1

Dæmi 1, (1-01-52)

a) $\bar{F} = m\bar{a}$, finna $[F] = [ma] = M\frac{L}{T^2}$

b) $K = \frac{1}{2}mv^2$, $[K] = M\frac{L^2}{T^2}$ líka almennt fyrir orku

c) $p = mv$, $[p] = M\frac{L}{T}$ skriðþungi

d) $W = mas$, $[W] = M\frac{L}{T^2}L = M\frac{L^2}{T^2}$ vinna með sömu vídd og orka

e) $L = mvr$, $[L] = M\frac{L}{T}L = M\frac{L^2}{T}$ hverfiþungi

2

auka)

$E = hf$, vitum að $[f] = \frac{1}{T}$

og $[E] = M\frac{L^2}{T^2}$

$$\rightarrow [h] = \left[\frac{E}{f}\right] = M\frac{L^2}{T^2}\cdot T = M\frac{L^2}{T}$$

Þannig að víddin á Plancks-fastanum er sama og vídd hverfiþunga

3

Dæmi 2, (1-01-54)

$[V] = L^3$, $[\rho] = \frac{M}{L^3}$, $[t] = T$

a)

$$\left[\int \rho\, dV\right] = [\rho V] = M$$

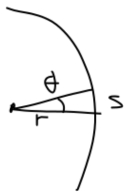
$$\left[\frac{dV}{dt}\right] = \left[\frac{V}{t}\right] = \frac{L^3}{T}$$

4

$$\left[\rho \frac{dV}{dt}\right] = \left[\frac{\rho V}{t}\right] = \frac{M}{L^3}L^3\frac{1}{T} = \frac{M}{T}$$

Dæmi 3, (1-01-55)

$s = r\theta$



$[r] = L$

$[s] = L$

$\rightarrow [\theta] = 1$

Horn er mælt í víddarlausum "einingum"

Dæmi 4, (1-02-68 (a) og (b)) (5)

a) $\bar{A} = 2{,}0\hat{i} - 4{,}0\hat{j} + \hat{k} = (2, -4, 1)$

$\bar{C} = 3{,}0\hat{i} + 4{,}0\hat{j} + 10{,}0\hat{k} = (3, 4, 10)$

Finna $\bar{A} \times \bar{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 1 \\ 3 & 4 & 10 \end{vmatrix}$

$= (-4 \cdot 10 - 4 \cdot 1, \; -2 \cdot 10 + 3 \cdot 1, \; 2 \cdot 4 - 3 \cdot (-4))$

$= (-44, -17, 20) = \bar{D}$

(6)

Skoðum

$\bar{D} \cdot \bar{A} = (-44, -17, 20) \cdot (2, -4, 1) = 0$

$\bar{D} \cdot \bar{C} = (-44, -17, 20) \cdot (3, 4, 10) = 0$

Þannig að vigurinn D er hornréttur á vigrana A og B, eins og verður að vera

b) $\bar{A} = (3, 4, 10)$, $\bar{A} \times \bar{C} = \begin{vmatrix} i & j & k \\ 3 & 4 & 10 \\ 2 & -4 & 1 \end{vmatrix}$

$\bar{C} = (2, -4, 1)$

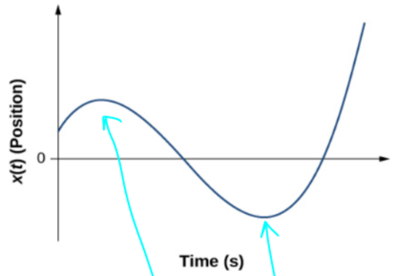
Því, $\bar{A} \times \bar{C} = -\bar{C} \times \bar{A}$

$= (44, 17, -20)$

1

Dæmi 1, (1-03-32)

Eigum að rissa upp v(t) graf
í samræmi við staðsetningargrafið
x(t)



$$v(t) = \frac{dx(t)}{dt}$$

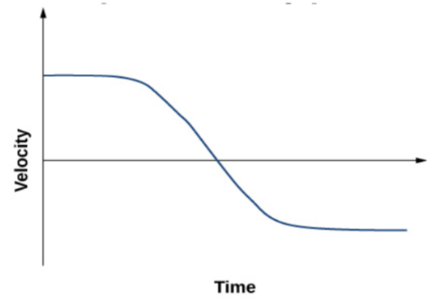
afleiðan í hverjum punkti x(t) grafsins
gefur hraðann, þ.e. við verðum að giska
á hallatöluna

Annars viðurkennist að ég er
ekki góður að teikna

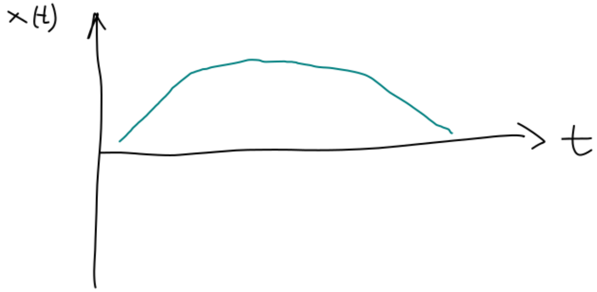
2

Dæmi 2, (1-03-33)

Hér eigum við að skissa
x(t) graf



$$v(t) = \frac{dx(t)}{dt}$$



3

Ég veit um fall með svipaða lögun, (svindlum á víddir)

$$v(t) = -\tanh(t)$$

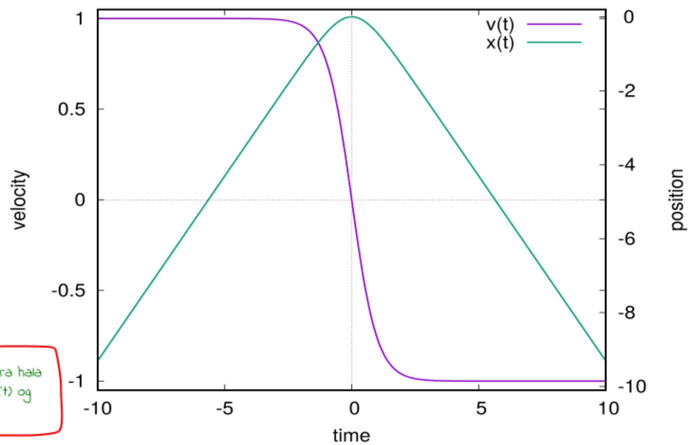
$$\rightarrow \quad x(t) = -\ln\left[\cosh(t)\right]$$

ræða fasta sem tínast í
óákveðinni heildun og diffrun

væri einfalt að skjóta
in nauðsynlegum föstum
með réttar víddir

Gnuplot og wxmaxima

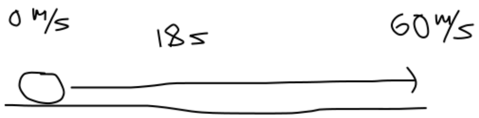
þetta sýnir að ég hef skammað einhverja vegna langra hala
sem voru réttir, en ég hef óvart ruglað þá saman x(t) og
v(t) gröfunum.



4

Dæmi 3, (1-03-42)

Finna hröðunina, (meðal hröðun)



Gerum ráð fyrir fastri hröðun

En við vitum ekki í raun
hvernig a(t) lítur út

$$\rightarrow \quad v = at$$

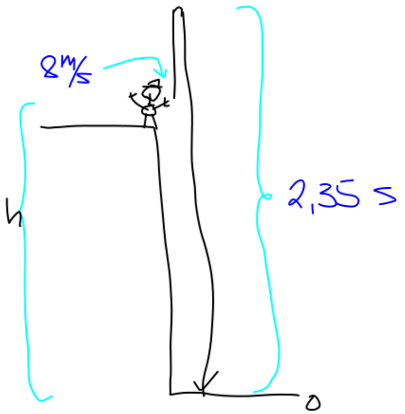
$$\rightarrow \quad a = \frac{v}{t} = \frac{60\,m/s}{18\,s} \simeq 3{,}33\ m/s^2$$

eða

$$\langle a \rangle = \frac{v_f - v_i}{t_f - t_i} = \frac{60\,m/s}{18\,s} \simeq 3{,}33\ m/s^2$$

5

Dæmi 4, (1-03-72)



Fyrir eina vídd leiddum við út

$$X(t) = X_0 + V_0(t-t_0) + \frac{a}{2}(t-t_0)^2$$

$$t_0 = 0\,, \quad t = 2{,}35\ \mathrm{s}$$

$$a = -9{,}81\ \mathrm{m/s^2}$$

$$V_0 = 8\ \mathrm{m/s}$$

$$x_0 = h\,, \quad x = 0$$

Finna hæðina h

$$0 = h + V_0 t + \frac{a}{2}t^2 \longrightarrow h = -V_0 t - \frac{at^2}{2}$$

6

$$h = -V_0 t - \frac{at^2}{2}$$

$$= -8 \cdot 2{,}35\ \mathrm{m} + \frac{9{,}81}{2}(2{,}35)^2\ \mathrm{m} \simeq 8{,}29\ \mathrm{m}$$

