Dæmi 1, (1-09-37)

$v_b = 400$ m/s

$m = 0{,}200$ kg

$M = 1{,}50$ kg

Kúlan festist í kubbnum

a) Finna hraða kubbs og kúlu eftir að hún festist, varðveisla skriðþunga

$$m v_b + M\cdot 0 = (m+M)v_f \rightarrow \boxed{v_f = \frac{m}{m+M}v_b}$$

$$v_f = \frac{0{,}2}{0{,}2+1{,}5}400 \text{ m/s} = 47{,}1 \text{ m/s}$$ til hægri

b) Atlag kubbs á kúlu?

$$\boxed{\bar{J}_m} = m\left\{v_f - v_b\right\} = m\left\{\frac{m v_b}{m+M} - v_b\right\}$$

$$= m v_b\left\{\frac{-M}{m+M}\right\} = \boxed{-\frac{mM}{m+M}v_b}$$

$$= 0{,}2\left\{47{,}1 - 400\right\} \text{ kg}\frac{\text{m}}{\text{s}} \approx -70{,}6 \text{ kgm/s}$$

til vinstri

c) Atlag kúlu á kubb

$$\boxed{\bar{J}_M} = M\left\{v_f - 0\right\} = \boxed{\frac{mM}{m+M}v_b}$$

$$= 70{,}6 \text{ kg}\frac{\text{m}}{\text{s}}$$ til hægri

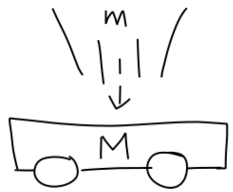
Því er ljóst að $\bar{J}_m + \bar{J}_M = 0$

d) Ef $\Delta t = 3$ms finna F_{ave}

$$\bar{F}_{ave} = \frac{\Delta\bar{p}}{\Delta t} = \frac{\bar{J}}{\Delta t} \rightarrow \boxed{|\bar{F}_{ave}| = \left(\frac{mM}{m+M}\right)\frac{v_b}{\Delta t}}$$

$$\approx 2{,}35\cdot 10^{4} \text{ N}$$

Dæmi 2, (1-09-42)



$\rightarrow v_i = 4{,}4 \frac{\text{m}}{\text{s}}$

Hvað má m vera mest til að lokahraðinn verði ekki minni en $v_f \geq 3{,}0\frac{\text{m}}{\text{s}}$

varðveisla skriðþunga

$$M v_i + m\cdot 0 = (M+m)v_f$$

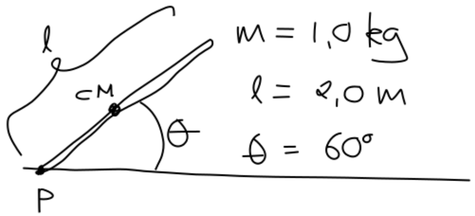
$$\rightarrow v_f = \frac{M v_i}{M+m} \rightarrow M + m = M\frac{v_i}{v_f}$$

$$\rightarrow \boxed{m = M\frac{v_i}{v_f} - M = M\left\{\frac{v_i}{v_f} - 1\right\}}$$

$$\approx 2000\left\{\frac{4{,}4}{3{,}0} - 1\right\} = 933{,}3 \text{ kg}$$

Dæmi 3, (1-10-67)

Finna ferð enda stangar þegar hún kemur í lárétta stöðu



$m = 1{,}0\ \text{kg}$

$l = 2{,}0\ \text{m}$

$\theta = 60^\circ$

Notum orkuvarðveislu

$$\boxed{E_T = \frac{m}{2} v_P^2 + \frac{I}{2}\omega^2 + mgh}$$

i) $E_T^i = 0 + 0 + mgh_{CM} = mg\frac{l}{2}\sin\theta$

f) $E_T^f = \frac{m}{2} v_P^2 + \frac{1}{2} I \omega^2 = 0 + \frac{I}{2}\omega^2$

$$I = I_{CM} + md^2 = \frac{ml^2}{3}$$

$$\rightarrow E_T^f = 0 + \frac{m}{6} l^2\omega^2 = \frac{m}{2}\,\frac{1}{3}\,l^2\omega^2$$

$$\rightarrow \frac{m}{2}\sin\theta \cdot gl = \frac{m}{2}\left[l^2\omega^2\frac{1}{3}\right]$$

