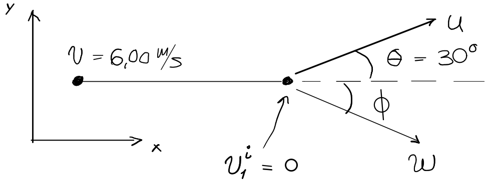
1

1-09-36

Identical pucks on a hocky air table $m_1 = m_2 = m$ Elastic collision, $\Delta E_T = 0$



$\Delta \bar{p} = 0$, no external force

Find w and ϕ

Conservation of momentum

(Y): $m u \sin\theta + m w \sin\phi = 0$ (1)

(X): $m v + 0 = m u \cos\theta + m w \cos\phi$ (2)

$\theta = \frac{\pi}{6} \rightarrow \sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

Conservation of energy

$\frac{1}{2} m v^2 + 0 = \frac{1}{2} m u^2 + \frac{1}{2} m w^2$ (3)

2

3 equations, we need to find ϕ and w (and u)

Simplify the equations

(1): $\frac{u}{2} + w\sin\phi = 0$

(2): $v = \frac{\sqrt{3}}{2} u + w\cos\phi$

(3): $v^2 = u^2 + w^2$

This is not a system of coupled linear equations, but many routs can be taken to find a solution. I want to start to find the angle ϕ

(1) $\rightarrow$ $u = -2w\sin\phi$ $\rightarrow$ (2)

$\rightarrow$ $v = -\sqrt{3}\, w\sin\phi + w\cos\phi = \underline{w\{\cos\phi - \sqrt{3}\sin\phi\}}$

(1) $\rightarrow$ (3) $\rightarrow$ $v^2 = 4\sin^2\phi \cdot w^2 + w^2 = \underline{w^2\{1 + 4\sin^2\phi\}}$

These 2 equation I combine to get an equation only for the angle ϕ

3

$\{\cos\phi - \sqrt{3}\sin\phi\}^2 = 1 + 4\sin^2\phi$

$\rightarrow$ $\cos^2\phi + 3\sin^2\phi - 2\sqrt{3}\cos\phi \cdot \sin\phi = 1 + 4\sin^2\phi$

use the fact that $\cos^2\phi + \sin^2\phi = 1$

$\rightarrow$ $\sin^2\phi = -\sqrt{3}\sin\phi \cdot \cos\phi$ $\rightarrow$ $\sin\phi = -\sqrt{3}\cos\phi$

$\rightarrow$ $\phi = \arctan(-\sqrt{3})$ $\rightarrow$ $\underline{\underline{\phi = -60^\circ}}$

This we use in (1) and (2)

$$\rightarrow \boxed{\begin{aligned} \frac{u}{2} - \frac{\sqrt{3}}{2} w &= 0 \\ \frac{\sqrt{3}}{2} u + \frac{w}{2} &= v \end{aligned}}\quad, \quad v = 6{,}00\ m/s$$

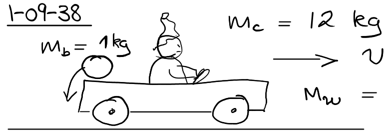
The solution of this set of linear equations gives

$u = (\sqrt{3})^3 \simeq 5{,}20\ m/s$

$\underline{\underline{w = 3\ m/s}}$

4

1-09-38



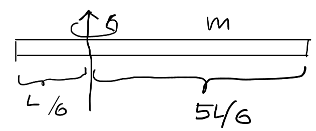
$m_c = 12\ kg$

$v = 3{,}3\ m/s$

$m_w = 4{,}0\ kg$

The child releases a ball at $t = 0$ with no velocity relative to the wagon. The momentum of the total system before $t = 0$ is the same as the momentum immediately after the release. There will be no change in the velocity of the wagon and the child.

