Potential energy - energy conservation, stöðuorka - orkuvarðveisla

(1)

As the football falls toward Earth, the work done on the football is now positive, because the displacement and the gravitational force both point vertically downward. The ball also speeds up, which indicates an increase in kinetic energy. Therefore, energy is converted from gravitational potential energy back into kinetic energy.

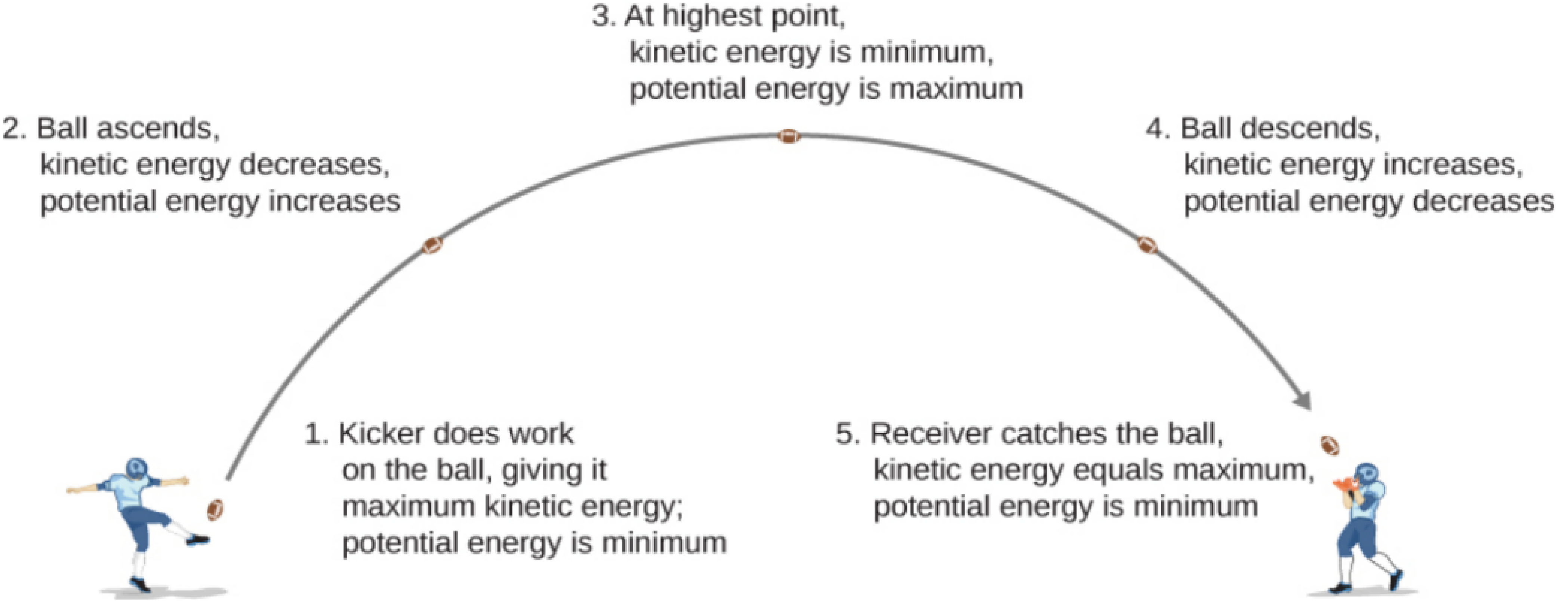

Figure 8.2 As a football starts its descent toward the wide receiver, gravitational potential energy is converted back into kinetic energy.

Based on this scenario, we can define the difference of potential energy from point A to point B as the negative of the work done:

$$\Delta U_{AB} = U_B - U_A = -W_{AB}. \quad (8.1)$$

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Define potential energy function $U(\bar{r})$

$$\boxed{\Delta U = U(\bar{r}) - U(\bar{r}_0)}$$

with reference point $\bar{r}_0$, stöðuorkufall - mættisorka - mættisorkufall

If no friction or air resistance, (closed system)

$$\boxed{\Delta K_{AB} = -\Delta U_{AB}}$$

valid for a system of particles

Different types of potential energy

Gravitational

Electrical

Spring .....