Hve langan tíma tekur fallið ef hraðinn er niður á við í upphafi, (þekkjum nú h)

$$0 = h + V_0 t + \frac{a}{2}t^2\,, \quad V_0 = -8\ \mathrm{m/s}$$

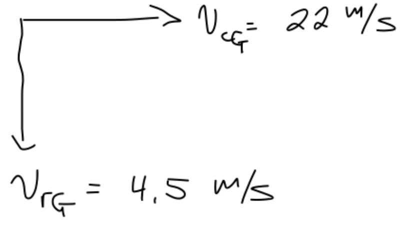
annars stigs jafna fyrir t, með lausn

$$t = \frac{-V_0 \pm \sqrt{V_0^2 - 4\frac{a}{2}h}}{2\frac{a}{2}} = \frac{8 \pm \sqrt{8^2 + \frac{4\cdot 9{,}81}{2} 8{,}29}}{9{,}81}\ \mathrm{s}$$

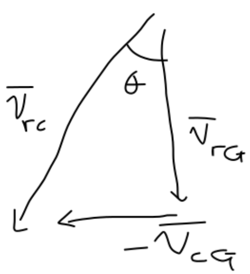
$$\simeq 2{,}18\ \mathrm{s} \text{ eða } \cancel{-0{,}55\ \mathrm{s}}$$

7

Dæmi 7, (1-04-72)



miðað við jörð



miðað við bíl

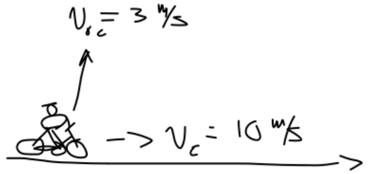
$$\bar{V}_{rG} = \bar{V}_{rc} + \bar{V}_{cG} \longrightarrow \bar{V}_{rc} = \bar{V}_{rG} - \bar{V}_{cG}$$

$$\longrightarrow |\bar{V}_{rc}| = \sqrt{V_{rG}^2 + V_{cG}^2} \simeq 22{,}5\ \mathrm{m/s}\,, \quad \theta = \arctan\left(\frac{V_{cG}}{V_{rG}}\right)$$

$$\longrightarrow \theta \simeq 1{,}37\,\mathrm{rad} \simeq 78{,}4^\circ$$

8

Dæmi 5, (1-04-42)



Finna braut miðað við jörð, þurfum upphafshraða miðað við jörð

$$V_{0i} = V_c\,, \quad V_{0j} = V_{0c}$$

$$\theta_0 = \arctan\left(\frac{V_{0c}}{V_c}\right)$$

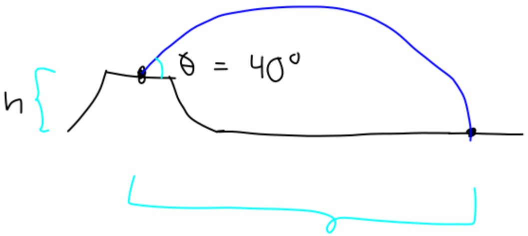
síðan höfum við úr fyrirlestri

$$y = x\tan\theta_0 - \frac{g x^2}{2(V_0\cos\theta_0)^2}$$

Gott að kanna markgildi....

$$= x\frac{V_{0c}}{V_c} - \frac{g x^2}{2\left[\sqrt{V_c^2 + V_{0c}^2}\cdot\cos\left[\arctan\left(\frac{V_{0c}}{V_c}\right)\right]\right]^2}$$

Dæmi 6, (1-04-52)



$h = 900\,m$ $\quad L = 1000\,m$

a) Finna v_0

Höfum

$$x = (v_0 \cos\theta)t$$

$$y = y_0 + (v_0 \sin\theta)t - \frac{g}{2}t^2$$

$$= h + (v_0 \sin\theta)t - \frac{g}{2}t^2$$

$$\rightarrow \quad t = \frac{x}{v_0 \cos\theta}, \quad y = h + x\tan\theta - \frac{g x^2}{2(v_0 \cos\theta)^2}$$

Viljum finna v_0 þ. $y = 0$, $x = L$

innsetning

$$0 = h + L\tan\theta - \frac{gL^2}{2\cos^2\theta \cdot v_0^2}$$

$$\rightarrow \quad h + L\tan\theta = \frac{gL^2}{2\cos^2\theta \cdot v_0^2}$$

$$\rightarrow \quad v_0 = \sqrt{\frac{gL^2}{2\cos^2\theta \cdot (h + L\tan\theta)}}$$

$$= \sqrt{\frac{9{,}81 \cdot 10^6}{2\cos^2(40) \cdot (900 + 1000\tan(40))}} \approx \underline{69{,}3\ m/s}$$

b) Flugtími

Munum eftir

$$y = y_0 + (v_0 \sin\theta)t - \frac{g}{2}t^2$$

Lending $\rightarrow$ $y = 0$

$$\rightarrow \quad 0 = h + t v_0 \sin\theta - \frac{g}{2}t^2$$

$$\rightarrow \quad t = \frac{-v_0\sin\theta \pm \sqrt{v_0^2 \sin^2\theta + \frac{g v_0 \sin\theta}{2}}}{-g}$$

$$t = \frac{69{,}3 \cdot \sin(40) \mp \sqrt{(69{,}3 \sin(40))^2 + \frac{9{,}81 \cdot 69{,}3 \sin(40)}{2}}}{9{,}81}$$

$\approx \underline{9{,}33\ s}$ eða $\cancel{-0{,}24}$ s

1

Problem 1, (1-05-22)

$$\bar{F}_1 + \bar{F}_2 + \bar{F}_3 = 0$$

$$\bar{F}_1 = \frac{75\,N}{\sqrt{2}}(1,-1)$$

$$\bar{F}_2 = \frac{150\,N}{\sqrt{2}}(1,-1)$$

$$\to \bar{F}_3 = -\bar{F}_1 - \bar{F}_2 = \frac{N}{\sqrt{2}}(-75-150,\ 75+150)$$

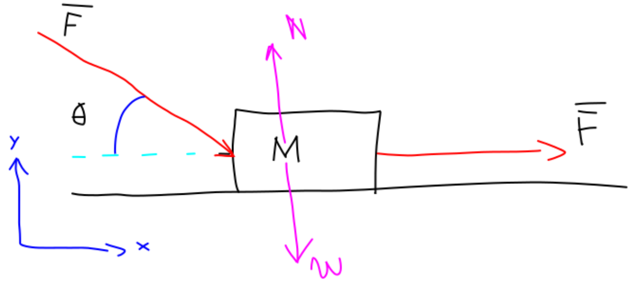
$$= \frac{225\,N}{\sqrt{2}}(-1,\ 1)$$

2

Problem 2, (1-05-40)

$M = 10\,kg$, $|\bar{F}| = 30\,N$

$\theta = 30°$



Find $\bar{a}$ for M

I add the forces N and w to realize that the y-component of F (on the left) ballances with them

y: $\quad N - W - F_y = 0 \to N = W + F\sin\theta$

$$= mg + F\sin\theta$$

3

x: $\quad (F_{net})_x = F\cos\theta + F = F(1+\cos\theta)$

$$\bar{F}_{net} = M\bar{a} \to \boxed{a_x = \frac{F(1+\cos\theta)}{M}}$$

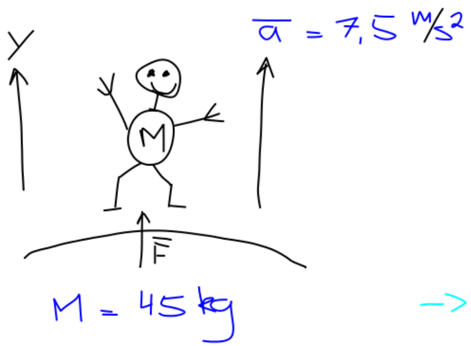
$$= \frac{30\,N\left[1+\cos\left(\frac{30\pi}{180}\right)\right]}{10\,kg}$$

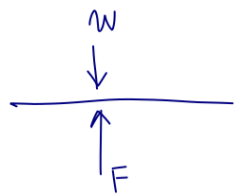
$$= 5{,}6\ m/s^2$$

4

Problem 3, (1-05-58)

Find F that gave her this initial a on the trampoline





$$F - W = F_{net} = Ma$$

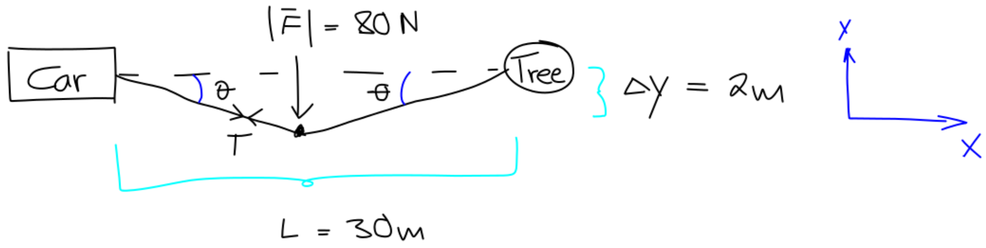
$$\to F - Mg = Ma$$

$$\to \boxed{F = M(a+g)} = 779\ N$$

(5)

Problem 4, (1-05-64)

Find the force on the car, T



Find θ

$$\tan\theta = \frac{\Delta y}{L/2} = \frac{2\Delta y}{L}$$

$$\rightarrow \quad \theta = \arctan\left(\frac{2\Delta y}{L}\right)$$

(6)

x:

$$(F_{net})_x = (T_R)_x - (T_L)_x = 0$$

$$\rightarrow \quad (T_R)_x = (T_L)_x$$

$$T_L \cos\theta = T_R \cos\theta \rightarrow T_L = T_R$$

y:

$$(F_{net})_y = (T_l)_y + (T_R)_L - F = 0$$

$$0 = T\sin\theta + T\sin\theta - F$$

$$\rightarrow \quad T = \frac{F}{2\sin\theta} = \frac{F}{2\sin\left(\arctan\left(\frac{2\Delta y}{L}\right)\right)}$$

