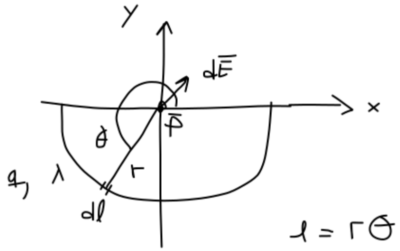
1

Dæmi 1, (11-05-84)



$$q = \pi r \lambda = L\lambda$$

q: heildarhleðsla boga

λ: hleðsla á lengd

L = πr: lengd boga

$$l = r\theta \;\rightarrow\; dl = r\,d\theta$$

Stefna $d\bar{E}$: $\hat{r} = (\cos(\theta - \pi), \sin(\theta - \pi)) = (\cos\theta, \sin\theta)$

$$\bar{E}(\bar{P}) = \frac{1}{4\pi\epsilon_0}\int \left[\frac{\lambda\, dl}{r^2}\right]\hat{r}$$ ← einingarvigurinn er innan heildisins

$$E_x = \frac{\lambda r}{4\pi\epsilon_0 r^2}\int_{\pi}^{2\pi} \cos\theta \cdot d\theta = \frac{\lambda}{4\pi\epsilon_0 r}\left[\sin(2\pi) - \sin(\pi)\right] = 0$$

$$E_y = \frac{-\lambda}{4\pi\epsilon_0 r}\int_{\pi}^{2\pi} \sin\theta \cdot d\theta = -\frac{\lambda}{4\pi\epsilon_0 r}\left[-\cos(2\pi) + \cos(\pi)\right]$$

2

$$\rightarrow\; E_y = -\frac{\lambda(-2)}{4\pi\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0 r}$$

og þar með í P er

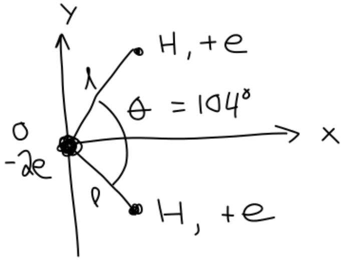
$$\bar{E} = \frac{\lambda}{2\pi\epsilon_0 r}(0, 1) = \frac{\lambda\,\hat{j}}{2\pi\epsilon_0 r}$$

miðað við hnitakerfið á rissmyndinni að framan

3

Dæmi 2, (11-05-107)

$$l = 0{,}9578\,\mathring{A} = 0{,}9578 \cdot 10^{-10}\,m$$



Reikna tvískutsvægið

$$\bar{P} = q\bar{d}$$

$$\ominus \xrightarrow{\bar{P}} \oplus$$

Leggjum saman tvö tvískautsvægi sem vigra

Þá styttist út vægið í y-stefnu, en eftir stendur

$$\bar{P} = \hat{i}\left[e\,l\,\cos\left(\frac{\theta}{2}\right)\cdot 2\right]$$

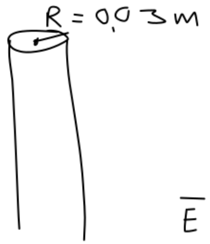
$$= \hat{i}\left[0{,}9578\,\mathring{A}\cdot 1{,}6022\cdot 10^{-19}\,C\cdot\cos\left(\frac{52\pi}{180}\right)\cdot 2\right]$$

$$= \hat{i}\left[1{,}8896\cdot 10^{-19}\,C\mathring{A}\right] = \underline{1{,}8896\cdot 10^{-29}\,Cm\,\hat{i}}$$

4

Dæmi 3, (11-06-50)

$$\lambda = -500\,\mu C/m = -5{,}0\cdot 10^{-4}\,C/m$$



a) Finna rafsviðið í fjarlægð r = 0.05 m
(fyrir utan óendanlega silfurleiðarann)
Í raun er allt tilbúið í 20. fyrirlestri eins og ég leysti dæmið þar (kerfin eru jafngild, hvers vegna?)

$$\bar{E} = \frac{\lambda\,\hat{r}}{2\pi\epsilon_0 r} = \frac{5\cdot 10^{-4}\,\frac{C}{m}}{4\pi\left[8{,}85\cdot 10^{-12}\,\frac{C^2}{m^2 N}\right]0{,}05\,m}\,\hat{r}$$

$$= \underline{-1{,}798\cdot 10^{8}\,\frac{N}{C}}$$

b) innan sívalnings, r = 2 cm

λ er á yfirborði leiðara. Þessi punktur er innan þess. Því er engin hleðsla innan Gauß-yfirborðsins
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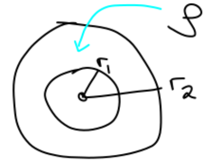
$$\underline{\bar{E} = 0}$$

Dæmi 4, (11-06-56)

Hlaðin kúluskel



$$\rho = \frac{r_1}{r}\rho_0$$

$r < r_1$ er engin hleðsla innan Gauß-yfirborðs

$$\bar{E} = 0$$

$r_1 < r < r_2$

$$Q_{enc}(r) = \int_{r_1}^{r} 4\pi r^2 \left(\frac{r_1}{r}\rho_0\right) dr$$

$$= 4\pi\rho_0 r_1 \int_{r_1}^{r} r\,dr = 4\pi\rho_0 r_1 \left[\frac{r^2}{2}\Big|_{r_1}^{r}\right]$$

$$= 2\pi r_1 \rho_0 \left[r^2 - r_1^2\right]$$

$$\oint \bar{E}\cdot d\bar{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\rightarrow \quad E\, 4\pi r^2 = \frac{2\pi}{\epsilon_0}\rho_0 r_1 \left[r^2 - r_1^2\right]$$

$$\rightarrow \quad \boxed{\bar{E} = \frac{\rho_0}{2\epsilon_0} r_1 \left[1 - \frac{r_1^2}{r^2}\right]\hat{r}} \qquad r_1 \leq r \leq r_2$$

$r > r_2$

$$Q = Q_{enc}(r_2) = 2\pi r_1 \rho_0 \left[r_2^2 - r_1^2\right]$$

$$\rightarrow \quad E\, 4\pi r^2 = \frac{2\pi}{\epsilon_0} r_1 \rho_0 \left[r_2^2 - r_1^2\right]$$

$$\rightarrow \quad \boxed{\bar{E} = \frac{\rho_0}{2\epsilon_0}\frac{r_1}{r^2}\left[r_2^2 - r_1^2\right]\hat{r}}$$