1

work -- vinna

$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \left|\vec{\mathbf{F}}\right| \left|d\vec{\mathbf{r}}\right| \cos\theta. \tag{7.1}$$

Then, we can add up the contributions for infinitesimal displacements, along a path between two positions, to get the total work.

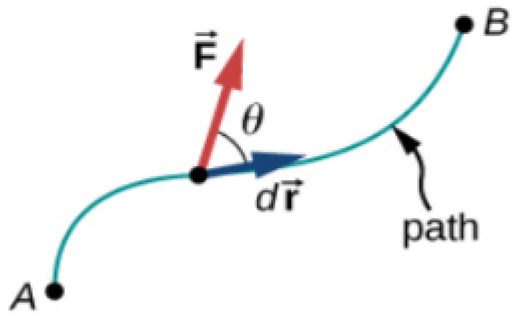
Work Done by a Force

The work done by a force is the integral of the force with respect to displacement along the path of the displacement:

$$W_{AB} = \int_{\text{path } AB} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}. \tag{7.2}$$

The vectors involved in the definition of the work done by a force acting on a particle are illustrated in Figure 7.2.

Any force



only the component along the path matters
inner product

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work done by a force can be negative, 0, positive

we will see this corresponds to the force taking energy out, or supplying to the system

we ask about the work of any force, with no focus on the net force

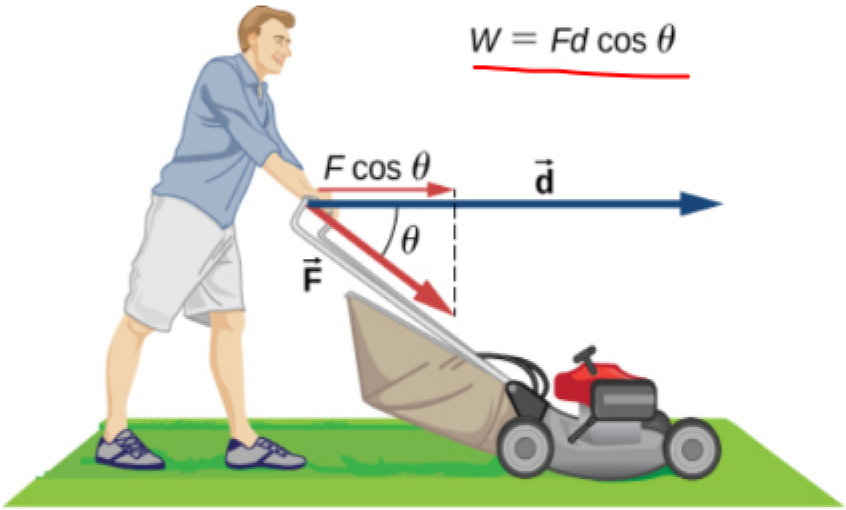
Constant force (special case)

$$W_{AB} = \int_A^B \bar{F} \cdot d\bar{r} = \int_A^B \left(F_x dx + F_y dy + F_z dz \right)$$

$$= F_x \int_A^B dx + F_y \int_A^B dy + F_z \int_A^B dz = F_x (x_B - x_A) + F_y (y_B - y_A) + F_z (z_B - z_A)$$

