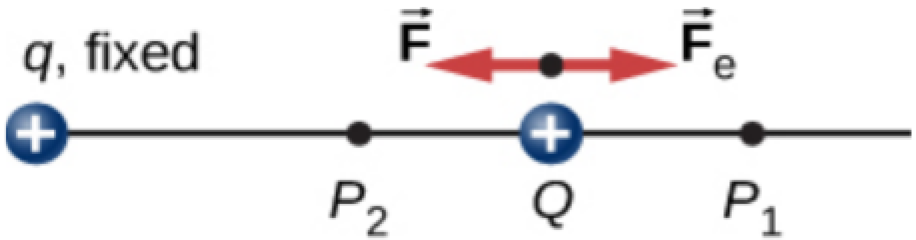
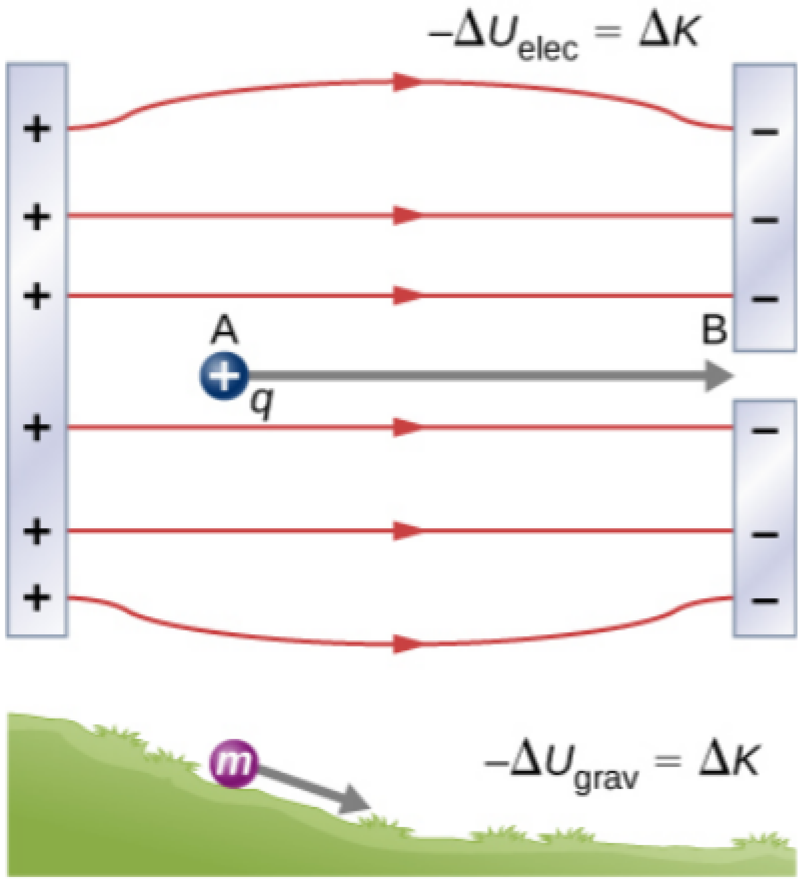
# Rafstöðumætti - spenna

Hægt er að hraða hleðslu með rafsviði

Skoðum hreyfanlega hleðslu Q nærri fastri hleðslu q




openstax

$\vec{F}_e$ er rafkraftur q á Q, $\vec{F}$ er ytri kraftur á Q
vinna $\vec{F}$ á Q vegna færslu frá $P_1$ til $P_2$ er