1-10-66



Find the momentum of inertia of the system with direct integration

$I = \int r^2 dm$ $\quad dm = \left(\frac{m}{L}\right) dx$

$\rightarrow I = \int_0^{L/6} x^2 dm + \int_0^{5L/6} x^2 dm = \frac{m}{L}\left\{\int_0^{L/6} x^2 dx + \int_0^{5L/6} x^2 dx\right\}$

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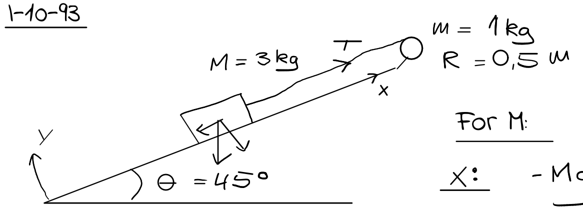
$$I = \frac{m}{L}\left[\frac{x^3}{3}\Big|_0^{L/6} + \frac{x^3}{3}\Big|_0^{5L/6} \right] = \frac{m}{L}\left\{ \frac{L^3}{3\cdot 6^3} + \frac{L^3\cdot 5^3}{3\cdot 6^3} \right\}$$

$$= mL^2\left\{ \frac{1}{3\cdot 6^3} + \frac{5^3}{3\cdot 6^3} \right\} = mL^2\left\{ \frac{1+125}{648} \right\}$$

$$= mL^2\left\{ \frac{63}{324} \right\} = m\frac{L^2}{12}\left\{ \frac{63}{27} \right\} = m\frac{L^2}{12}\cdot\frac{7}{3}$$

1-10-93



For M:

x: $-Mg\sin\theta + \mu N + T = am$ (1)

y: $-Mg\cos\theta + N = 0$ (2)

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For m:

$\tau_T = I\alpha$, (3) $\quad a = R\alpha$, (4) $\quad I = \frac{1}{2}mR^2$, (6) $\quad \tau_T = R\cdot T$ (5)

(2) -> (1): $\hookrightarrow -Mg\sin\theta + \mu Mg\cos\theta + T = aM$ (*)

(3-6) $\longrightarrow$ $-RT = I\frac{a}{R} = \frac{1}{2}mR^2\frac{a}{R} = \frac{m}{2}Ra$

$\rightarrow T = -\frac{m}{2}a$

The last equation can be used in (*):

$$-Mg\sin\theta + \mu Mg\cos\theta - \frac{m}{2}a = aM$$

$$\rightarrow -Mg\left\{\sin\theta - \mu\cos\theta\right\} = a\left\{M + \frac{m}{2}\right\}$$

$$\rightarrow a = -g\left\{\frac{M}{M+\frac{m}{2}}\right\}\left\{\sin\theta - \mu\cos\theta\right\} = -3.6\ m/s^2$$

7

1-11-30

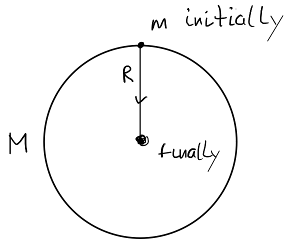
Ball with M = 40.0 kg rolls on a horizontal plane surface with velocity v = 6.0 m/s
How much work is needed to stop it

$I = \frac{2}{5}MR^2$ when turning around an axis through the center

$$E_k = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2, \qquad \omega R = v$$

$$E_k = \frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 = \frac{7}{10}Mv^2$$

1-11-52



M = 2,0 kg

R = 0,60 m

m = 0,05 kg

$\omega_i = 2\cdot 2\pi\ s^{-1}$

Find ω_f

after m has slid to the center

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we assume the sliding of m does not change the angular momentum of the system --> L is conserved:

$$I_i\omega_i = I_f\omega_f$$

$$\frac{1}{2}R^2\left\{M+m\right\}\omega_i = \frac{1}{2}MR^2\omega_f$$

$$\rightarrow \left[M+m\right]\omega_i = M\omega_f$$

$$\rightarrow \omega_f = \frac{M+m}{M}\omega_i = (1,025)\,\omega_i = 12,88\ Hz$$