1

Hringsnúningur með fastri hröðun, (sértilfelli)

Notum einfaldlega samsvörunina við lýsingu línulegar hreyfingar með fastri hröðun

Angular displacement from average angular velocity	$\theta_f = \theta_0 + \bar{\omega}t$
Angular velocity from angular acceleration	$\omega_f = \omega_0 + \alpha t$
Angular displacement from angular velocity and angular acceleration	$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
Angular velocity from angular displacement and angular acceleration	$\omega_f^2 = {\omega_0}^2 + 2\alpha(\Delta\theta)$

Table 10.1 Kinematic Equations

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	Linear	Rotational
Position	x	θ
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

Rotational	Translational
$\theta_f = \theta_0 + \bar{\omega}t$	$x = x_0 + \bar{v}t$
$\omega_f = \omega_0 + \alpha t$	$v_f = v_0 + at$
$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x_f = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$	$v_f^2 = v_0^2 + 2a(\Delta x)$

Table 10.2 Rotational and Translational Kinematic Equations

Hreyfiorka í hringhreyfingu -- rotational kinetic energy
Hverfitregða -- moment of inertia

Snúningur um fastan ás

Hugsum hlutsem snýst sem samsettan úr fjölda massa

$$\boxed{K} = \sum_j \frac{1}{2} m_j v_j^2 = \sum_j \frac{1}{2} m_j (r_j \omega_j)^2$$

$\rightarrow$ $\omega_j = \omega$ fyrir alla massana

$$\rightarrow \quad K = \frac{1}{2}\left[\sum_j m_j r_j^2\right]\omega^2 = \boxed{\frac{1}{2} I \omega^2}$$

$$\underline{I} = \sum_j m_j r_j^2 \rightarrow \int r^2 dm$$

hverfitregða fyrir safn punktmassa eða hlut

Hverfitregða nokkurra hluta um fastan ás

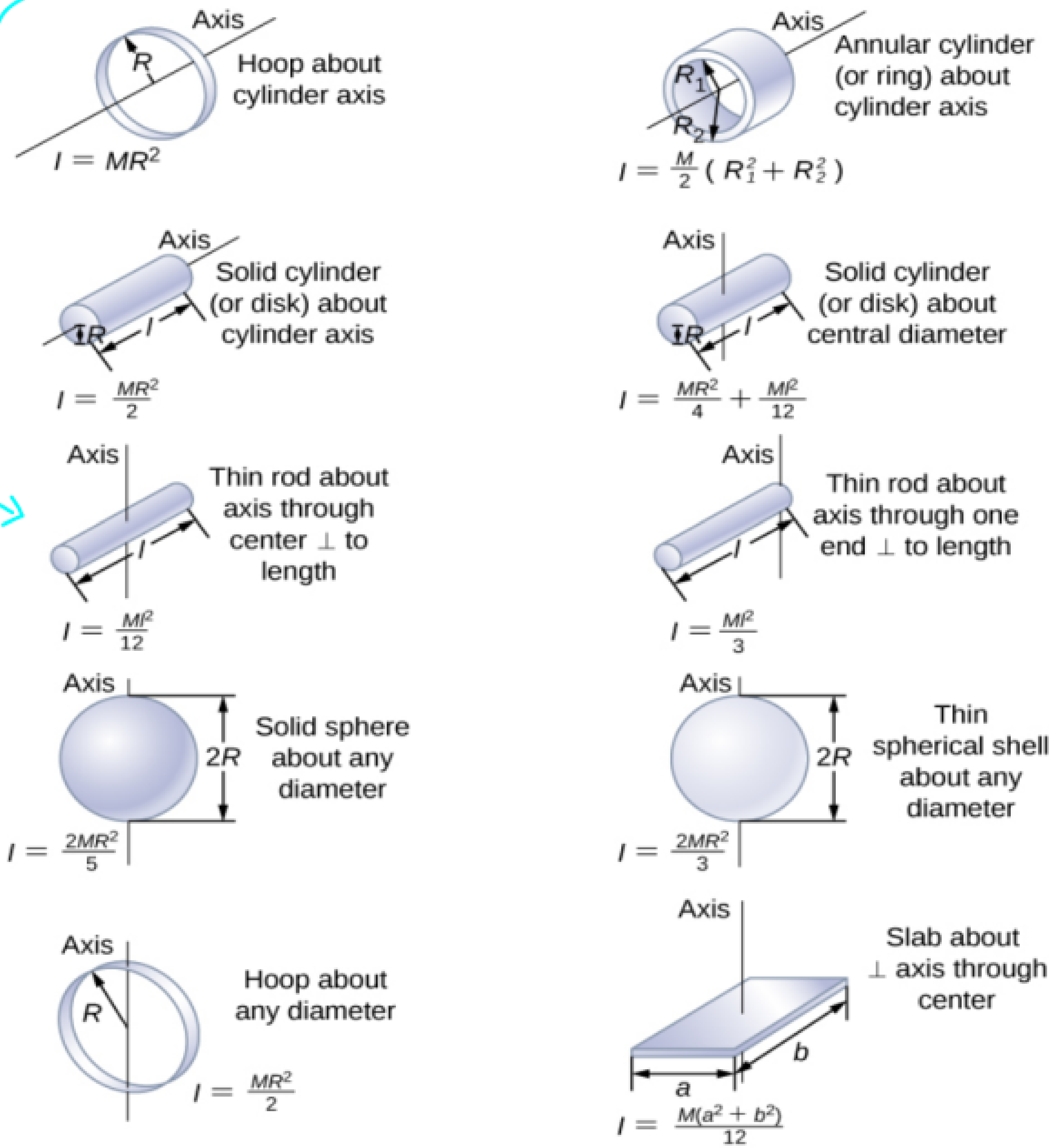
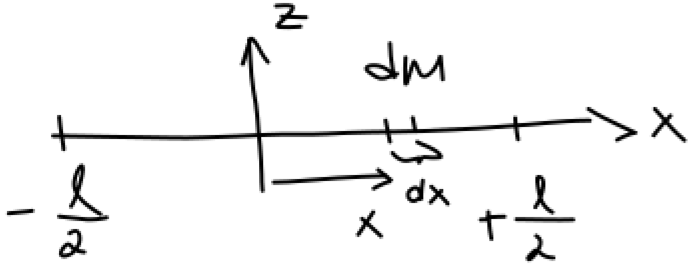


