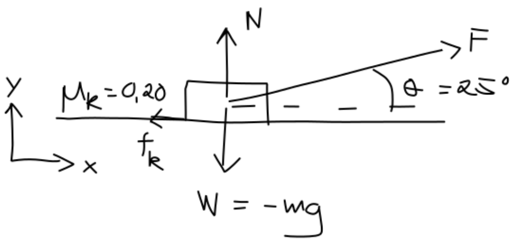
Dæmi 1 (1-07-34)



$\bar{v}$: Constant

First we start by finding $F = |\bar{F}|$

$$y: \quad N + F\sin\theta - mg = 0 \quad \rightarrow \quad N = mg - F\sin\theta$$

$$x: \quad -\mu_k N + F\cos\theta = 0$$

$$y \rightarrow x: \quad -\mu_k\{mg - F\sin\theta\} + F\cos\theta = 0$$

only F is unknown here

$$F\{\mu_k \sin\theta + \cos\theta\} = \mu_k mg$$

$$\boxed{F = \frac{\mu_k mg}{\mu_k \sin\theta + \cos\theta}}$$

a) $$W_F = F\cdot\cos\theta\cdot L = \frac{\mu_k mg\cos\theta\cdot L}{\mu_k\sin\theta + \cos\theta}$$

b) Find the work of the friction force

$$W_{f_k} = -\mu_k N\cdot L = -\mu_k\{mg - F\sin\theta\}L$$

$$W_{f_k} = -\mu_k\left\{mg - \frac{\mu_k mg\sin\theta}{\mu_k\sin\theta + \cos\theta}\right\}L$$

$$= -\mu_k\left\{\frac{\mu_k mg\cos\theta}{\mu_k\sin\theta + \cos\theta}\right\}L$$

So we see that the work of the friction force is exactly opposite to the work of the external F.

c) The total work of both external forces is 0, as the system is not accelerated. Interesting is to check that the results for $\theta = 0$ are familiar...

Dæmi 2 (1-07-42) x-y-plane $\bar{F} = \left(x, \frac{y^2}{3m}\right) 50\ \frac{N}{m}$

Calculate the work of F for the translation from A = (3,4) to B = (6,8)

$$W = \int_A^B \bar{F}\cdot d\bar{r} = \int_A^B \{F_x dx + F_y dy\}$$

$$= \int_3^6 F_x dx + \int_4^8 F_y dy = 50\int_3^6 x\,dx + 50\int_4^8 \frac{y^2}{3} dy$$

$$= 675\ \text{Nm} + \frac{22400}{9}\ \text{Nm}$$

$$= 3163{,}9\ \text{Nm}$$

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Dæmi 3 (1-08-24)

$$F(x) = \left(\frac{3{,}0}{x}\right) N$$

on the positive x-axis

a)

$$\boxed{W} = \int_A^B F(x)\,dx = \int_2^5 \frac{3}{x}\,dx = 3\ln(x)\Big|_2^5$$

$$= 3\left[\ln(5) - \ln(2)\right]$$

$$= 3\ln\left(\frac{5}{2}\right) = \boxed{3\ln\left(\frac{5}{2}\right) Nm}$$

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b) Can we find U such that $F = -\frac{\partial}{\partial x} U$

Guess $U(x) = -3\ln(ax) + U_0$ with $a > 0$ and U_0 as constants

$$\rightarrow F = -\frac{\partial}{\partial x}\left\{-3\ln(ax) + U_0\right\} = \frac{3a}{ax} = \frac{3}{x}$$

It is not possible to use $x = \infty$ as a reference point, but a is free and fixes a reference point, f. ex.

$a = 1\ m^{-1}$ gives $U(1) = 0$

The logarithm has very special properties....

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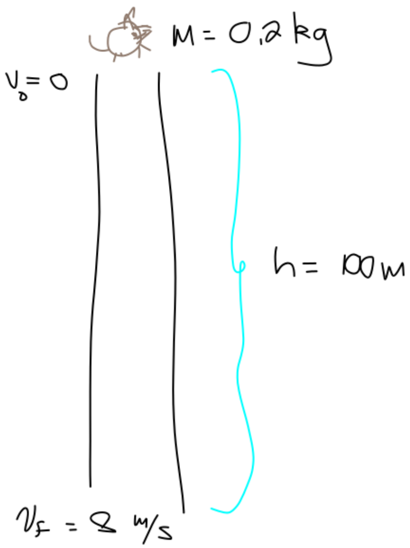
Dæmi 4 (1-08-32) Conservation of energy



No friction or air resistance

$$v = \sqrt{2gh}$$

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}m\,2gh = mgh \ !$$

with friction the energy

$$mgh - \frac{1}{2}mv_f^2$$

is taken out of the system, thus the friction does the work

$$\boxed{W_f = \frac{1}{2}mv_f^2 - mgh}$$

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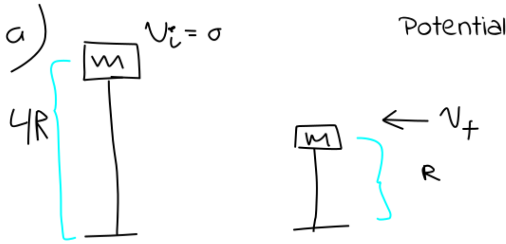
Dæmi 5 (1-08-42) Energy conservation

a)



Potential energy is linear in h

potential energy of $3R \rightarrow E_{km}$

$$\rightarrow \frac{1}{2}mv_f^2 = mg3R$$

$$\rightarrow v_f^2 = 6gR$$

a) $\rightarrow v_f = \sqrt{6gR}$

b)

$$F_c = m\frac{v_f^2}{R}$$

$= m6g$ to the right