$$\rightarrow g\sin\theta = l\omega^2\frac{1}{3}$$

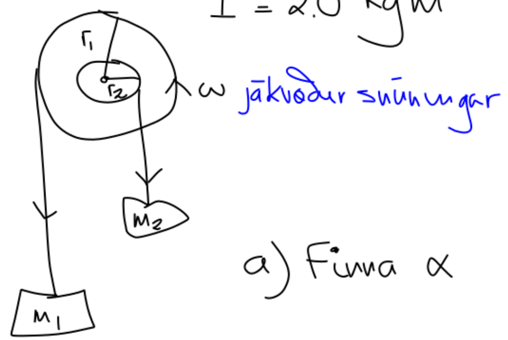
$$\rightarrow \omega^2 = \frac{g\cdot 3\cdot\sin\theta}{l} \rightarrow \omega = \sqrt{3\,\frac{g\sin\theta}{l}}$$

$$\boxed{v_{endi} = l\omega = \sqrt{3\,gl\sin\theta}}$$

$$= 7{,}14\ \text{m/s}$$

Dæmi 4, (1-10-92)

$I = 2{,}0\ \text{kgm}^2$



$r_1 = 0{,}5\ \text{m}$

$r_2 = 0{,}2\ \text{m}$

$m_1 = 1{,}0\ \text{kg}$

$m_2 = 2{,}0\ \text{kg}$

a) Finna α

$$\bar{\tau} = \bar{r}\times\bar{F}\ ,\quad L = I\omega$$

Annað lögmál Newtons fyrir snúning

$$\frac{d\bar{L}}{dt} = \sum_i \bar{\tau}_i \rightarrow I\alpha = \tau_1 - \tau_2$$

$$\rightarrow \boxed{\alpha = \frac{(m_1 r_1 - m_2 r_2)g}{I}} = 0{,}49\ 1/s^2$$

Jákvæður snúningur $m_1 \downarrow$ og $m_2 \uparrow$

b) $\boxed{a_1 = -r_1\alpha} = -0{,}25\ \text{m/s}^2$

$\boxed{a_2 = r_2\alpha} = 0{,}1\ \text{m/s}^2$

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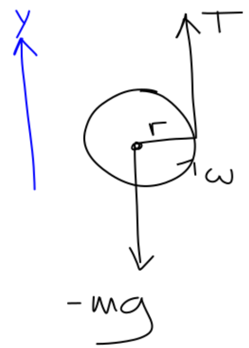
Dæmi 5, (1-11-26)



Finna hröðun a

Annað lögmál Newtons fyrir hring og línulega hreyfingu

$$\frac{d\bar{L}}{dt} = \bar{\tau} \quad , \quad \bar{F} = m\bar{a}$$

$$I\frac{d}{dt}\omega = \tau \quad \longrightarrow \quad I\alpha = \tau$$

$$\tau = Tr = \frac{1}{2}mr^2\alpha = \frac{1}{2}mr^2\left(\frac{a}{r}\right)$$

$$\longrightarrow \boxed{T = \frac{m}{2}a}$$

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$$F = ma$$

$$\longrightarrow -mg + T = -ma$$

$$\longrightarrow -mg + \frac{1}{2}ma = -ma$$

$$\longrightarrow g - \frac{a}{2} = a \quad \longrightarrow \quad \boxed{a = \frac{2}{3}g}$$

með stefnu niður á við

Hröðunin er minni en g, þar sem í þyngdarhröðunin verkar bæði á tregðumassann og hverfitregðuna. Þetta dæmi má líka leysa með því að nota punktinn á jójóinu þar sem bandið kemur að sem snúningsás í stað massamiðju. Þá er það þyngdarkrafturinn sem fær vægi á jójóið. Þá þarf að nota setn. Steiners á I

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Dæmi 6, (1-11-53)

$$M = 2.0 \cdot 10^{30}\,\text{kg}$$

Þyngdarhrun sólar $R_i \longrightarrow R_f$

$$R_i = 7.0 \cdot 10^{8}\,\text{m}$$

$$R_f = 3.5 \cdot 10^{6}\,\text{m}$$

$$T_i = 28\,d$$

$$I = \frac{2}{5}MR^2$$

finna $T_f = \frac{2\pi}{\omega_f}$

Varðveisla hverfiþunga $\quad I_i\omega_i = I_f\omega_f$

$$\longrightarrow \omega_f = \left(\frac{R_i}{R_f}\right)^2\omega_i$$

$$\longrightarrow \frac{2\pi}{T_f} = \left(\frac{R_i}{R_f}\right)^2\frac{2\pi}{T_i}$$

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$$\longrightarrow \boxed{T_f = \left(\frac{R_f}{R_i}\right)^2 T_i}$$

$$= \left(\frac{3.5 \cdot 10^{6}}{7.0 \cdot 10^{8}}\right) T_i$$

$$\simeq 2.5 \cdot 10^{-5} \cdot 28\,d = 7.0 \cdot 10^{-4}\,d$$

$$\simeq \underline{60.5\;s}$$