Figure 10.20 Values of rotational inertia for common shapes of objects.

Reynum



$$dm = \frac{M}{l}\,dx$$

$$I = \int r^2\,dm$$

$$= \int x^2 \frac{M}{l}\,dx$$

$$= \frac{M}{l}\int_{-l/2}^{l/2} x^2\,dx$$

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$$I = \frac{M}{\ell}\int_{-\ell/2}^{\ell/2} x^2\,dx = \frac{M}{\ell}\,\frac{x^3}{3}\Bigg|_{-\ell/2}^{\ell/2} = \frac{M}{3\ell}\left[\left(\frac{\ell}{2}\right)^3 - \left(-\frac{\ell}{3}\right)^3\right]$$

$$= \frac{M}{3\ell}\,\frac{\ell^3}{8}\,2 = \frac{1}{12}M\ell^2$$

Ef snúningsásinn væri í gegnum annan endann fæst $I = \frac{1}{3}M\ell^2$
og almennar

Parallel-Axis Theorem

Let m be the mass of an object and let d be the distance from an axis through the object's center of mass to a new axis. Then we have

$$I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2. \qquad (10.20)$$

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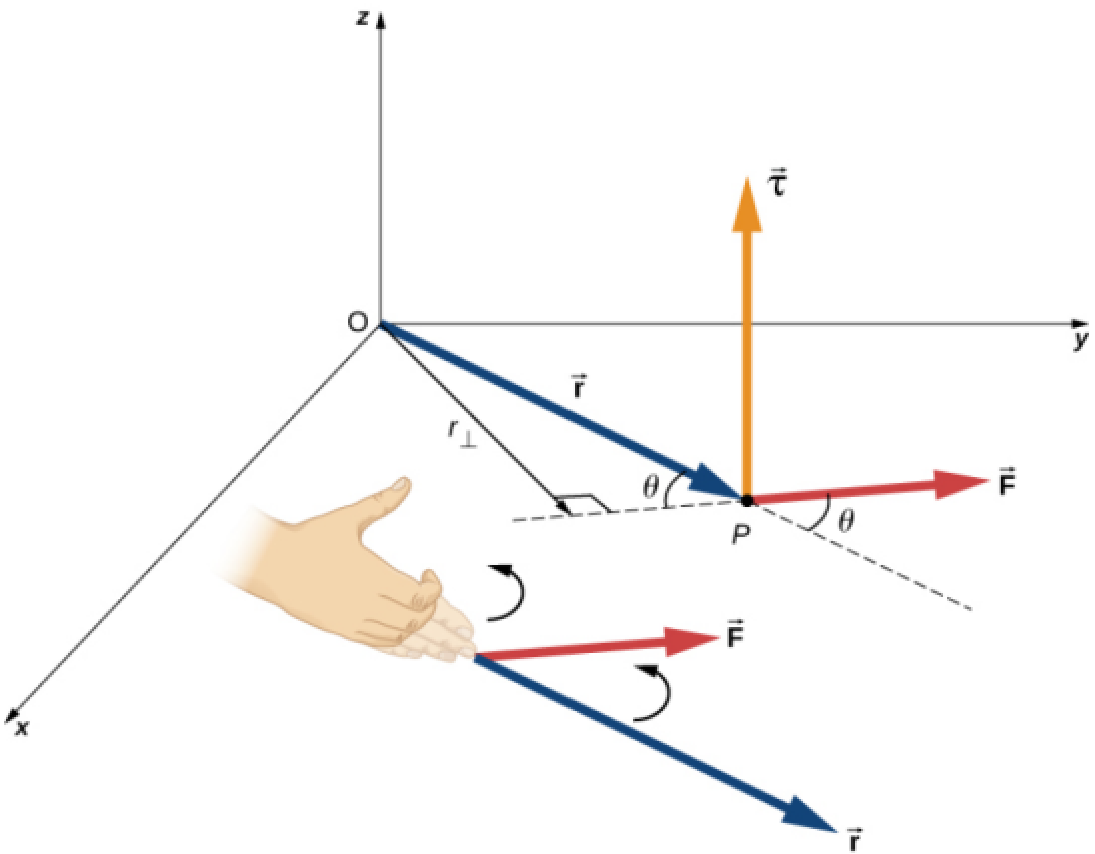
Huygens - Steiner

vægi -- torque

Torque

When a force $\vec{\mathbf{F}}$ is applied to a point P whose position is $\vec{\mathbf{r}}$ relative to O (Figure 10.32), the torque $\vec{\tau}$ around O is

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}. \tag{10.22}$$



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Annað lögmál Newtons fyrir hringhreyfingu um fastan ás

Newton's Second Law for Rotation