Typical scales of energy of phenomenas

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Annual world energy use	4.0×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
1 barrel crude oil	5.9×10^{9}
1 metric ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
Tennis ball at 100 km/h	22
Mosquito $(10^{-2}$ g at 0.5 m/s$)$	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

Table 8.1 Energy of Various Objects and Phenomena

Power in Iceland (2014)

Hydropower stations	1984 MW
Geothermal	665 MW
Fuel	117 MW

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Conservative and nonconservative forces - geymnir og ógeymnir kraftar

Conservative Force

The work done by a conservative force is independent of the path; in other words, the work done by a conservative force is the same for any path connecting two points:

$$W_{AB,\text{path-1}} = \int_{AB,\text{path-1}} \vec{\mathbf{F}}_{\text{cons}} \cdot d\vec{\mathbf{r}} = W_{AB,\text{path-2}} = \int_{AB,\text{path-2}} \vec{\mathbf{F}}_{\text{cons}} \cdot d\vec{\mathbf{r}}. \quad 8.8$$

The work done by a non-conservative force depends on the path taken.

Equivalently, a force is conservative if the work it does around any closed path is zero:

$$W_{\text{closed path}} = \oint \vec{\mathbf{F}}_{\text{cons}} \cdot d\vec{\mathbf{r}} = 0. \quad 8.9$$

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Globally stated

Locally stated (in 2D) $\bar{F} \cdot d\bar{r} = F_x dx + F_y dy$

has to be exact differencial: $\boxed{\dfrac{dF_x}{dy} = \dfrac{dF_y}{dx}}$

Ex. 8.5

a) $\bar{F} = a(xy^3, yx^3) \rightarrow \frac{dF_x}{dy} = 3axy^2, \frac{dF_y}{dx} = 3ax^2y$

$\rightarrow$ not conservative

b) $\bar{F} = a\left(\frac{y^2}{x}, 2y\ln\left(\frac{x}{b}\right)\right) \rightarrow \frac{dF_x}{dy} = 2a\frac{y}{x}$

$$\frac{dF_y}{dx} = 2ay \cdot \frac{1}{x/b} \cdot \frac{1}{b} = 2a\frac{y}{x}$$

$\rightarrow$ Conservative

c) $\bar{F} = \frac{a}{x^2+y^2}(x, y) \rightarrow \frac{dF_x}{dy} = -\frac{ax2y}{(x^2+y^2)^2}$

(wXmaxima) $\frac{dF_y}{dx} = -\frac{ay2x}{(x^2+y^2)^2}$

$\rightarrow$ Conservative

Potential energy can only be found for conservative forces

we defined the increase in potential energy as the negative work done by the force

$$dU = -\overline{F} \cdot d\overline{\ell} = -F_\ell \, d\ell$$

$$\rightarrow \quad F_\ell = -\frac{dU}{d\ell}$$

hlutafleiða

partial derivative ..

gradient,

stigull

For 2D we thus have $\boxed{\overline{F} = -\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}\right) = -\overline{\nabla} U}$

..and of course

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \quad \text{as} \quad \frac{\partial^2 U}{\partial x \partial y} = \frac{\partial^2 U}{\partial y \partial x}$$

Check

$$U = \frac{1}{2}kx^2 \longrightarrow F = -\frac{\partial U}{\partial x} = -kx$$

Hookes law

$$U = \frac{1}{2}k\left[x^2 + y^2\right] \longrightarrow \bar{F} = -k(x, y)$$

Potential bowl, quantum dot

$$U = \frac{1}{4}cx^4 \longrightarrow F = -cx^3$$

Conservation of energy

single particle at the moment

Conservation of Energy

The mechanical energy E of a particle stays constant unless forces outside the system or non-conservative forces do work on it, in which case, the change in the mechanical energy is equal to the work done by the non-conservative forces:

$$W_{\mathrm{nc},AB} = \Delta(K + U)_{AB} = \Delta E_{AB}. \tag{8.12}$$

External conservative forces can add energy to, or extraxt energy from a system by the work done on it, positive or negative. In a closed system with no external forces the energy is conserved.