(7)

$$\arctan x = \arcsin\left[\frac{x}{\sqrt{1+x^2}}\right]$$

$$\rightarrow \quad T = \frac{F}{2\sin\left[\arctan\left(\frac{2\Delta y}{L}\right)\right]}$$

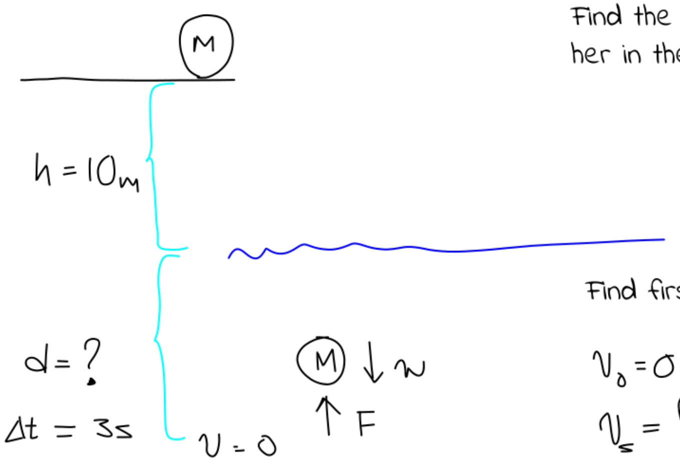
$$= \frac{F\sqrt{1+\left(\frac{2\Delta y}{L}\right)^2}}{2\left(\frac{2\Delta y}{L}\right)} \simeq 303\ N$$

(8)

Problem 5, (1-05-76)

No air resistance $\qquad M = 80\ kg$

Find the average force stopping her in the water



Find first speed at surface

$v_0 = 0$

$v_s = \sqrt{2gh}$

9

Meðalhröðun

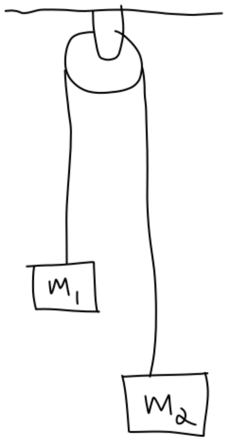
$$\langle \bar{a} \rangle = \frac{0 - \sqrt{2gh}}{\Delta t}$$

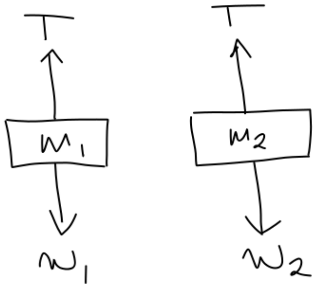
$$\langle \bar{F} \rangle = -M \frac{\sqrt{2gh}}{\Delta t}$$

$$= -80 \cdot \frac{\sqrt{2 \cdot 9{,}81 \cdot 10}}{3} \approx \underline{374\,N}$$

Dæmi 1, (1-06-42) ①

a) Find the acceleration a





Same T

$$\underbrace{a_1 = -a_2}_{\text{properties of the rope}}$$

m_1: $T - w_1 = m_1 a$

m_2: $T - w_2 = -m_2 a$

②

$m_1 - m_2$ $\rightarrow$ $-w_1 + w_2 = m_1 a + m_2 a$

$\rightarrow$ $-m_1 g + m_2 g = a(m_1 + m_2)$

$\rightarrow$ $\boxed{a = \dfrac{m_2 - m_1}{m_2 + m_1} g}$

Eq. m_1 indicates a is defined positive when pointed upwards. The Eq. for a shows this remains as long as $m_2 > m_1$

③

b) Find T, now add m_1 and m_2

$$2T - w_1 - w_2 = -a(m_2 - m_1) \quad \text{nota } a$$

$\rightarrow$ $2T - (m_1 + m_2) g = -\dfrac{(m_2 - m_1)^2}{m_1 + m_2} g$

$\rightarrow$ $2T - \dfrac{(m_1 + m_2)^2}{m_1 + m_2} g = -\dfrac{(m_2 - m_1)^2}{m_1 + m_2} g$

$\rightarrow$ $\boxed{T} = \dfrac{g/2}{m_1 + m_2}\left\{(m_1 + m_2)^2 - (m_2 - m_1)^2\right\}$

$= \dfrac{g/2}{m_1 + m_2} 4 m_1 m_2 = \boxed{\dfrac{2 g m_1 m_2}{m_1 + m_2}}$

④

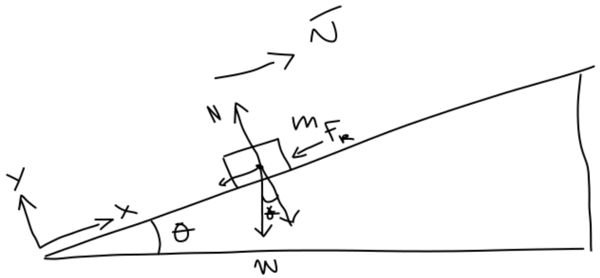
c) Find a and T for $m_1 = 2$ kg, $m_2 = 4$ kg, $g = 9{,}81\ m/s^2$

$a = \dfrac{m_2 - m_1}{m_2 + m_1} g = \dfrac{2}{6} g \simeq \underline{3{,}27\ m/s^2}$

$T = \dfrac{2 m_1 m_2}{m_1 + m_2} g = \dfrac{16 kg}{6} g \simeq \underline{26{,}2\ N}$

Dæmi 2, 1-06-54

5



Find the acceleration opposite to the motion of the snowboarder uphill

$$\mu_k = 0,1$$

$$\theta = 5°$$

$$y: \quad N - mg\cos\theta = 0 \quad \rightarrow \quad N = mg\cos\theta$$

$$x: \quad -mg\sin\theta - \mu_k N = ma$$

$$-mg\sin\theta - \mu_k mg\cos\theta = ma$$

6

m cancles out of the last Eq. and we are left with

$$-g\sin\theta - \mu_k g\cos\theta = a$$

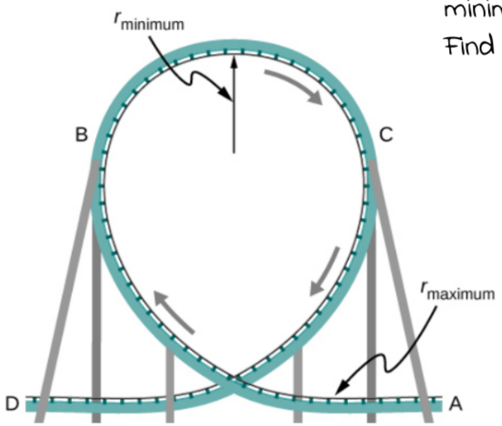
$$\rightarrow \quad a = -g\left[\sin\theta + \mu_k\cos\theta\right]$$

$$\simeq -g\{0,19\} \simeq -1,83 \ m/s^2$$

if $g = 9,81 \ m/s^2$

Dæmi 3, 1-06-72

7



The needed centripetal acceleration at minimum r has to be greater then g. Find the speed needed to give 1.5g

$$r_m = 15,0 \ m$$

$$a_c = \frac{v^2}{r_m}$$

$$\rightarrow \quad v^2 = a_c r_m$$

$$v = \sqrt{1,5 g r_m}$$

$$= 14,9 \ m/s$$

ef $g = 9,81 \ m/s^2$

Dæmi 4, 1-06-86

8

Spherical bacteria $d = 2.00 \ \mu m$ falling in water

$\rho_b = 1.10 \cdot 10^3 \ kg/m^3$, Find v_T

Stokes: $F_s = 6\pi r \eta v$

$r = 1,00 \ \mu m$

$F_s = bv, \quad b = 6\pi r\eta$

$\eta_{H_2O,\ 20°C} = 1.0016 \ mPa\cdot s$

$\simeq 1,00 \cdot 10^{-3} \ \frac{kg}{m\cdot s}$

$$v_T = \frac{mg}{b} = \frac{mg}{6\pi r\eta}$$

$$m_b = m = \frac{4\pi}{3} r^3 \rho_b$$

$$\rightarrow \quad v_T = \frac{4\pi r^3 \rho_b g}{18\pi r\eta} = \frac{2r^2\rho_b g}{9\eta}$$

$$= \frac{2\cdot(10^{-6})^2 \cdot 1.10\cdot 10^3 \cdot 9.81}{9\cdot 10^{-3}} \simeq 2,18\cdot 10^{-6} \ m/s$$

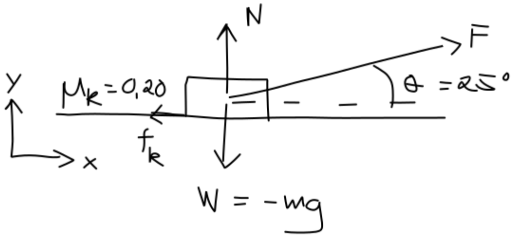
$$\simeq 2,18 \ \mu m/s$$

..en, flotkraftur --> m = ...