$$W_{12} = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}$$

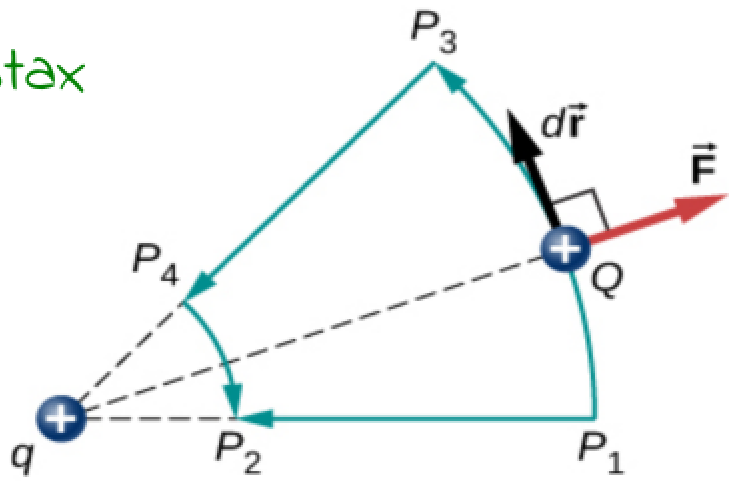
veljum $F = -F_e$

$$\vec{F} = -\vec{F}_e = -\frac{kQq}{r^2}\hat{r}$$

breyting stöðuorku Q er þá

$$\Delta U = -W$$

openstax




Vinnan fyrir $P_4 \to P_2$ og $P_1 \to P_3$
er 0, því $\vec{F} \cdot d\vec{r} = 0$ þar
$W_{34} = W_{12} = -W_{21}$

$$\to \boxed{\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = 0}$$

$$W_{12} = kqQ \int_{r_1}^{r_2} \frac{1}{r^2} \hat{\mathrm{r}} \cdot \hat{\mathrm{r}} dr = kqQ \frac{1}{r_2} - kqQ \frac{1}{r_1}$$

$$\Delta U = - \int_{r_{\text{ref}}}^{r} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$$

$$\boxed{U(r) = k\frac{qQ}{r} - U_{\text{ref}}}$$

oft er hægt að velja $U_{ref}$ í óendanlegri fjarlægð en ekki alltaf (t.d. gengur ekki fyrir línuhleðslu og sívalning)

# Uppröðun hleðslu kostar vinnu - stöðuorka rafhleðslna

$$W_{12\ldots N} = \frac{k}{2}\sum_{i}^{N}\sum_{j}^{N}\frac{q_i q_j}{r_{ij}} \text{ for } i \neq j$$

, $W_e = \frac{1}{2}\sum_{i=1}^{N} q_i V_i$

openstax

$$V_i = \frac{U_i}{q_i}$$

$$W_e = \frac{1}{2}\int_V \rho V \, dV'$$

$$W_e = \frac{1}{2}\epsilon_0 \int_V E^2 dV'$$

Fyrir samfellda hleðsludreifingu $\rho$

$U_i$ er rafstöðuorka $q_i$ vegna hinna hleðslanna

Rafstöðumætti og mættismunur - spennumunur

openstax

## Electric Potential

The electric potential energy per unit charge is

$$V = \frac{U}{q}. \quad 7.4$$

## Electric Potential Difference

The **electric potential difference** between points $A$ and $B$, $V_B - V_A$, is defined to be the change in potential energy of a charge $q$ moved from $A$ to $B$, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1 \text{ V} = 1 \text{ J/C}$$

## Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

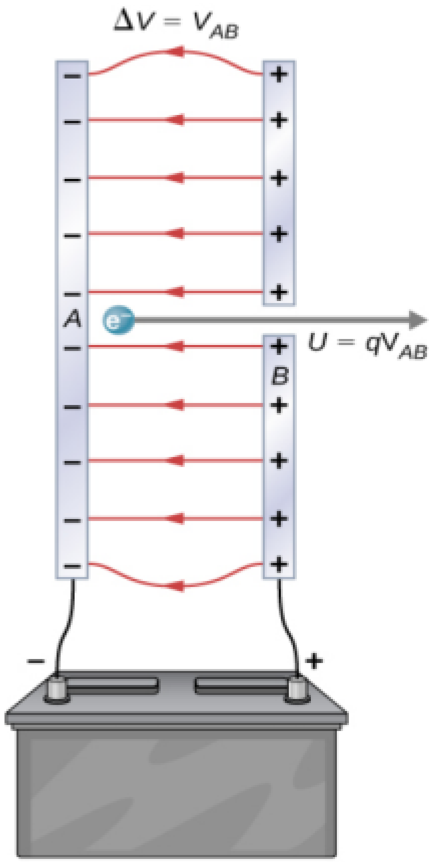
$$\Delta V = \frac{\Delta U}{q} \text{ or } \Delta U = q\Delta V. \quad 7.5$$