Ex. 8.7

The system is the pendulum and the gravitational force

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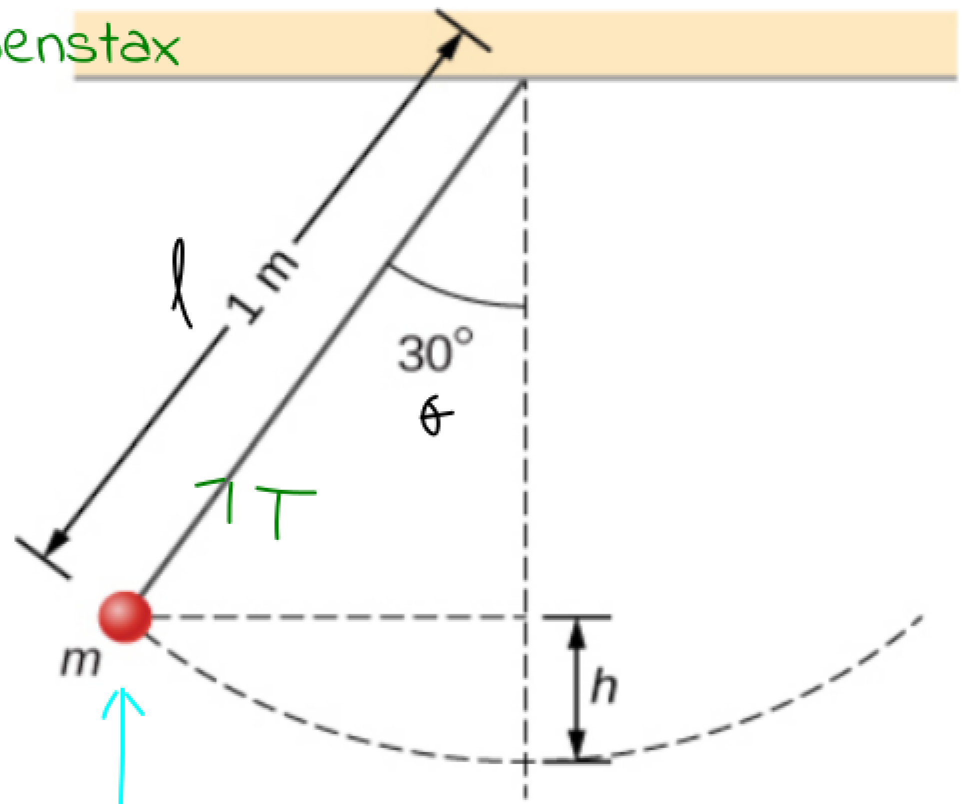

$v = 0$

i

$h = 0$

f

$$\Delta\left[K + U\right] = 0$$

Tensiion T does no work, always perpendicular to the path

$$\bar{T} \cdot d\bar{r} = 0$$

$$K = \frac{1}{2} m v^2$$

$$U = mgh$$

$K_i = 0$ as $v_i = 0$

$U_f = 0$ as $h_f = 0$

$$U_i = mgh_i = K_f = \frac{1}{2}mv_f^2$$

$$\rightarrow mgh_i = \frac{1}{2}mv_f^2 \longrightarrow gh_i = \frac{1}{2}v_f^2$$

$$\rightarrow v_f = \sqrt{2gh_i}$$

Note

$l = h + l\cos\theta$

$\rightarrow h = l - l\cos\theta$

$\rightarrow U = mgh = mgl(1-\cos\theta)$, $\rightarrow F_\theta = -\frac{\partial U}{l\,\partial\theta} = -mg\sin\theta$

arclength

$$s = \ell\theta$$

$$v = \frac{ds}{dt} = \ell\frac{d\theta}{dt} \rightarrow a = \ell\frac{d^2\theta}{dt^2}$$

$$ma = \ell\frac{d^2\theta}{dt^2} = \ell\ddot{\theta}, \qquad F = -mg\sin\theta$$

one variable, θ, --> Equation of motion for the pendulum is

$$ma = F \rightarrow m\ell\ddot{\theta} = -mg\sin\theta$$

$$\rightarrow \boxed{\ddot{\theta} + \frac{g}{\ell}\sin\theta = 0}$$

nonlinear

For small θ (in radians)