①

Dæmi 1 (1-07-34)

$\bar{v}$: Constant



First we start by finding $F = |\bar{F}|$

Y: $\quad N + F\sin\theta - mg = 0 \rightarrow N = mg - F\sin\theta$

X: $\quad -\mu_k N + F\cos\theta = 0$

Y -> X $\quad -\mu_k\{mg - F\sin\theta\} + F\cos\theta = 0$

only F is unknown here

②

$$\rightarrow F\{\mu_k \sin\theta + \cos\theta\} = \mu_k mg$$

$$\rightarrow \boxed{F = \frac{\mu_k mg}{\mu_k \sin\theta + \cos\theta}}$$

a)
$$W_F = F \cdot \cos\theta \cdot L = \frac{\mu_k mg\cos\theta \cdot L}{\mu_k \sin\theta + \cos\theta}$$

b) Find the work of the friction force

$$W_{f_k} = -\mu_k N \cdot L = -\mu_k\{mg - F\sin\theta\}L$$

③

$$\rightarrow W_{f_k} = -\mu_k\left\{mg - \frac{\mu_k mg\sin\theta}{\mu_k \sin\theta + \cos\theta}\right\}L$$

$$= -\mu_k\left\{\frac{\mu_k mg\cos\theta}{\mu_k \sin\theta + \cos\theta}\right\}L$$

So we see that the work of the friction force is exactly opposite to the work of the external F.

c) The total work of both external forces is 0, as the system is not accelerated. Interesting is to check that the results for $\theta = 0$ are familiar...

④

Dæmi 2 (1-07-42) $\quad$ x-y-plane $\quad \bar{F} = \left(x, \frac{y^2}{3m}\right) 50\ \frac{N}{m}$

Calculate the work of F for the translation from A = (3,4) to B = (6,8)

$$W = \int_A^B \bar{F} \cdot d\bar{r} = \int_A^B \{F_x dx + F_y dy\}$$

$$= \int_3^6 F_x dx + \int_4^8 F_y dy = 50\int_3^6 x dx + 50\int_4^8 \frac{y^2}{3} dy$$

$$= 675\ \text{Nm} + \frac{22400}{9}\ \text{Nm}$$

$$= 3163{,}9\ \text{Nm}$$

⑤

Dæmi 3 (1-08-24)

$$F(x) = \left(\frac{3{,}0}{x}\right) N$$

on the positive x-axis

$$0 \longrightarrow x$$

a)

$$\boxed{W} = \int_A^B F(x)\,dx = \int_2^5 \frac{3}{x}\,dx = 3\ln(x)\Big|_2^5$$

$$= 3\left[\ln(5) - \ln(2)\right]$$

$$= 3\ln\left(\frac{5}{2}\right) = \boxed{3\ln\left(\frac{5}{2}\right) Nm}$$

⑥

b) Can we find U such that $F = -\frac{\partial}{\partial x} U$

Guess $U(x) = -3\ln(ax) + U_0$ with $a>0$ and U_0 as constants

$$\rightarrow \quad F = -\frac{\partial}{\partial x}\left[-3\ln(ax) + U_0\right] = \frac{3a}{ax} = \frac{3}{x}$$

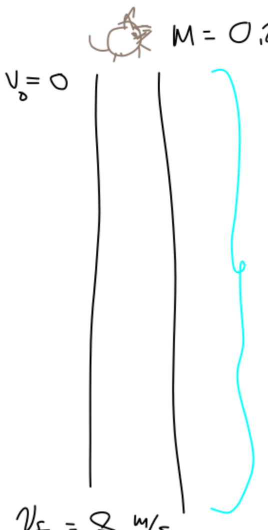
It is not possible to use $x = \infty$ as a reference point, but a is free and fixes a reference point, f. ex.

$a = 1\ m^{-1}$ gives $U(1) = 0$

The logarithm has very special properties....

⑦

Dæmi 4 (1-08-32) Conservation of energy



No friction or air resistance

$$V = \sqrt{2gh}$$

$$K_f = \frac{1}{2}mV^2 = \frac{1}{2}m\,2gh = mgh \ !$$

with friction the energy

$$mgh - \frac{1}{2}mV_f^2$$

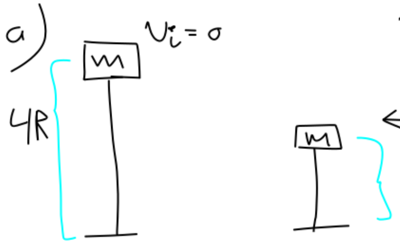
is taken out of the system, thus the friction does the work

$$\boxed{W_f = \frac{1}{2}mV_f^2 - mgh}$$

⑧

Dæmi 5 (1-08-42) Energy conservation

a)



Potential energy is linear in h

potential energy of $3R \longrightarrow E_{kin}$

$$\rightarrow \frac{1}{2}mV_f^2 = mg3R$$

$$\rightarrow V_f^2 = 6gR$$

a)

$$\rightarrow V_f = \sqrt{6gR}$$

b)

$$F_c = m\frac{V_f^2}{R}$$

$$= m6g \quad \text{to the right}$$

(1)

Dæmi 1, (1-09-37)

$v_b = 400$ m/s
$m = 0{,}200$ kg
$M = 1{,}50$ kg

Kúlan festist í kubbnum

a) Finna hraða kubbs og kúlu eftir að hún festist,
varðveisla skriðþunga

$$m v_b + M \cdot 0 = (m+M) v_f \rightarrow \boxed{v_f = \frac{m}{m+M} v_b}$$

$$v_f = \frac{0{,}2}{0{,}2+1{,}5} 400 \text{ m/s} = 47{,}1 \text{ m/s} \quad \text{til hægri}$$

(2)

b) Atlag kubbs á kúlu?

$$\boxed{\bar{J}_m} = m\{v_f - v_b\} = m\left\{\frac{m v_b}{m+M} - v_b\right\}$$

$$= m v_b \left\{\frac{-M}{m+M}\right\} = \boxed{-\frac{mM}{m+M} v_b}$$

$$= 0{,}2\{47{,}1 - 400\} \text{ kg}\frac{\text{m}}{\text{s}} \approx -70{,}6 \text{ kgm/s}$$

til vinstri

(3)

c) Atlag kúlu á kubb

$$\boxed{\bar{J}_M} = M\{v_f - 0\} = \boxed{\frac{mM}{m+M} v_b}$$

$$= 70{,}6 \text{ kg}\frac{\text{m}}{\text{s}} \quad \text{til hægri}$$

því er ljóst að $\bar{J}_m + \bar{J}_M = 0$

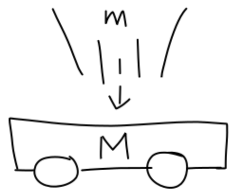
d) Ef $\Delta t = 3$ms finna F_{ave}

$$\bar{F}_{ave} = \frac{\Delta \bar{p}}{\Delta t} = \frac{\bar{J}}{\Delta t} \rightarrow \boxed{|F_{ave}| = \left(\frac{mM}{m+M}\right)\frac{v_b}{\Delta t}}$$

$$\approx 2{,}35 \cdot 10^4 \text{ N}$$

(4)

Dæmi 2, (1-09-42)



$\rightarrow v_i = 4{,}4 \frac{\text{m}}{\text{s}}$

Hvað má m vera mest til að lokahraðinn verði ekki minni en $v_f \geq 3{,}0 \frac{\text{m}}{\text{s}}$

varðveisla skriðþunga

$$M v_i + m \cdot 0 = (M+m) v_f$$

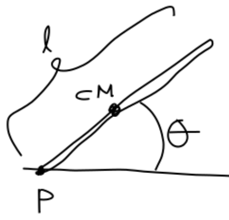
$$\rightarrow v_f = \frac{M v_i}{M+m} \rightarrow M + m = M\frac{v_i}{v_f}$$

$$\rightarrow \boxed{m} = M\frac{v_i}{v_f} - M = \boxed{M\left\{\frac{v_i}{v_f} - 1\right\}}$$

$$\approx 2000\left\{\frac{4{,}4}{3{,}0} - 1\right\} = 933{,}3 \text{ kg}$$

Dæmi 3, (1-10-67)

Finna ferð enda stangar þegar hún kemur í lárétta stöðu



$m = 1{,}0$ kg

$l = 2{,}0$ m

$\theta = 60°$

Notum orkuvarðveislu

$$\boxed{E_T = \frac{m}{2} v_P^2 + \frac{I}{2}\omega^2 + mgh}$$

i) $$E_T^i = 0 + 0 + mgh_{CM} = mg\frac{l}{2}\sin\theta$$

f) $$E_T^f = \frac{m}{2} v_P^2 + \frac{1}{2} I \omega^2 = 0 + \frac{I}{2}\omega^2$$

$$I = I_{CM} + md^2 = \frac{ml^2}{3}$$

$$\rightarrow E_T^f = 0 + \frac{m}{6} l^2 \omega^2 = \frac{m}{2} \frac{1}{3} l^2 \omega^2$$

$$\rightarrow \frac{m}{2}\sin\theta \cdot gl = \frac{m}{2}\left[l^2\omega^2 \frac{1}{3}\right]$$