# Rafeindavolt (eV) - orkueining

## Electron-Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron-volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$1\text{ eV} = (1.60 \times 10^{-19}\text{ C})(1\text{ V}) = (1.60 \times 10^{-19}\text{ C})(1\text{ J/C}) = 1.60 \times 10^{-19}\text{ J}.$$




Jónunarorka einu rafeindar vetnisatóms er 13.6 eV

Massaorka rafeindar er 511 keV

$k_B = 8.617 \times 10^{-5}$ eV/K $=$ 0.08617 meV/K

--> $k_B T = 0.26$ meV fyrir $T = 3.0$ K

$k_B T = 25$ meV fyrir $T = 20$ °C

openstax

Spenna og rafsvið

$$U_P = -\int_R^P \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$$

$$U_P = -q\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}$$

$$\boxed{V_P = -\int_R^P \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}}}$$

openstax

### Electric Potential *V* of a Point Charge

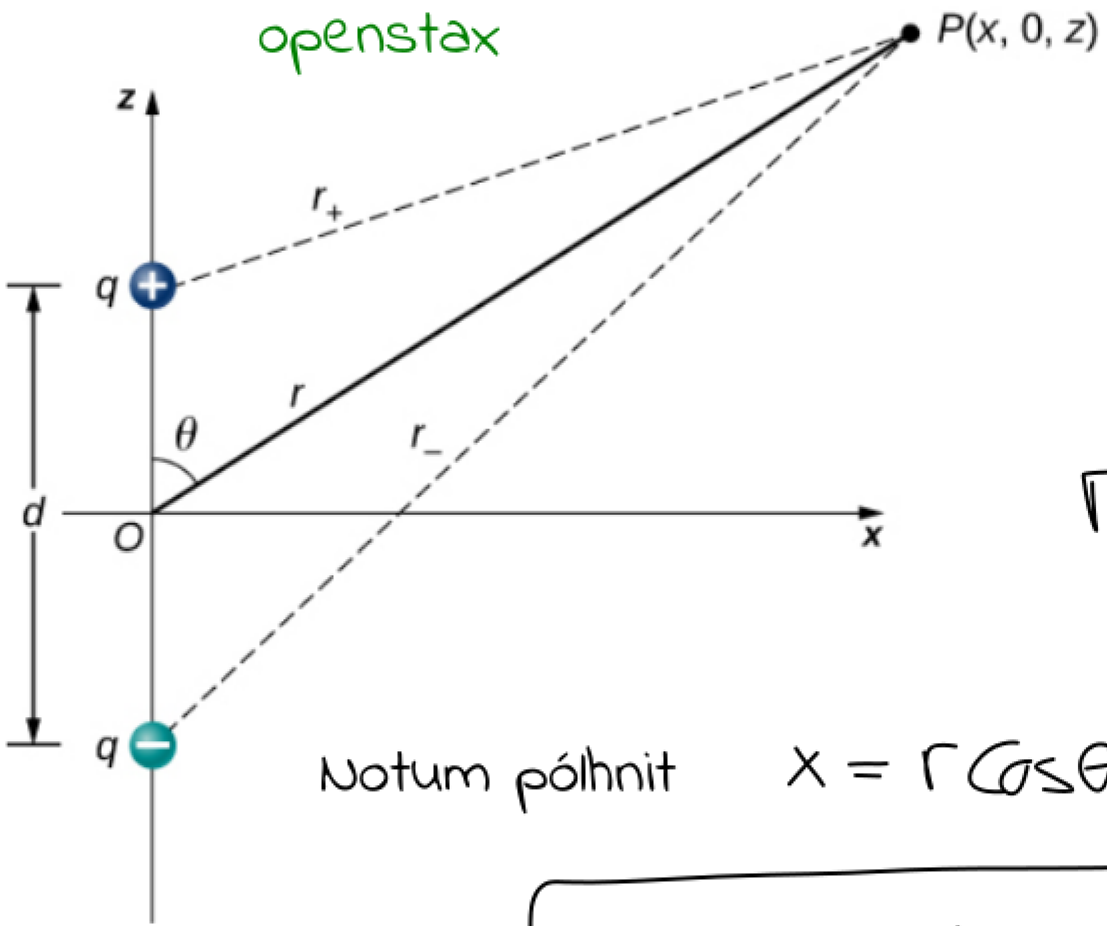
The electric potential $V$ of a point charge is given by