If more than one torque acts on a rigid body about a fixed axis, then the sum of the torques equals the moment of inertia times the angular acceleration:

$$\sum_i \tau_i = I\alpha. \tag{10.25}$$

Samanborið við

$$\boxed{\bar{F} = m\bar{a}}$$

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en eigum enn eftir að sjá framsetningu sem hægt er að bera saman við

$$\boxed{\bar{F} = \frac{d}{dt}\bar{p}}$$

Vinna og afl fyrir hringheyfingu um fastan ás

$$\bar{s} = \bar{\theta} \times \bar{r}, \quad d\bar{s} = d(\bar{\theta} \times \bar{r}) = d\bar{\theta} \times \bar{r}$$

$$W = \int \sum \bar{F} \cdot d\bar{s} = \int \sum \bar{F} \cdot (d\bar{\theta} \times \bar{r}) = \int d\bar{\theta} \cdot (\bar{r} \times \sum \bar{F})$$

p.s. $\bar{a} \cdot (\bar{b} \times \bar{c}) = \bar{b} \cdot (\bar{c} \times \bar{a})$

Notum $\bar{r} \times \sum \bar{F} = \sum \bar{\tau}$

$$\rightarrow \quad W = \int \sum \bar{\tau} \cdot d\bar{\theta}$$

Tökum betur saman

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Work-Energy Theorem for Rotation

The work-energy theorem for a rigid body rotating around a fixed axis is

$$W_{AB} = K_B - K_A \tag{10.29}$$

where

$$K = \frac{1}{2} I \omega^2$$

and the rotational work done by a net force rotating a body from point A to point B is

$$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta. \tag{10.30}$$

Afl - power

$$\boxed{P = \frac{dW}{dt}} = \frac{d}{dt}(\tau\theta) = \tau \frac{d\theta}{dt} = \boxed{\tau\omega}$$