$$= \bar{F} \cdot (\bar{r}_B - \bar{r}_A)$$

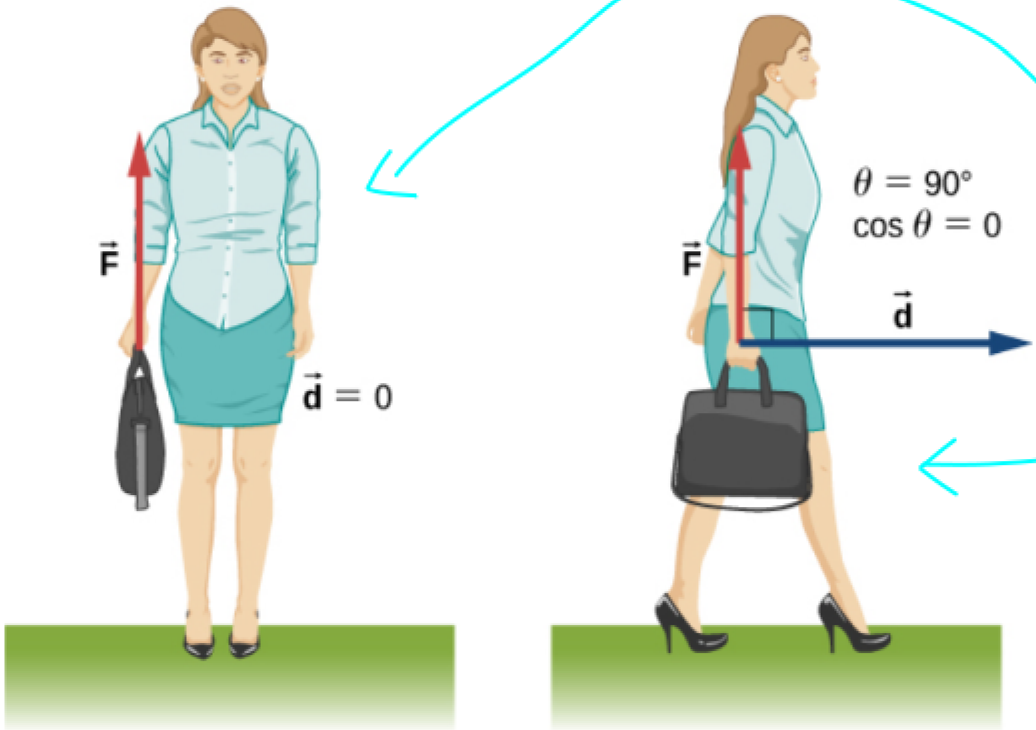
3



(a)

$$\bar{F} \cdot \bar{d} \neq 0 \qquad dW_N = \bar{d} \cdot \bar{N} = 0$$

$$\bar{d} \neq 0$$



(b) (c)

$$w = 0$$

$$\bar{d} = 0$$

$$\bar{F} \cdot \bar{d} = 0$$

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The friction force does negative work on the lawn mower

$$W_{\text{fr}} = \int_A^B \vec{\mathbf{f}}_k \cdot d\vec{\mathbf{r}} = -f_k \int_A^B |dr| = -f_k\, |l_{AB}|,$$

$\vec{F}_k \cdot d\vec{r} < 0$

It depends on the length of the total path

Moving a couch horizontally

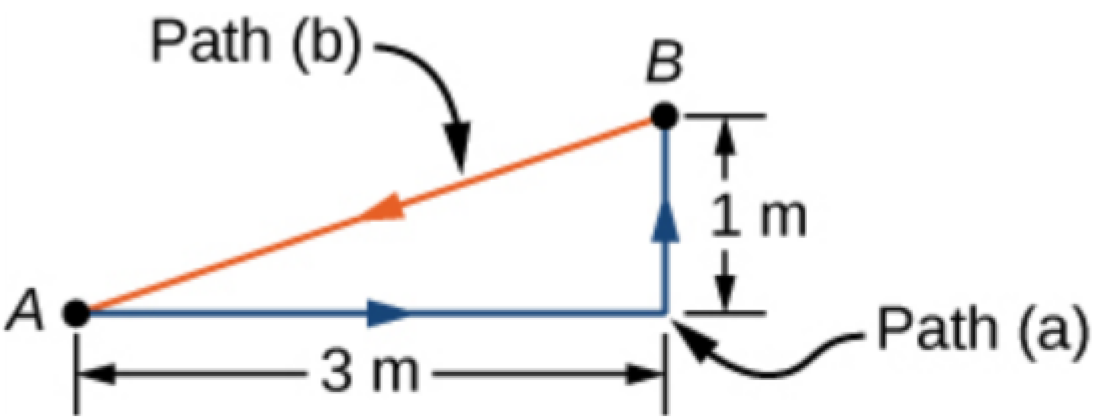

Figure 7.4 Top view of paths for moving a couch.

Two paths:

Path (a) A --> B, open path

Path (b) A --> B --> A, closed

$\mu_k = 0.6$

$|N| = 1\,kN$

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work done by the friction force