$$V = \frac{kq}{r} \text{(point charge)} \tag{7.8}$$

where $k$ is a constant equal to $8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$.

## Tvískaut

openstax




$$V_P = V_+ + V_-$$

$$= k\left[\frac{q}{r_+} - \frac{q}{r_-}\right]$$

$$r_\pm = \sqrt{x^2 + \left(z \mp \frac{d}{2}\right)^2}$$

Notum pólhnit $\quad x = r\cos\theta \,, \quad y = r\sin\theta$

$$r_\pm = \sqrt{r^2\sin^2\theta + \left(r\cos\theta \mp \frac{d}{2}\right)^2}$$

$$= r\sqrt{\sin^2\theta + \left(\cos \mp \frac{d}{2r}\right)^2}$$

$$r_\pm = r\sqrt{\underbrace{\sin^2\theta + \cos^2\theta}_{=1} \mp \frac{d}{r}\cos\theta + \left(\frac{d}{2r}\right)^2}$$

$$= r\sqrt{1 \mp \frac{d}{r}\cos\theta + \left(\frac{d}{2r}\right)^2}$$

viljum skoða fjærsviðið þegar $r >> d$

$$\to r_\pm \simeq r\sqrt{1 \mp \frac{d}{r}\cos\theta}$$

viljum líka nota

$$\frac{1}{\sqrt{1 \mp \alpha}} \simeq 1 \pm \frac{\alpha}{2}$$

ef $\alpha << 1$

$$\to V_p = k\left\{\frac{q}{r}\left(\cancel{1} + \frac{d\cos\theta}{2r}\right) - \frac{q}{r}\left(\cancel{1} - \frac{d\cos\theta}{2r}\right)\right\}$$

$$= k\,\frac{qd\cos\theta}{r^2}$$

Skilgreinum tvískautsvægi $\bar{p} = q\bar{d}$ $\rightarrow$ $V_p = k\dfrac{\bar{p}\cdot\hat{r}}{r^2}$