Vinna fæst út úr kerfinu með vægi (eða sett í kerfið)

velta án skriks

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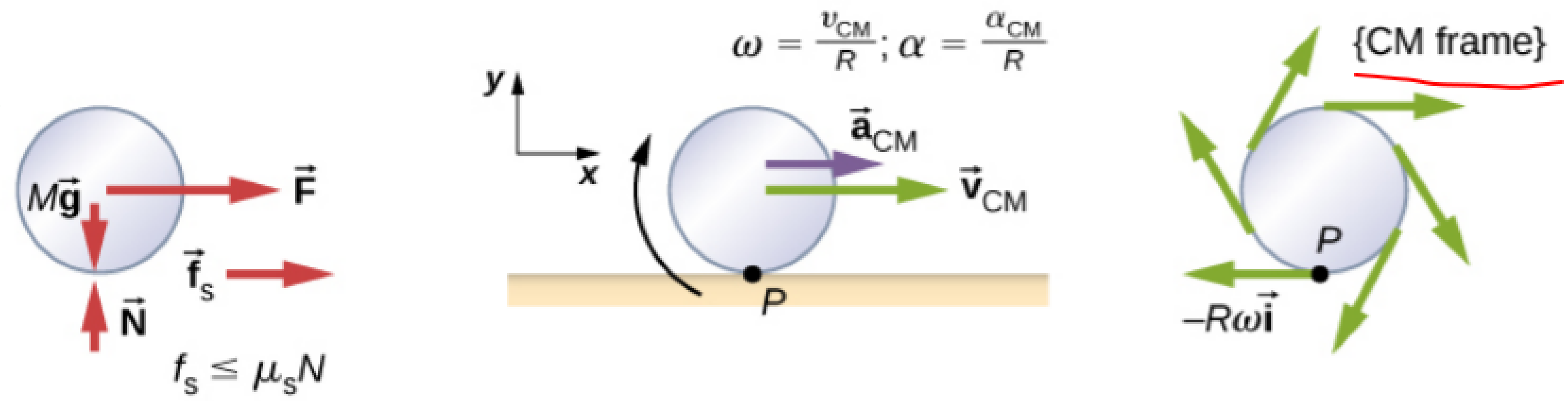
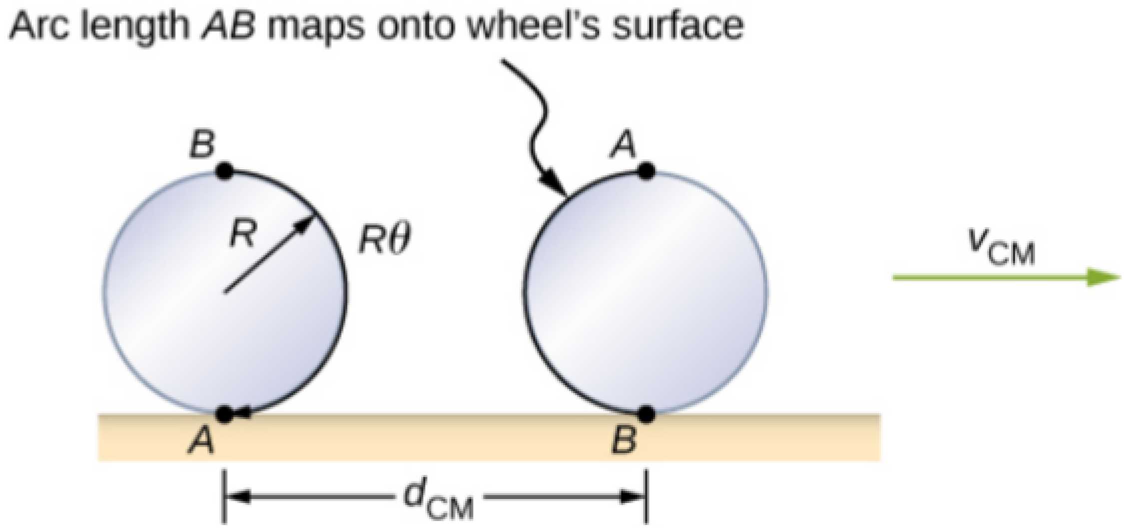