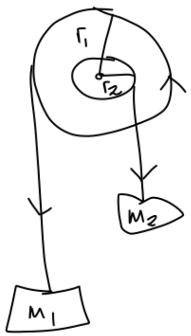
$$\rightarrow g\sin\theta = l\omega^2 \frac{1}{3}$$

$$\rightarrow \omega^2 = \frac{g \cdot 3 \cdot \sin\theta}{l} \rightarrow \omega = \sqrt{3\frac{g\sin\theta}{l}}$$

$$\boxed{v_{endi} = l\omega = \sqrt{3gl\sin\theta}}$$

$$= 7{,}14 \text{ m/s}$$

Dæmi 4, (1-10-92)



jákvæður snúningar

$I = 2{,}0 \text{ kgm}^2$

$r_1 = 0{,}5$ m

$r_2 = 0{,}2$ m

$m_1 = 1{,}0$ kg

$M_2 = 2{,}0$ kg

a) Finna α

$$\bar{\tau} = \bar{r} \times \bar{F}, \quad L = I\omega$$

Annað lögmál Newtons fyrir snúning

$$\frac{d\bar{L}}{dt} = \sum_i \bar{\tau}_i \rightarrow I\alpha = \tau_1 - \tau_2$$

$$\rightarrow \boxed{\alpha = \frac{(m_1 r_1 - m_2 r_2)g}{I}} = 0{,}49 \ 1/s^2$$

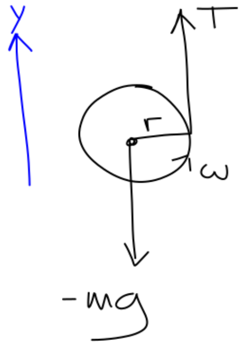
Jákvæður snúningur $m_1 \downarrow$ og $m_2 \uparrow$

b)

$$\boxed{a_1 = -r_1\alpha} = -0{,}25 \text{ m/s}^2$$

$$\boxed{a_2 = r_2\alpha} = 0{,}1 \text{ m/s}^2$$

Dæmi 5, (1-11-26)



Finna hröðun a

Annað lögmál Newtons fyrir hring og línulega hreyfingu

$$\frac{d\bar{L}}{dt} = \bar{\tau}, \qquad \bar{F} = m\bar{a}$$

$$I\frac{d}{dt}\omega = \tau \;\rightarrow\; I\alpha = \tau$$

$$\tau = Tr = \frac{1}{2}mr^2\alpha = \frac{1}{2}mr^2\left(\frac{a}{r}\right)$$

$$\rightarrow \boxed{T = \frac{m}{2}a}$$

$$\bar{F} = ma$$

$$\rightarrow -mg + T = -ma$$

$$\rightarrow -mg + \frac{1}{2}ma = -ma$$

$$\rightarrow g - \frac{a}{2} = a \;\rightarrow\; \boxed{a = \frac{2}{3}g}$$

með stefnu niður á við

Hröðunin er minni en g, þar sem í þyngdarhröðunin verkar bæði á tregðumassann og hverfitregðuna. Þetta dæmi má líka leysa með því að nota punktinn á jójóinu þar sem bandið kemur að sem snúningsás í stað massamiðju. Þá er það þyngdarkrafturinn sem fær vægi á jójóið. Þá þarf að nota setn. Steiners á I

Dæmi 6, (1-11-53)

$$M = 2{,}0\cdot 10^{30}\,\text{kg}$$

þyngdarhrun sólar $R_i \rightarrow R_f$

$$R_i = 7{,}0\cdot 10^{8}\,\text{m}$$

$$R_f = 3{,}5\cdot 10^{6}\,\text{m}$$

$$T_i = 28\,\text{d}$$

$$I = \frac{2}{5}MR^2$$

finna $T_f = \dfrac{2\pi}{\omega_f}$

varðveisla hverfiþunga $\quad I_i\omega_i = I_f\omega_f$

$$\rightarrow \omega_f = \left(\frac{R_i}{R_f}\right)^2\omega_i$$

$$\rightarrow \frac{2\pi}{T_f} = \left(\frac{R_i}{R_f}\right)^2\frac{2\pi}{T_i}$$

$$\rightarrow \boxed{T_f = \left(\frac{R_f}{R_i}\right)^2 T_i}$$

$$= \left(\frac{3{,}5\cdot 10^{6}}{7{,}0\cdot 10^{8}}\right)T_i$$

$$\simeq 2{,}5\cdot 10^{-5}\cdot 28\,\text{d} = 7{,}0\cdot 10^{-4}\,\text{d}$$

$$\simeq \underline{60{,}5\ \text{s}}$$

1

Dæmi 1, (1-14-74)

$$M = 75{,}0 \text{ kg}$$

V: rúmmál, tóm lungu

$V+V_L$: rúmmál + lungu, lungu full

Lungu tóm 3% ofan -> hlutfall = 0,97

-II- full 5% -II- -> -II- = 0,95

$$V\rho_E = M$$

$$[V+V_L]\rho_F \simeq M$$

$$[V+V_L]\rho_F = V\rho_E$$

$$\rho_E = 970\ \frac{\text{kg}}{\text{m}^3},\quad \rho_F = 950\ \frac{\text{kg}}{\text{m}^3}$$

2

$$[V+V_L]\rho_F = V\rho_E \longrightarrow V_L = \frac{V}{\rho_F}[\rho_E - \rho_F]$$

$$\longrightarrow \boxed{V_L} = V\frac{\rho_E-\rho_F}{\rho_F} = \frac{M}{\rho_E}\left[\frac{\rho_E}{\rho_F}-1\right]$$

$$\approx \frac{75{,}0}{970}\left[\frac{970}{950}-1\right] = 1{,}63\cdot 10^{-3}\,\text{m}^3$$

$$\boxed{= 1{,}63\ \text{L}}$$

Örugglega ekki ofmat. Eðlilegt væri að sjá 2.5 - 5.0 L

3

Dæmi 2, (1-14-90)

ρ = Fasti, Finna $\Delta p(v_1, A_1, A_2, \rho)$

Enginn hæðarmunur, jafna Bernoullis fyrir þrengingu

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2$$

Massavarðveisla

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2}v_1$$

$$\longrightarrow [p_1 - p_2] = \tfrac{1}{2}\rho[v_2^2 - v_1^2]$$

$$[p_1 - p_2] = \tfrac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)^2 v_1^2 - v_1^2\right]$$

4

$$[p_1 - p_2] = \tfrac{1}{2}\rho\left[\left(\frac{A_1}{A_2}\right)^2 v_1^2 - v_1^2\right]$$

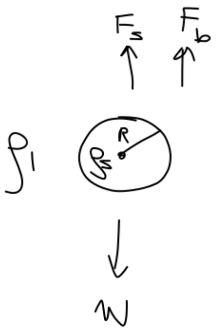
$$= \tfrac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$\boxed{= \tfrac{1}{2}\rho v_1^2\left[\frac{A_1^2 - A_2^2}{A_2}\right]}$$

Þannig að án þrengingar helst þrýstingurinn óbreyttur í þessu einfalda líkani

Dæmi 3, (1-14-93)

Fall í vökva, flotkraftur og viðnámskraftur



$$F_s + F_b - W = 0$$

$$6\pi R \eta v_T + V\rho_1 g - V\rho_s g = 0$$

$$V = \frac{4\pi}{3} R^3$$

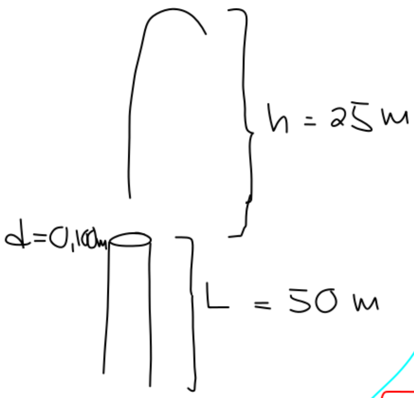
$$6\pi R \eta v_T + \frac{4\pi}{3} R^3 \rho_1 g - \frac{4\pi}{3} R^3 \rho_s g = 0$$

$$\rightarrow \quad 3\eta v_T + \frac{2}{3} R^2 g\left[\rho_1 - \rho_2\right] = 0$$

$$\rightarrow \quad \boxed{v_T = \frac{2R^2 g}{9}\left[\rho_s - \rho_1\right]}$$

Dæmi 4, (1-14-100)

Olía spítist upp úr röri. Finna hvort flæðið í rörinu sé jafnt (lagskipt)



$\rho = 900\ \mathrm{kg/m^3}$

$\eta = 1{,}00\ \frac{\mathrm{N}}{\mathrm{m^2}}\mathrm{s}$

$$N_R = \frac{2\rho v r}{\eta}$$

Í rörinu er sama hraði og við stútinn (massa varðveisla, ósamþjappanlegur)

$$v = \sqrt{2gh}, \quad r = \frac{d}{2}$$

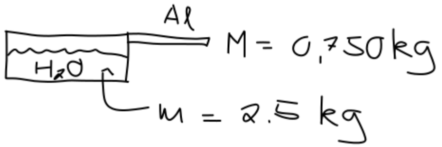
$$N_R = \frac{2 \cdot 900 \cdot \sqrt{2 \cdot 9{,}81 \cdot 25}\ \frac{0{,}1}{2}}{1{,}00}$$

$$\approx \boxed{1993}$$

jafnt flæði

(1)

Dæmi 1, (11-01-74)