$$\sin\theta \simeq \theta - \frac{\theta^3}{6} + \cdots$$

linear

$$\boxed{\ddot{\theta} + \frac{g}{\ell}\theta \simeq 0}$$

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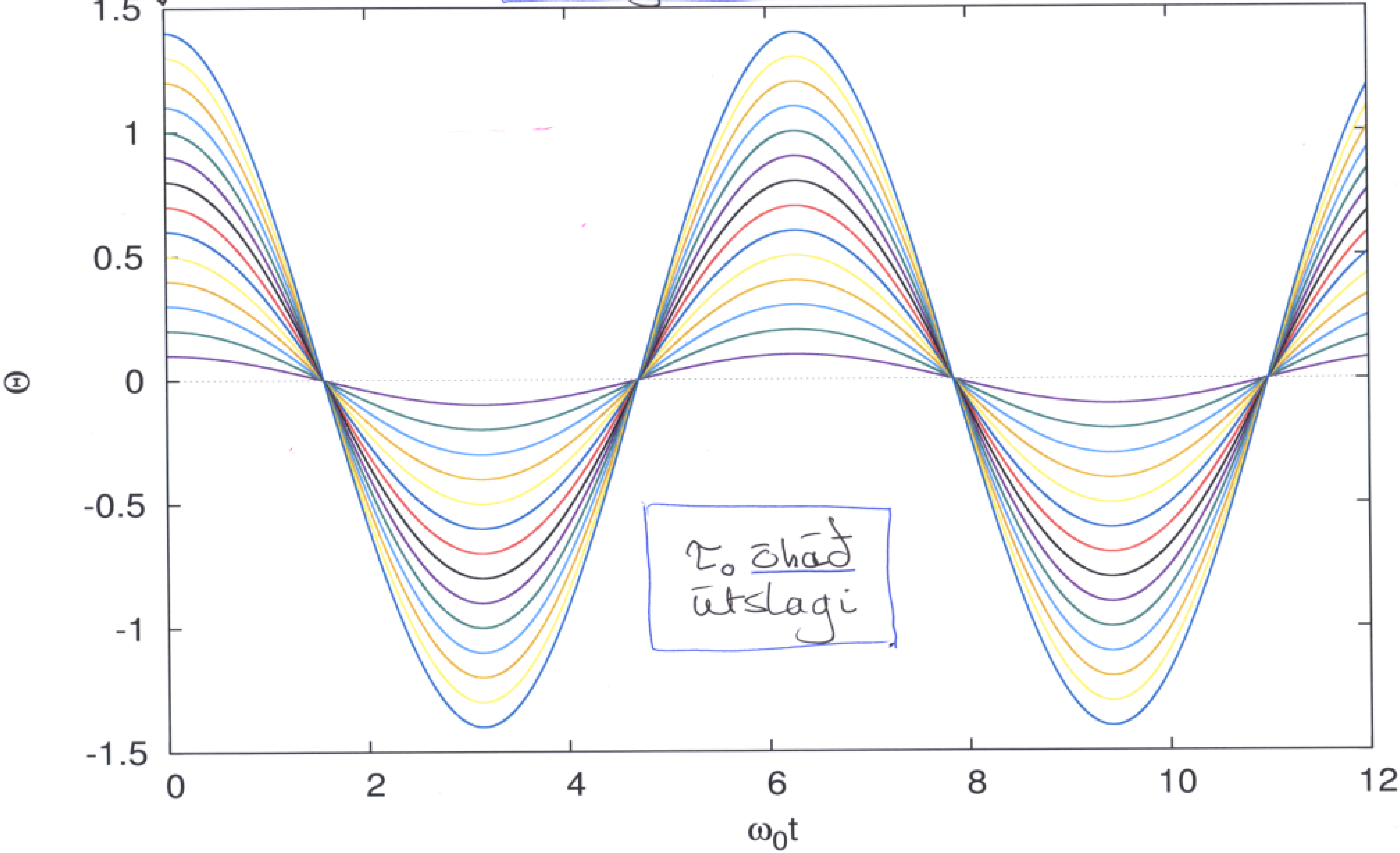
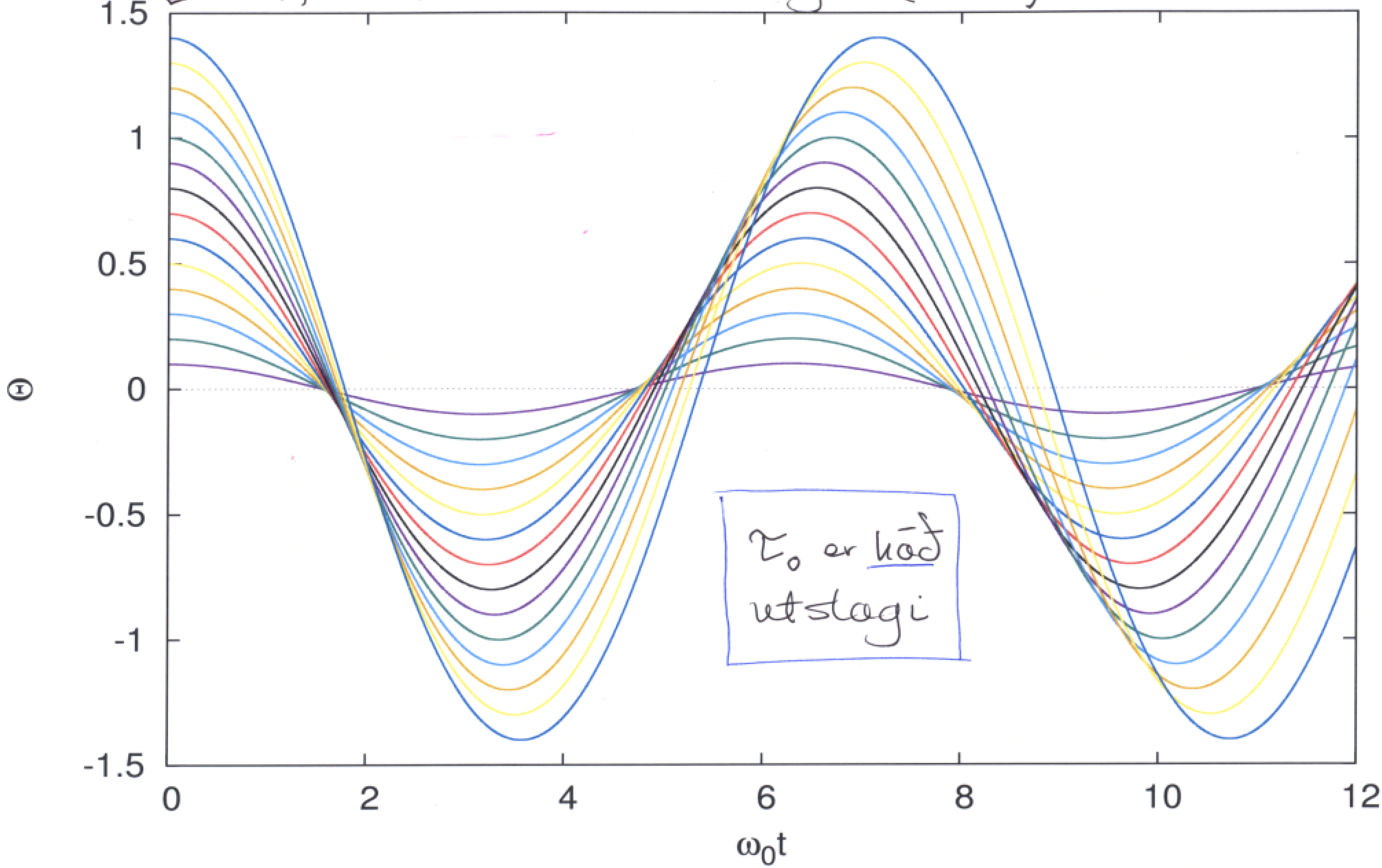
$\theta(0)$, $\dot{\theta}(0) = 0$ línulegur sveifill $\omega_0 = \sqrt{\frac{g}{\ell}}$