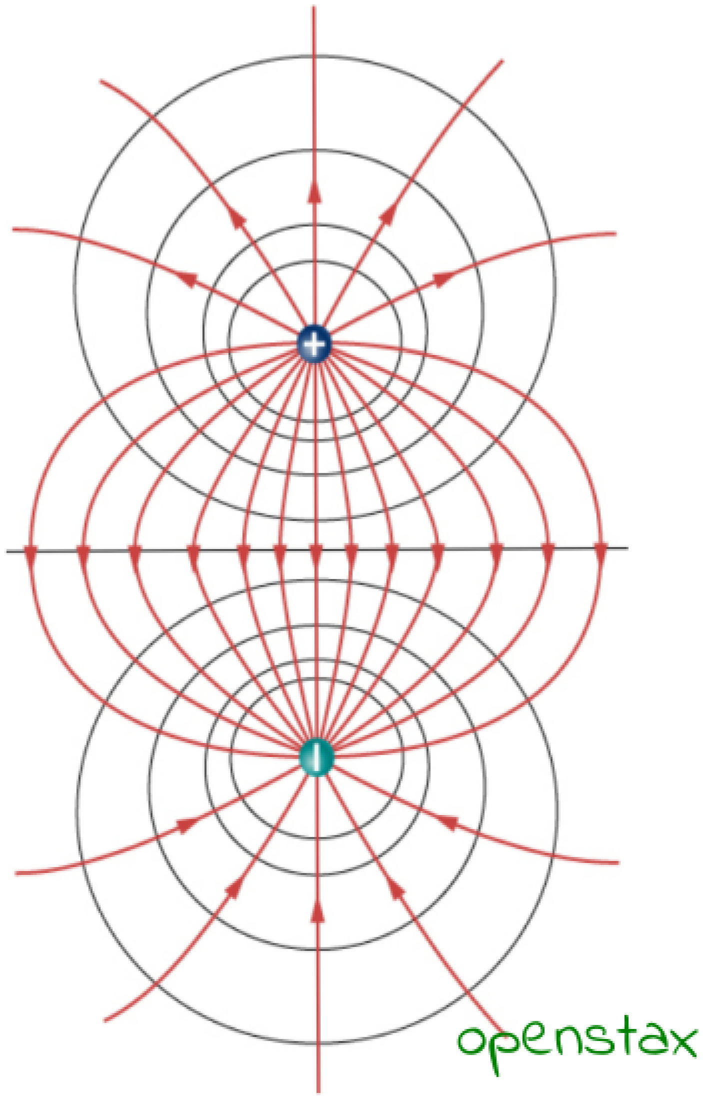
Tvískautið hefur því aðfelluform $V \sim \dfrac{1}{r^2}$ þegar $r \to \infty$
meðan stök hleðsla hefur $V \sim \dfrac{1}{r}$

Tvískautið er með rafsvið og rafmætti sem ekki eru stefnusnauð

Tvískaut og hærri skaut koma mikið fyrir í sameindum og hafa mikil áhrif á efnafræði þeirra

Tímaháð tvískaut geta geislað rafsegulbylgjum tímaháð einskaut getur það ekki

Hér sjást rafsviðslínur og jafnspennufletir sem við komum að rétt bráðum



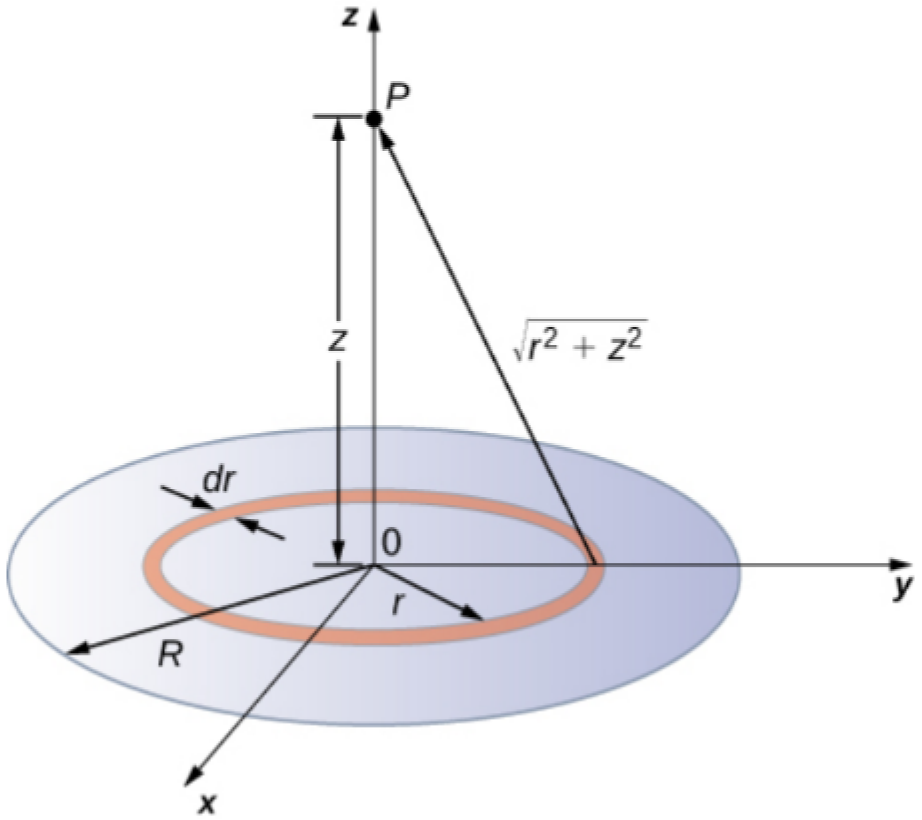
# Rafmætti samfelldrar hleðsludreifingar

