

$$\to \boxed{V(r) = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r}{R}\right) + V_i \quad r > R}$$

$$\boxed{V(r) = V_i \quad r < R}$$

5



11-08-42

Find the energy in a single-sphere capacitor, $R = 2.0\,m$, $V = 10.0\,V$

Spherical capacitor

$$C = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}, \quad \text{Check Ex. 8.3}$$

or take the Limit $R_2 \to \infty$

6

$$C = 4\pi\epsilon_0 \frac{R_1}{1 - \frac{R_1}{R_2}} \xrightarrow[R_2 \to \infty]{} 4\pi\epsilon_0 R_1$$

So, for a single sphere capacitor we have

$$C = 4\pi\epsilon_0 R, \qquad U_c = \frac{1}{2} V^2 C = \frac{1}{2} V^2 4\pi\epsilon_0 R$$

$$= \frac{1}{2}\, 10^2\, V^2\, 4\pi\epsilon_0 \cdot 2.0\, m$$

ϵ_0: $8.85 \cdot 10^{-12} \frac{C^2}{N m^2}$

$$= 1.1 \cdot 10^{-8}\, J$$

11-09-60

12 V and 100 Ah, 80 W Lights

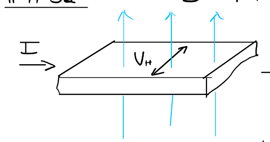
$80\,W$ at $12\,V$ $\to$ $P = I \cdot V$ $\to$ $I = \frac{P}{V} = \frac{80}{12} = 6.667\,A$

$$\to T = \frac{100\,Ah}{6.667} = 15\,h$$

1

II-11-52

$B = 1\,T$



$\longrightarrow I = 2A$

Hall sensor

$V_H = 1{,}5\,\mu V$

Find B that gives $V_H = 2\,\mu V$ for $I = 1{,}7\,A$

(..) parameters that are particular to the Hall sensor

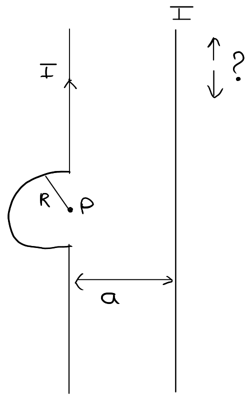
$$V_H = \frac{IB\ell}{neA} = IB\left(\frac{\ell}{neA}\right) \longrightarrow \left(\frac{\ell}{neA}\right) = \frac{V_H}{IB} = \frac{1{,}5\,\mu V}{2A \cdot 1T}$$

$$= 0{,}75\,\frac{\mu V}{AT}$$

$$\longrightarrow B = \frac{V_H}{I\left(\frac{\ell}{neA}\right)} = \frac{2}{1{,}7 \cdot 0.75}\,T = \underline{1{,}57\,T}$$

2

II-12-21



Find a such that $B(P) = 0$

Magnetic field due to the arch is

$$B(P) = \frac{\mu_0 I}{4\pi R}\cdot\pi = \frac{\mu_0 I}{4R} \quad \text{into the page}$$

burfum I up the page to counteract this

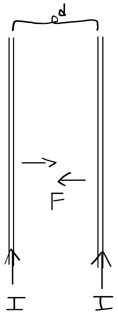
$$B = \frac{\mu_0 I}{2\pi a}$$

so we need

$$\frac{\mu_0 I}{4R} = \frac{\mu_0 I}{2\pi a} \longrightarrow 4R = 2\pi a \longrightarrow \underline{a = \frac{2R}{\pi}}$$

3

II-12-30



$d = 0{,}25\,m$

$I = 50\,A$

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

a) Attraction for parallel currents (check energy density..)

$$\frac{F}{\ell} = \frac{4\pi \cdot 10^{-7}\,(50)^2}{2\cdot\pi\cdot 0{,}25} \approx 2\cdot 10^{-3}\,N/m$$

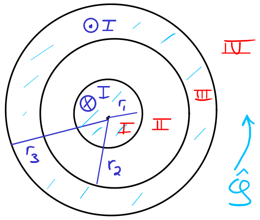
b) Attraction

c) For antiparallel currents there is a repulsion between the currents (best understood from the energy density of the system, and how it changes as d is varied..)

4

II-12-48

Find B caused by the homogeneous currents



a) I, $r < r_1$

b) II, $r_1 < r < r_2$

c) III, $r_2 < r < r_3$

d) IV, $r > r_3$

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

(I): $I_{enc}(r) = I\left(\frac{r}{r_1}\right)^2$

$$\longrightarrow 2\pi r B = \mu_0 I \left(\frac{r}{r_1}\right)^2 \longrightarrow B = \frac{\mu_0 I}{2\pi r_1^2}\,r$$

and

$$\boxed{\vec{B} = -\frac{\mu_0 I}{2\pi r_1^2}\,r\,\hat{\varphi}}$$

Linear in r

II : $2\pi r B = \mu_0 I \rightarrow \boxed{\bar{B} = -\frac{\mu_0 I}{2\pi r}\hat{\varphi}}$

IV : $\bar{B} = 0$ as $I_{enc} = 0$

III : $\bar{B} = \bar{B}_1 + \bar{B}_2$

$$\bar{B}_1 = -\frac{\mu_0 I}{2\pi r}\hat{\varphi}$$

For $\bar{B}_2$:

$$2\pi r B_2 = \mu_0 I_{enc}, \quad I_{enc} = I\frac{(r^2-r_2^2)}{(r_3^2-r_2^2)}$$

$$\rightarrow B_2 = \frac{\mu_0 I}{2\pi r}\frac{(r^2-r_2^2)}{(r_3^2-r_2^2)}$$

and the total in III

$$\boxed{\bar{B} = \frac{\mu_0 I}{2\pi r}\left\{-1 + \frac{(r^2-r_2^2)}{(r_3^2-r_2^2)}\right\}\hat{\varphi}},$$

and

$B(r_3) = 0$