$\omega_0 = \sqrt{\frac{g}{l}}$

$\theta(0)$, $\dot{\theta}(0) = 0$ ólínulegur ($\sin\theta$)



1D motion

$$E = K + U(x)$$

$$K = \frac{m}{2} v^2 = E - U(x)$$

$$\rightarrow \quad v = \frac{dx}{dt} = \sqrt{\frac{2(E - U(x))}{m}}$$

$$\rightarrow \quad dt = \frac{dx}{\sqrt{\frac{2(E - U(x))}{m}}}$$

Integrate

$$t = \int_0^t dt' = \int_{x_0}^{x} \frac{dx'}{\sqrt{\frac{2(E - U(x'))}{m}}}$$

Solution for the path found from energy conservation. often difficult to invert to x(t), and not good for numerical calculations but can still give information

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Potential energy -- stability (1D)

Free fall

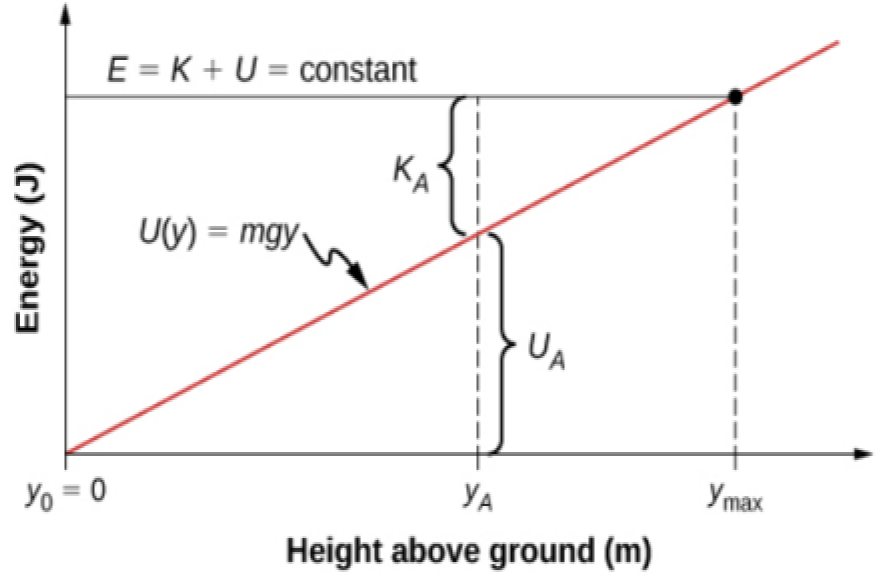
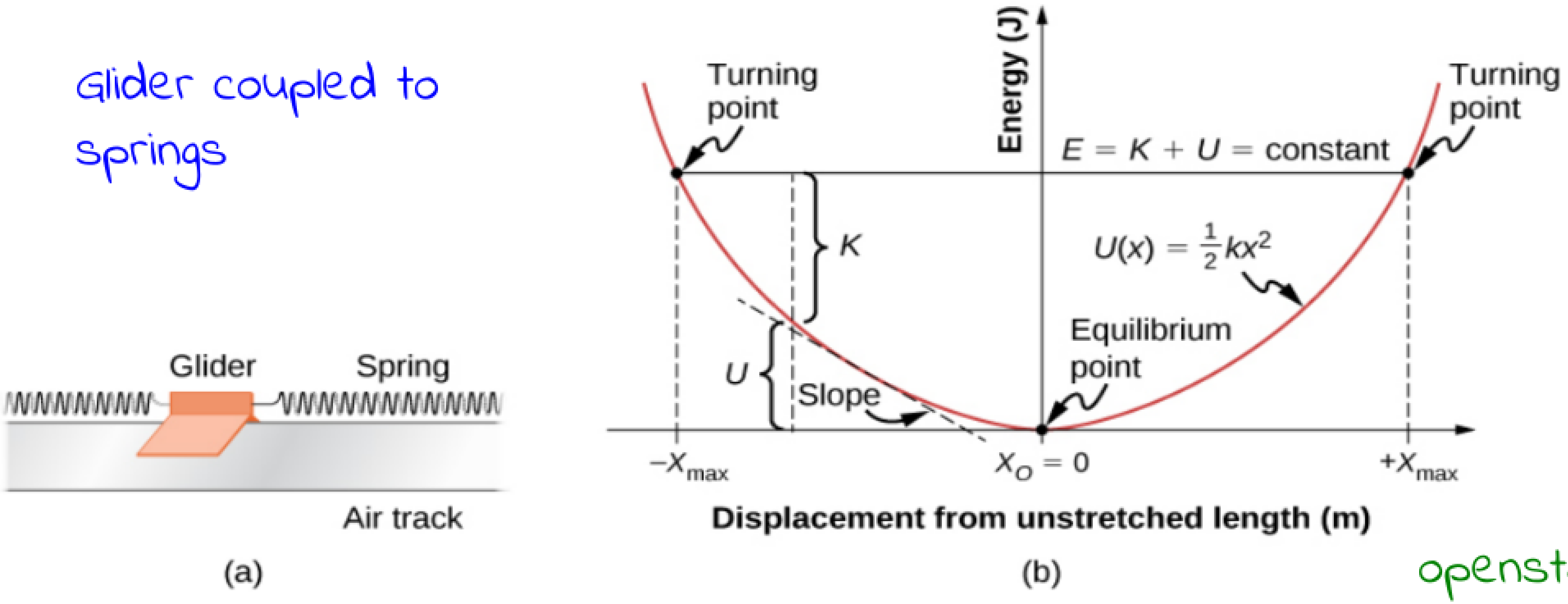


Figure 8.10 The potential energy graph for an object in vertical free fall, with various quantities indicated.