openstax

$$V_P = k\int \frac{dq}{r}$$

$$dq = \begin{cases} \lambda\, dl & \text{(one dimension)} \\ \sigma\, dA & \text{(two dimensions)} \\ \rho\, dV & \text{(three dimensions)} \end{cases}$$

Skoðum skífu eða disk




$$dV_p = k\frac{dq}{\sqrt{z^2+r^2}}$$

$$dq = \sigma \cdot 2\pi r\, dr$$

$$V_p = k2\pi\sigma \int_0^R \frac{r\,dr}{\sqrt{z^2+r^2}}$$

$$= k2\pi\sigma\left[\sqrt{z^2+R^2} - \sqrt{z^2}\right]$$

Rafsvið frá rafmætti

openstax

### Relationship between Voltage and Uniform Electric Field

In equation form, the relationship between voltage and uniform electric field is

$$E = -\frac{\Delta V}{\Delta s}$$

where $\Delta s$ is the distance over which the change in potential $\Delta V$ takes place. The minus sign tells us that $E$ points in the direction of decreasing potential. The electric field is said to be the gradient (as in grade or slope) of the electric potential.

$$\boxed{\vec{\mathbf{E}} = -\vec{\nabla} V}$$

$$\vec{\nabla} = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$$

í Kartískum hnitum

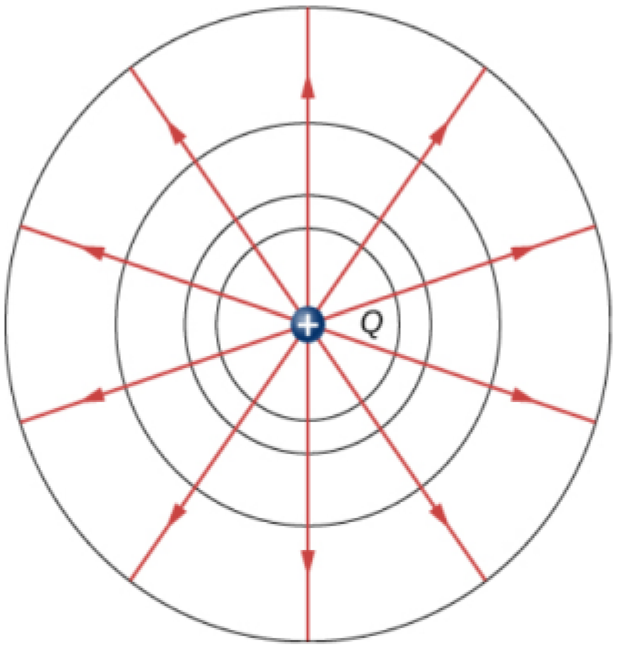
$$\text{Cylindrical: } \vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