$T_i = 30{,}0\,°C$

$T_f = 100{,}0\,°C$

+ uppgufun

a) Orka Q Tafla 1.3

$C_{H_2O} = 4186 \frac{J}{kg\,°C}$ nálgun, ekki alveg fasti á þessu T-bili

$C_{Al} = 900 \frac{J}{kg\,°C}$

$$Q_{suða} \approx \left[mC_{H_2O} + MC_{Al} \right] \Delta T$$

(2)

Uppgufun $(L_v)_{H_2O} = 2256 \frac{kJ}{kg}$

$Q_T = Q_{suða} + Q_v$, $Q_v = m(L_v)_{H_2O}$

$$Q_T = \left[mC_{H_2O} + MC_{Al} \right](T_f - T_i) + m(L_v)_{H_2O}$$

$$= \left[2{,}5\,kg \cdot 4186 \frac{J}{kg\,K} + 0{,}750\,kg \cdot 900 \frac{J}{kg\,K} \right] 70\,K$$

$$+ 2{,}5\,kg \cdot 2256 \cdot 10^3 \frac{J}{kg} \approx 6{,}42\ MJ$$

(3)

b) Ef hitarinn er 500 W, hve langan tíma þarf?

$$P \cdot \Delta t = Q_T \rightarrow \Delta t = \frac{Q_T}{P}$$

$$\Delta t = \frac{6.42 \cdot 10^6\,J}{500\,J/s} = 1{,}28 \cdot 10^4\,s$$

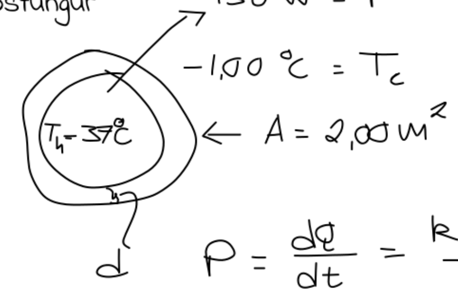
$\approx$ 3hr og 34 mín

(4)

Dæmi 2, (11-01-96)

Hver er meðalþykkt spiks?

Rostungur



$k = 0{,}2 \frac{W}{m^2\,°C}$

$$P = \frac{dQ}{dt} = \frac{kA(T_h - T_c)}{d}$$

$$d = \frac{kA(T_h - T_c)}{P} = \frac{0{,}2 \cdot 2{,}00 \cdot 38}{150}\,m \approx 0{,}10\,m$$

$= 10\,cm$

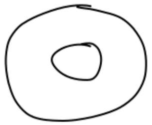
5

Dæmi 3, (11-02-34)

a) p_{gauge} í dekki?, $T = 25°C$

með 3,60 mol gasi $= n$

í $V = 30{,}0\ L$



$$pV = nRT \rightarrow p = \frac{nRT}{V}$$

$$p = \frac{3{,}60\,\text{mol} \cdot 0{,}0821\,\frac{L\cdot atm}{mol\cdot K}\,(273+25)K}{30{,}0\ L}$$

$$\approx 2{,}94\ atm \rightarrow p_{gauge} = 1{,}94\ atm$$

6

b) Finna p ef bætt er við 1,00 L gass

sem var í 1 atm og 25°C

Gera ráð fyrir $\Delta T = 0$, $\Delta V = 0$

bæta við 1L $\qquad p_0 = 1\,atm$

$\qquad \delta V = 1L$

$$\rightarrow \Delta n = \frac{p_0\,\delta V}{RT}$$

$$p = \frac{(n+\Delta n)}{V}RT = \frac{\left(n+\frac{p_0\,\delta V}{RT}\right)RT}{V}$$

$$= n\frac{RT}{V} + \Delta p, \quad \Delta p = \frac{p_0\,\delta V}{V}$$

$$\Delta p = \frac{1\,atm\,1L}{30\,L} \approx 0{,}033\ atm$$

7

Dæmi 4, (11-02-46)

Escape velocity $v_{esc} = 11{,}1\ km/s$

$$M_{O_2} = 32{,}0\,\frac{g}{mol} = \frac{0{,}032\,kg}{mol}$$

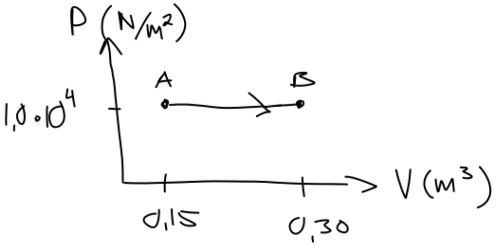
fyrir hvaða T er $v_{esc} = v_{rms}$

$$v_{rms} = \sqrt{\frac{3RT}{M}} \rightarrow v_{esc}^2 = \frac{3RT_{esc}}{M_{O_2}}$$

$$\rightarrow T_{esc} = v_{esc}^2 \cdot \frac{M_{O_2}}{3R} = \left(11{,}1\cdot10^3\ m/s\right)^2 \frac{0{,}032\,\frac{kg}{mol}}{3\cdot 8{,}31\,\frac{J}{mol\,K}}$$

$$\approx 1{,}58\cdot10^5\,K$$

Dæmi 1, (11-03-42)



Jafnþrýstiferli, finna ΔE_{int}

1. lögmál varmafræðinnar

$$dE_{int} = dQ - dW$$
$$= dQ - p\,dV$$

Fyrst ætlaði ég að gera ráð fyrir að gasið væri kjörgas, en sú æfing með ábendingu nemanda sýndi mér að tölurnar af grafinu eiga ekki vel við kjörgas, því mun ég aðeins nota upplýsingarnar af grafinu og varmafræði

$$\Delta E_{int} = \Delta Q - p\Delta V$$
$$= 3100\,J - 1{,}0 \cdot 10^4 \cdot 0{,}15\,J = 1600\,J$$

Hluti varmans ΔQ = 3100 J sem settur er í gasið við fastan þrýsting leiðir til þess að það framkvæmir vinnu á umhverfinu við að þenjast út. Sú orka er því ekki tiltæk til að auka við innri orku kerfisins.

Dæmi 2, (11-03-74)

$T_1 \to T_2$

Kjörgas, nærjafnvægis óvermið ferli, sýna að sé vinna gassins

$$W = \frac{nR}{\gamma - 1}\left[T_2 - T_1\right]$$

$$dQ = 0$$
$$dW = p\,dV$$

vinna gassins er $dW = p\,dV$

Höfum $pV^{\gamma} = \text{fasti}$ og $pV = nRT \to TV^{\gamma-1} = \text{fasti}$

$$dT\,V^{\gamma-1} + T(\gamma-1)V^{\gamma-2}dV = 0$$

$$\to dV = \frac{V\,dT}{T(\gamma-1)} \to p\,dV = \frac{nRT\,dT}{T(\gamma-1)} = \frac{nR}{\gamma-1}dT$$

$$W = \int_{T_1}^{T_2} dW = \frac{nR}{\gamma-1}\int_{T_1}^{T_2} dT = \frac{nR}{\gamma-1}\left[T_2 - T_1\right]$$

5

Dæmi 3, (11-04-60)

a) 1og H_2O bráðna við 0°C $\qquad \Delta Q = mL_f$ inn í H_2O

Metum
$$\left.\begin{aligned} \Delta S_{H_2O} &= \frac{\Delta Q}{T} \\ \Delta S_{air} &= -\frac{\Delta Q}{T} \end{aligned}\right\} \longrightarrow \Delta S_{umhv} = 0$$

T breytist ekki, gerist hægt --> $\Delta S_{universe} = 0$

b) best er að skoða Ex. 4.7 og allar réttlætingar þar á aðferðinni. 1og íss bráðna í 20 °C

$T_A = 0\,°C$

$T_B = 20\,°C$

$$\Delta S_{H_2O} = m\left[\frac{L_f}{T_A} + c\int_{T_A}^{T_B} \frac{dT}{T}\right]$$

6

$$\Delta S_{H_2O} = m\left[\frac{L_f}{T_A} + c\ln\left(\frac{T_B}{T_A}\right)\right]$$

Varminn sem loftið tapar til íssins er

$$\Delta Q_{air} = m\left\{L_f + c(T_B - T_A)\right\}$$

$$\rightarrow \Delta S_{air} = -\frac{\Delta Q_{air}}{T_B} = -m\left[\frac{L_f}{T_B} + \frac{c(T_B - T_A)}{T_B}\right]$$

Ef $T_B > T_A$ þá eru allir liðirnir stærri en 0

$$\rightarrow \boxed{\Delta S_{umhv}} = \Delta S_{H_2O} + \Delta S_{air} = m\left\{L_f\left(\frac{1}{T_A} - \frac{1}{T_B}\right) + c\left[\ln\left(\frac{T_B}{T_A}\right) - \left(\frac{T_B - T_A}{T_B}\right)\right]\right\}$$

$\Delta S_{umhv} > 0$

skoðum betur á næstu síðu

7

T_A

$T_B = T_A + \delta T, \qquad \delta T << T_A, T_B$

$$\frac{1}{T_A} - \frac{1}{T_B} = \frac{1}{T_A} - \frac{1}{T_A + \delta T} > 0$$

$$\ln\left(\frac{T_B}{T_A}\right) - \frac{T_B}{T_B} + \frac{T_A}{T_B} = \ln\left(1 + \frac{\delta T}{T_A}\right) - \frac{\delta T}{T_A + \delta T}$$