Glider coupled to springs



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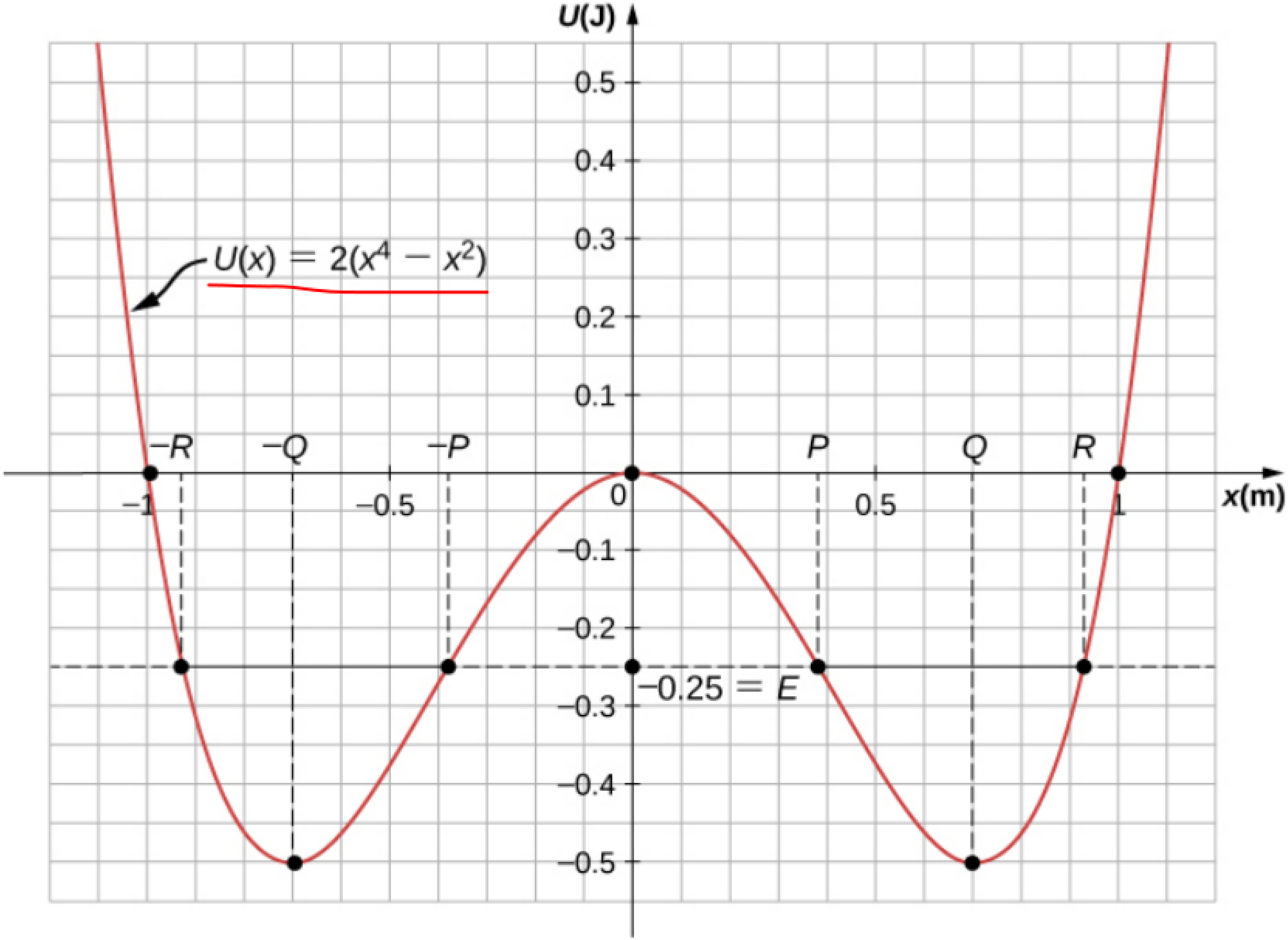

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