(a) Forces on the wheel

(b) Wheel rolls without slipping

(c) Point P has velocity vector in the negative direction with respect to the center of mass of the wheel



$$\bar{v}_P = -R\omega\hat{i} + \bar{v}_{CM}\hat{i} = 0$$

$$\rightarrow \quad v_{CM} = R\omega$$

$$a_{CM} = R\alpha$$

$$d_{CM} = R\theta$$

velta niður halla

$$\sum F_x = ma_x; \quad \sum F_y = ma_y.$$

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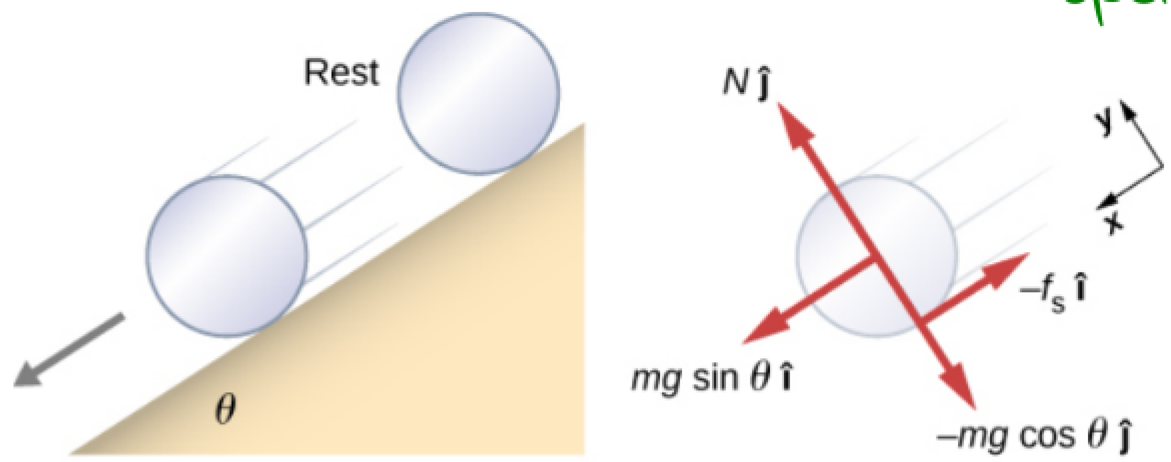

Rolling without slipping

Free-body diagram

x:
$$mg\sin\theta - f_s = m(a_{CM})_x = ma_{CM}$$

y:
$$N - mg\cos\theta = 0$$

$$a_{CM} = g\sin\theta - \frac{f_s}{m} \quad (3)$$

$$f_s = \frac{I_{CM}\alpha}{r}$$

Annað lögmál Newtons fyrir snúning

$$\sum \tau_{CM} = I_{CM}\alpha \longrightarrow f_s r = I_{CM}\alpha \quad (4)$$

$$a_{CM} = r\alpha$$

4 $\rightarrow$ $$f_s = \frac{I_{CM}\alpha}{r} = \frac{I_{CM}a_{CM}}{r^2}$$

3 $\rightarrow$ $$a_{CM} = g\sin\theta - \frac{I_{CM}a_{CM}}{mr^2}$$

$\rightarrow$ $$a_{CM}\left[1 + \frac{I_{CM}}{mr^2}\right] = g\sin\theta$$

$\rightarrow$ $$a_{CM} = \frac{mg\sin\theta}{m + \frac{I_{CM}}{r^2}}$$

Sívalningur

$$I_{CM} = \frac{mr^2}{2} \rightarrow \boxed{a_{CM} = \frac{mg\sin\theta}{m + \frac{mr^2}{2r^2}}} = \boxed{\frac{2}{3}g\sin\theta}$$

Síðan fæst

$$f_s = \frac{I_{CM}\alpha}{r} = \frac{mg I_{CM} \sin\theta}{mr^2 + I_{CM}}$$

$$f_s \leq \mu_s N = \mu_s mg \cos\theta$$

$$\rightarrow \boxed{\mu_s \geqslant \frac{\tan\theta}{1 + \frac{mr^2}{I_{CM}}} = \frac{1}{3}\tan\theta}$$

Hröðun sívalningsins niður hallann er minni en hlutar sem rinni niður án viðnáms
þar sem sívalningurinn hefur massa og hverfitregðu
Fyrir vissan halla getum við metið hvaða μ_s þarf til að hann skriki ekki