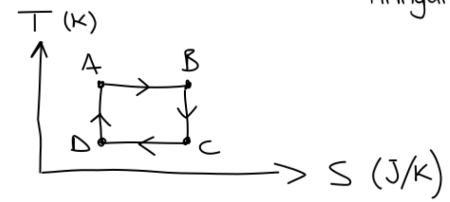
$$\approx \frac{\delta T}{T_A} + o\left(\left(\frac{\delta T}{T_A}\right)^2\right) - \frac{\delta T}{T_A + \delta T} \; \underline{> 0}$$

þar sem við notum $\ln(1+x) = x - x^2/2 + x^3/3 + \ldots$, ef $x << 1$

8

Dæmi 4, (11-4-61)

Hringur Carnots



ΔQ_{AB}

a) Q_H ? $\qquad \Delta S_H = \frac{\Delta Q_H}{T_H} \rightarrow \Delta Q_H = T_H \Delta S_H = 1200\,J$

b) Q_c ? $\qquad \Delta Q_c = T_c \Delta S_c = 600\,J$

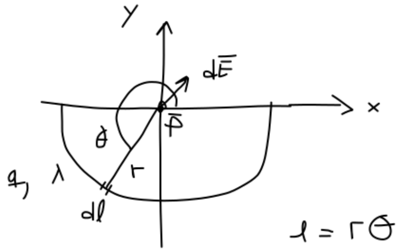
c) W = Flötur ferlis $= 300 \cdot 2{,}0 = 600\,J$ $\qquad$ eða

d) $e = 1 - \frac{T_c}{T_H} = 0{,}5$ $\qquad W = Q_H - Q_c = 600\,J$

(1)

Dæmi 1, (11-05-84)

$$q = \pi r \lambda = L\lambda$$



q: heildarhleðsla boga

λ: hleðsla á lengd

L = πr: lengd boga

$$l = r\theta \rightarrow dl = r\,d\theta$$

Stefna $d\bar{E}$: $\hat{r} = (\cos(\theta-\pi), \sin(\theta-\pi)) = (\cos\theta, \sin\theta)$

$$\bar{E}(\bar{P}) = \frac{1}{4\pi\epsilon_0}\int \left[\frac{\lambda\,dl}{r^2}\right]\hat{r}$$ ← einingarvigurinn er innan heildisins

$$E_x = \frac{\lambda r}{4\pi\epsilon_0 r^2}\int^{2\pi} \cos\theta \cdot d\theta = \frac{\lambda}{4\pi\epsilon_0 r}\left[\sin(2\pi) - \sin(\pi)\right] = 0$$

$$E_y = \frac{-\lambda}{4\pi\epsilon_0 r}\int_{\pi}^{2\pi}\sin\theta\cdot d\theta = -\frac{\lambda}{4\pi\epsilon_0 r}\left[-\cos(2\pi)+\cos(\pi)\right]$$

(2)

$$\rightarrow E_y = -\frac{\lambda(-2)}{4\pi\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0 r}$$

og þar með í P er

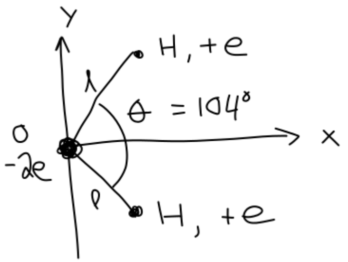
$$\bar{E} = \frac{\lambda}{2\pi\epsilon_0 r}(0, 1) = \frac{\lambda\,\hat{j}}{2\pi\epsilon_0 r}$$

miðað við hnitakerfið á rissmyndinni að framan

(3)

Dæmi 2, (11-05-107)

$$l = 0{,}9578\,\mathring{A} = 0{,}9578\cdot10^{-10}\,m$$



Reikna tvískutsvægið

$$\bar{p} = q\bar{d} \qquad \ominus \xrightarrow{\bar{p}} \oplus$$

Leggjum saman tvö tvískautsvægi sem vigra

þá styttist út vægið í y-stefnu, en eftir stendur

$$\bar{p} = \hat{i}\left[e\,l\cos\left(\frac{\theta}{2}\right)\cdot 2\right]$$

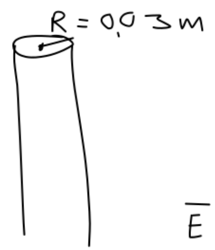
$$= \hat{i}\left[0{,}9578\,\mathring{A}\cdot 1{,}6022\cdot10^{-19}C\cdot\cos\left(\frac{52\pi}{180}\right)\cdot 2\right]$$

$$= \hat{i}\left[1{,}8896\cdot10^{-19}C\mathring{A}\right] = 1{,}8896\cdot10^{-29}\,Cm\,\hat{i}$$

(4)

Dæmi 3, (11-06-50)

$$\lambda = -500\,\mu C/m = -5{,}0\cdot10^{-4}\,C/m$$



a) Finna rafsviðið í fjarlægð r = 0.05 m
(fyrir utan óendanlega silfurleiðarann)

Í raun er allt tilbúið í 20. fyrirlestri eins og ég leysti dæmið þar (kerfin eru jafngild, hvers vegna?)

$$\bar{E} = \frac{\lambda\,\hat{r}}{2\pi\epsilon_0 r} = \frac{5\cdot10^{-4}\frac{C}{m}\,\hat{r}}{2\pi\left\{8{,}85\cdot10^{-12}\frac{C}{m^2N}\right\}0{,}05m}$$

$$= -1{,}798\cdot10^{8}\,\frac{N}{C}$$

b) innan sívalnings, r = 2 cm

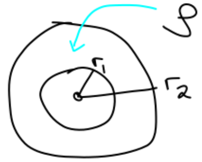
λ er á yfirborði leiðara. Þessi punktur er innan þess. Því er engin hleðsla innan Gauß-yfirborðsins

-->

$$\bar{E} = 0$$

Dæmi 4, (11-06-56)

Hlaðin kúluskel



$$\rho = \frac{r_1}{r}\rho_0$$

$r < r_1$ er engin hleðsla innan Gauß-yfirborðs

$$\bar{E} = 0$$

$r_1 < r < r_2$

$$Q_{enc}(r) = \int_{r_1}^{r} 4\pi r^2 \left(\frac{r_1}{r}\rho_0\right) dr$$

$$= 4\pi\rho_0 r_1 \int_{r_1}^{r} r\,dr = 4\pi\rho_0 r_1 \left[\frac{r^2}{2}\Big|_{r_1}^{r}\right]$$

$$= 2\pi r_1 \rho_0 \left[r^2 - r_1^2\right]$$

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\rightarrow \quad E\, 4\pi r^2 = \frac{2\pi}{\epsilon_0}\rho_0 r_1 \left[r^2 - r_1^2\right]$$

$$\rightarrow \quad \boxed{\bar{E} = \frac{\rho_0}{2\epsilon_0} r_1 \left[1 - \frac{r_1^2}{r^2}\right]\hat{r}} \qquad r_1 \leq r \leq r_2$$

$r > r_2$

$$Q = Q_{enc}(r_2) = 2\pi r_1 \rho_0 \left[r_2^2 - r_1^2\right]$$

$$\rightarrow \quad E\, 4\pi r^2 = \frac{2\pi}{\epsilon_0} r_1 \rho_0 \left[r_2^2 - r_1^2\right]$$

$$\rightarrow \quad \boxed{\bar{E} = \frac{\rho_0}{2\epsilon_0}\frac{r_1}{r^2}\left[r_2^2 - r_1^2\right]\hat{r}}$$

1

Dæmi 1, (11-07-56)

$$V(x,y,z) = -xy^2 z + 4xy$$

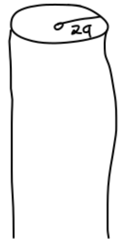
$$\bar{E}(x,y,z) = -\bar{\nabla}\, V(x,y,z) = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) V(x,y,z)$$

$$= \underline{\left(y^2 z - 4y,\ 2xy - 4x,\ xy^2\right)}$$

Stigullinn verkar á skalar stærð $V(x,y,z)$ og varpar henni á vigurstærð $\bar{E}(x,y,z)$ Hlutafleiða verkar aðeins á þá stærð, sem hún er með tilliti til, en lætur hinar óbreyttar.