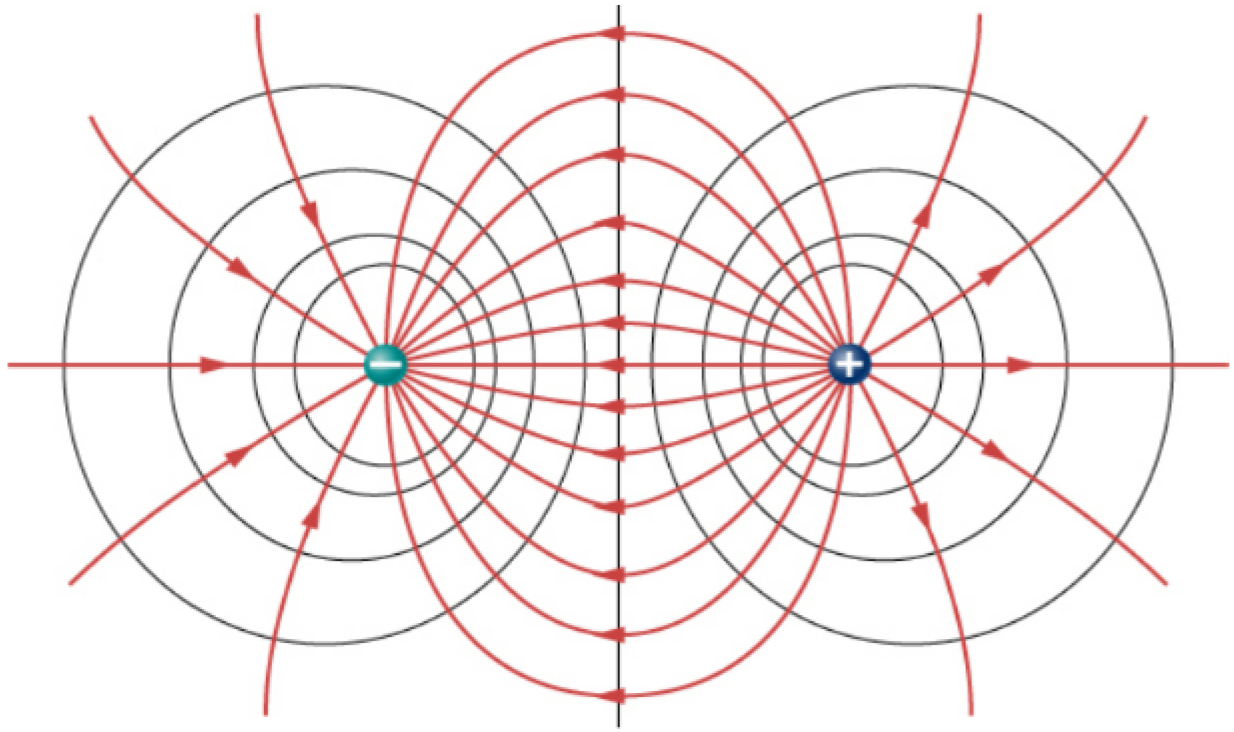
$$\text{Spherical: } \vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}$$

$\bar{E}$ er vigur, V er skalar og $\bar{\nabla}$ er afleiðuvirki sem varpar skalar í vigur

## Jafnspennufletir og rafsvið

Rafsvið er alltaf hornrétt á jafnspennufleti. Jafnspennuflötur er flötur þar sem V er fasti



openstax

Í jafnvægi er góður leiðari jafnspennuflötur

Þess vegna er hleðsla á hlöðnum leiðara í jafnvægi ekki endilega jafndreifð

$$k\frac{q_1}{R_1} = k\frac{q_2}{R_2} \rightarrow \frac{q_1}{R_1} = \frac{q_2}{R_2}$$