Orkuvarðveisla í veltu

Heildarorkan er

$$\boxed{E_T = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2 + mgh}$$

Hverfiþungi - angular momentum

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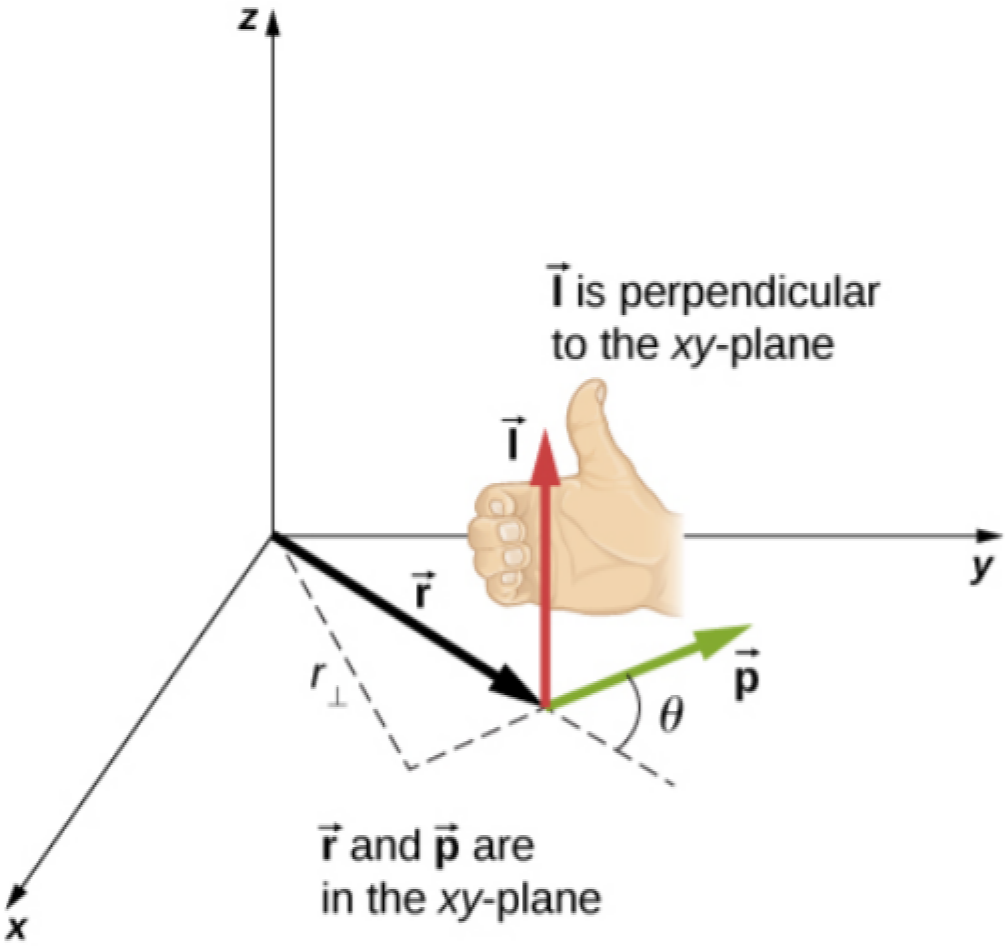
Angular Momentum of a Particle

The **angular momentum** $\vec{\mathbf{l}}$ of a particle is defined as the cross-product of $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$, and is perpendicular to the plane containing $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$:

$$\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}. \tag{11.5}$$

Táknum

$$\boxed{\overline{L} = \overline{r} \times \overline{p}}$$



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$$\bar{L} = \bar{r} \times \bar{p}$$

$$\frac{d\bar{L}}{dt} = \frac{d\bar{r}}{dt} \times \bar{p} + \bar{r} \times \frac{d\bar{p}}{dt}$$

$$= \underbrace{\bar{v} \times m\bar{v}}_{=0} + \bar{r} \times \frac{d\bar{p}}{dt}$$

$$= \bar{r} \times \bar{F} = \bar{\tau}$$

$$\rightarrow \boxed{\frac{d\bar{L}}{dt} = \sum \bar{\tau}}$$

Hreyfijafna fyrir hverfiþunga

varðveisla hverfiþunga

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Law of Conservation of Angular Momentum