2

Dæmi 2, (11-07-64)



óendanlegur leiðandi sívalningur með yfirborðshleðsluþéttleika σ. Samkvæmt Ex. 7.16 er ekki vænlegt að nota

$$V_p = k\int \frac{dq}{r} = k\int \frac{\sigma\, dA}{r}$$

fyrir sívalningssamhverfa óendanlega hleðslu vegna ósamleitinna heilda. Í stað munum við eftir í 20. fyrirlestri, blaðsíðu 9, að

$$\bar{E} = \frac{\lambda\, \hat{r}}{2\pi \epsilon_0 r}$$

var rafsvið línuhleðslu utan hennar í fjarlægð r. Við þurfum því að nota að

$$2\pi(2a)\sigma = \lambda \longrightarrow \boxed{\bar{E} = \frac{2a\sigma\, \hat{r}}{\epsilon_0 r}}$$

fyrir $r > 2a$

3

$$V(r) - V_R = -\int_R^r \bar{E}\cdot d\bar{l} = -\int_R^r \frac{2a\sigma}{\epsilon_0 r'}\, dr'$$

$$= -\frac{2a\sigma}{\epsilon_0} \ln(r')\Big|_R^r = -\frac{2a\sigma}{\epsilon_0} \ln\left(\frac{r}{R}\right)$$

þar sem R er viðmiðunarpunktur fyrir rafmættið (ákveðum hann seinna). Innan rörs, $r < 2a$, $E = 0$, samkvæmt lögmáli Gauß (engin hleðsla)

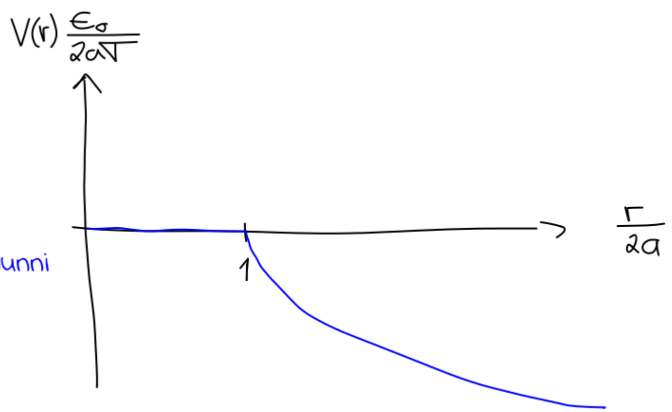
$$\longrightarrow \underline{V(\bar{r}) = \text{fasti} = C} \qquad \underline{r < 2a}$$

Rafmættið er samfellt í $r = 2a$ --> $R = 2a$ er heppilegt val

4

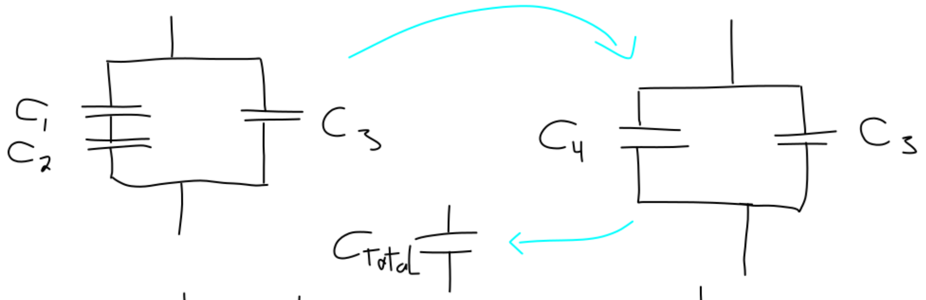
Ef $R = 2a$ --> $C = 0$ og því

$$V(r) = \begin{cases} 0 & \text{ef } r < 2a \\ -\frac{2a\sigma}{\epsilon_0} \ln\left(\frac{r}{2a}\right) & \end{cases}$$



Brotið í V er ósamfella í sviðinu E sem tengist beint yfirborðshleðslunni

Dæmi 3, (11-08-36)

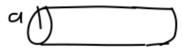


$$\frac{1}{C_4} = \frac{1}{C_1} + \frac{1}{C_2} \longrightarrow C_4 = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$\rightarrow C_4 = \frac{C_1 C_2}{C_2 + C_1}, \quad C_{Total} = C_4 + C_3 = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

Dæmi 4, (11-09-32)

Straumþéttleiki $|J(r)| = Cr^2$



$$I = \int \bar{J}(\bar{r}) \cdot d\bar{A}$$

$$I = 2\pi \int_0^a J(r)\, r\, dr = 2\pi \int_0^a C r^2 r\, dr$$

$$= 2\pi C \int_0^a r^3 dr = 2\pi C \frac{a^4}{4} = \frac{\pi C a^4}{2}$$

og einingarnar passa samkvæmt því sem gefið er fyrir C A/m^4

1

Dæmi 1, (11-11-60)

$$\bar{B} = (B_x, 0, B_z) = (0{,}5,\ 0{,}0,\ 0{,}8)\,\mathrm{T}$$

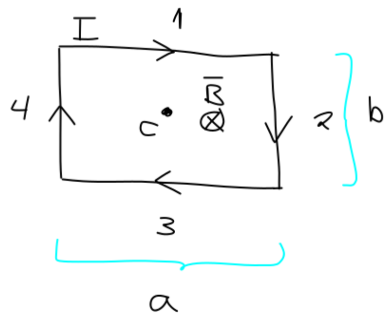
$$\bar{v} = (v_x, v_y, 0) = (3{,}0,\ 4{,}0,\ 0{,}0)\cdot 10^6\ \mathrm{m/s}$$

Finna $\bar{F} = q\,\bar{v} \times \bar{B}$

$$\bar{F} = q\begin{vmatrix} i & j & k \\ v_x & v_y & 0 \\ B_x & 0 & B_z \end{vmatrix} = q\left(v_y B_z,\ -v_x B_z,\ -B_x v_y\right)$$

2

Dæmi 2, (11-12-20)



Finna segulsviðið í punktinum C í miðri lykkjunni $\bar{B}_C$

Hægrihandarreglan gefur okkur að segulsviðið í C er inn í síðuna og $\bar{B}_4 = \bar{B}_2$, $\bar{B}_1 = \bar{B}_3$ í C

Ég nota jöfnur (12.5-6) með nauðsynlegri aðlögun

$$B_3 = \frac{\mu_0 I}{2\pi}\int_0^{a/2} \frac{R\,dx}{(x^2+R^2)^{3/2}}, \quad R = \frac{b}{2}$$

notandi

$$\int \frac{dx}{(x^2+R^2)^{3/2}} = \frac{x}{R^2(x^2+R^2)^{1/2}} + C$$

3

$$B_3 = \frac{\mu_0 I R}{2\pi}\ \frac{a\sqrt{4R^2+a^2}}{4R^4+a^2R^2} = \frac{\mu_0 I}{\pi b}\ \frac{1}{\sqrt{1+\left(\frac{b}{a}\right)^2}}$$

og á svipaðan hátt

$$B_4 = \frac{\mu_0 I}{\pi a}\ \frac{1}{\sqrt{1+\left(\frac{a}{b}\right)^2}}$$

$$\boxed{B_{Total}} = \frac{2\mu_0 I}{\pi b}\ \frac{1}{\sqrt{1+\left(\frac{b}{a}\right)^2}} + \frac{2\mu_0 I}{\pi a}\ \frac{1}{\sqrt{1+\left(\frac{a}{b}\right)^2}}$$

$$\boxed{= \frac{2\mu_0 I}{\pi a b}\sqrt{a^2+b^2}}$$

4

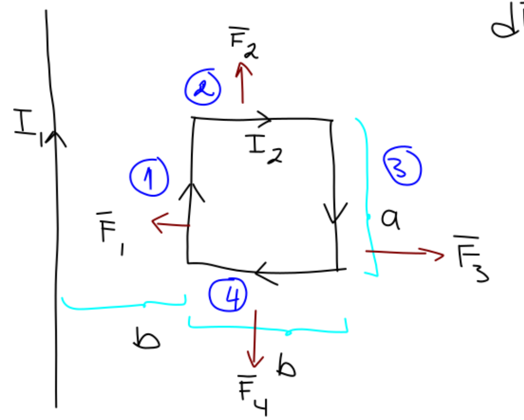
Dæmi 3, (11-12-34)

Finna heildarkraftinn á lykkjuna

$$d\bar{F} = I\,d\bar{l} \times \bar{B}$$

$$\boxed{\bar{F}_2 + \bar{F}_4 = 0}$$

$$\boxed{\bar{F}_1 + \bar{F}_3 \neq 0}$$



Á leiðurum 1 og 3 er segulsvið vegna I1 fast

5

$$F_1 = I_2 a \frac{\mu_0 I_1}{2\pi b}$$

$$F_3 = I_2 a \frac{\mu_0 I_1}{2\pi\, 2b} \quad \rightarrow \quad F_1 > F_3$$

$$F_1 - F_3 = \frac{\mu_0 I_1 I_2 a}{2\pi}\left(\frac{1}{b} - \frac{1}{2b}\right)$$

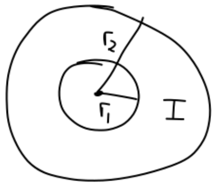
$$= \frac{\mu_0 I_1 I_2}{4\pi}\left(\frac{a}{b}\right)$$

$$\rightarrow \boxed{\bar{F}_{total} = -\frac{\mu_0 I_1 I_2}{4\pi}\left(\frac{a}{b}\right)\hat{i}}$$

með stefnu að beina vírnum með I_1

6

Dæmi 4, (11-12-46)



$$\oint \bar{B}\cdot d\bar{l} = \mu_0 I$$

Fyrir $r < r_1$ $\quad I_{enc} = 0 \rightarrow \boxed{\bar{B} = 0}$

Fyrir $r > r_2$

$$2\pi r B = \mu_0 I$$

$$\boxed{\bar{B}(r) = \frac{\mu_0 I}{2\pi r}\hat{\theta}}$$

7

Fyrir $r_1 < r < r_2$

$$I_{enc}(r) = I\left(\frac{r^2 - r_1^2}{r_2^2 - r_1^2}\right)$$

$$\rightarrow \quad 2\pi r B = \mu I\left(\frac{r^2 - r_1^2}{r_2^2 - r_1^2}\right)$$

$$\rightarrow \quad \boxed{\bar{B}(r) = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - r_1^2}{r_2^2 - r_1^2}\right)\hat{\theta}}$$