$$q = \sigma(4\pi R^2)$$

$$\sigma_1 R_1 = \sigma_2 R_2$$

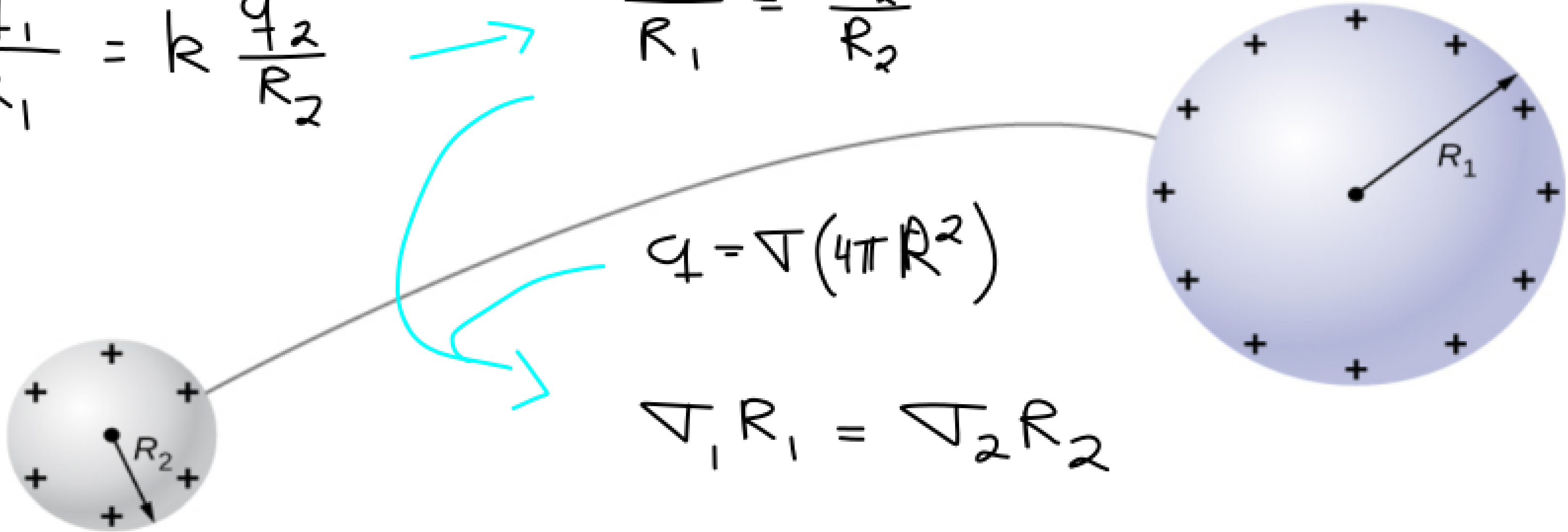
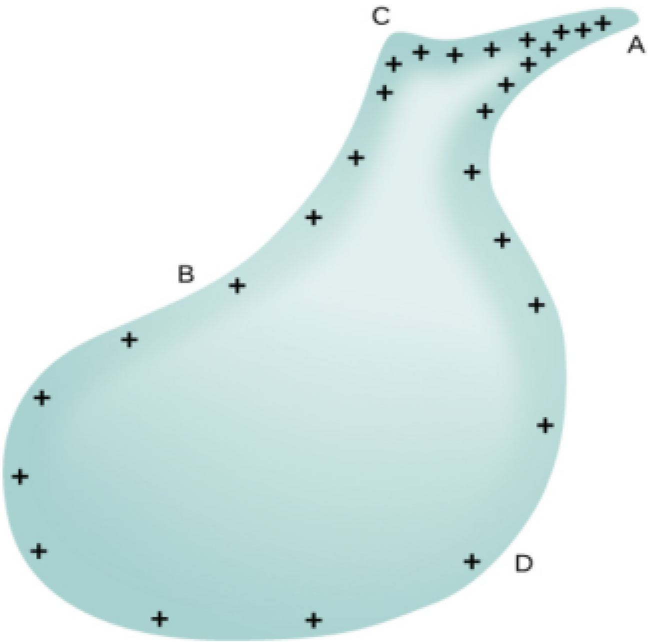



**Figure 7.39** Two conducting spheres are connected by a thin conducting wire.




Ef við tengjum R við krappageisla sjáum við að mest hleðslan safnast fyrir þar sem krappageislinn er minnstur

Eldingavarar