The angular momentum of a system of particles around a point in a fixed inertial reference frame is conserved if there is no net external torque around that point:

$$\frac{d\vec{\mathbf{L}}}{dt} = 0 \qquad 11.10$$

or

$$\vec{\mathbf{L}} = \vec{\mathbf{l}}_1 + \vec{\mathbf{l}}_2 + \cdots + \vec{\mathbf{l}}_N = \text{constant}. \qquad 11.11$$

Alltaf þarf að taka fram hvaða viðmiðunarpunkt er átt við fyrir hverfiþunga

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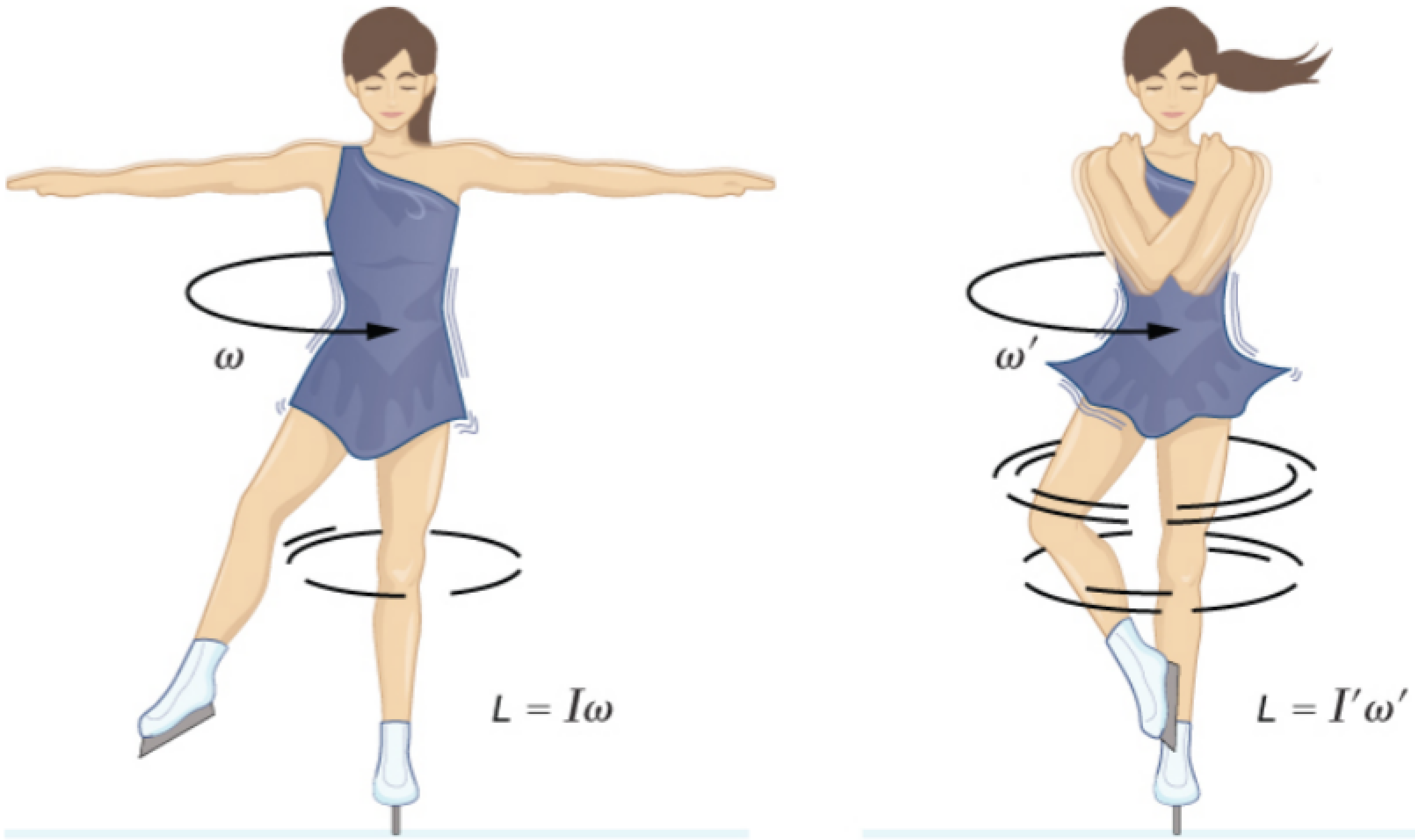

Varðveisla hverfiþungans leiðir til þess að hornferðin breytist þegar stúlkan breytir hverfiþunganum