a: $W = -0{,}6\,(1\,kN)\cdot(3m + 1m) = \underline{-2{,}4\,kJ}$

b: $W = -0{,}6\,(1\,kN)\cdot(3m + 1m + \sqrt{10}\,m) = \underline{-4{,}3\,kJ}$

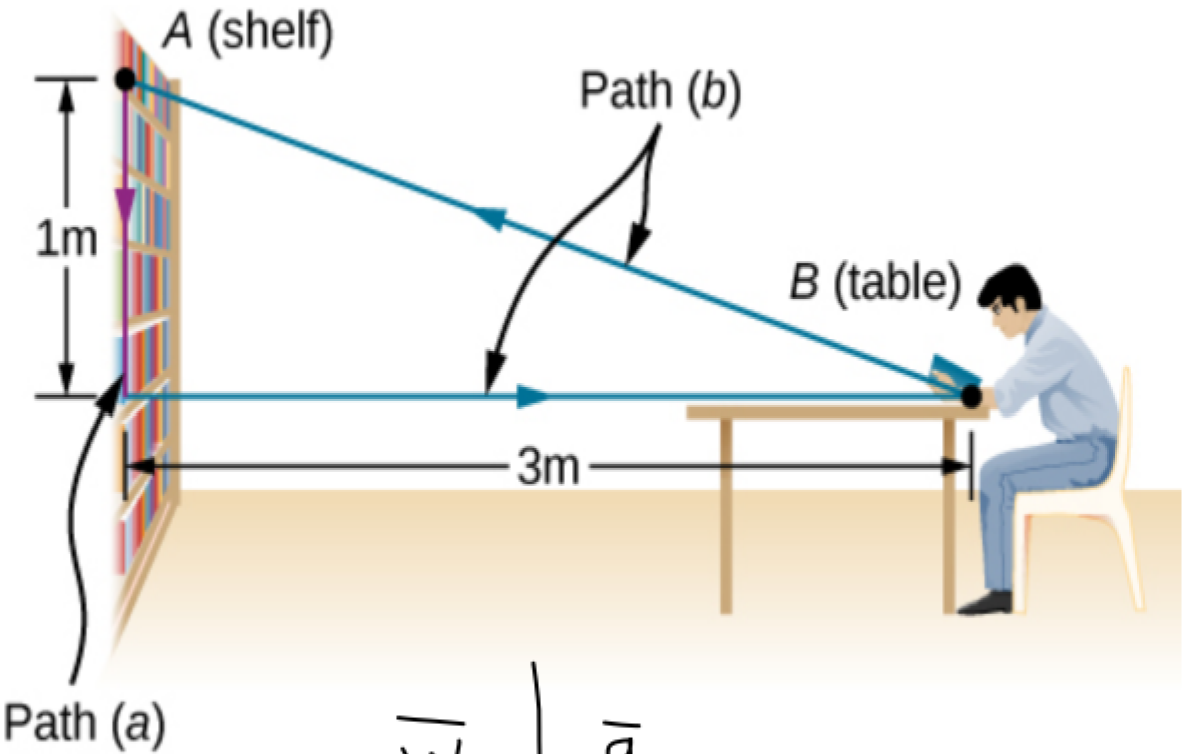
$$\boxed{\oint \bar{F}_k \cdot d\bar{r} \neq 0}$$

as the force of friction is not conservative, ekki geyminn, see more soon...

It is dissipative, takes energy out of the system
(open systems)

Shelving a book

$$W = \int_A^B \bar{F} \cdot d\bar{r}$$



$\bar{w} \downarrow \bar{g}$

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Constant force of gravity

its work on the book

$$W_{AB} = -mg(y_B - y_A)$$

$$= mg(y_A - y_B) > 0$$

$$W_{ABA} = 0$$

Gravity is conservative force (geyminn), only the endpoints of the path matter

Variable force

Path: $y = (0{,}5\,\mathrm{m}^{-1})\,x^2$

$$\bar{F} = \left(5\,\tfrac{N}{m}\right) y\,\hat{i} + \left(10\,\tfrac{N}{m}\right) x\,\hat{j}$$

$$= \left(5\,\tfrac{N}{m}\,y\,,\ 10\,\tfrac{N}{m}\,x\right)$$

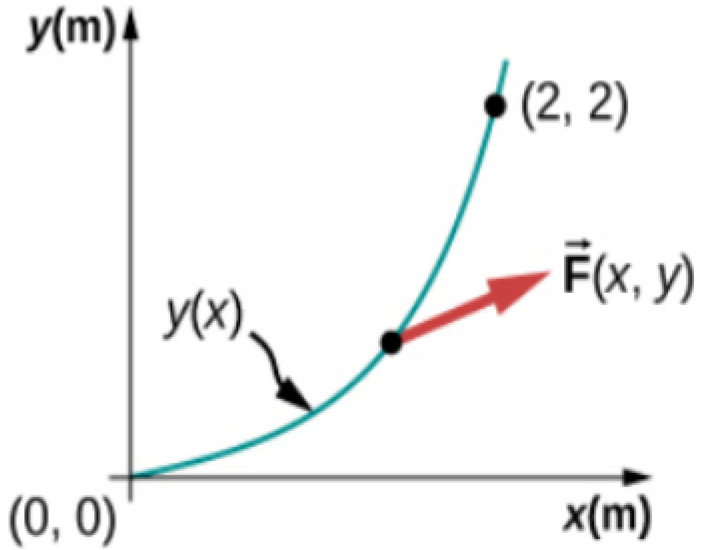

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Figure 7.6 The parabolic path of a particle acted on by a given force.

Parametrize the path (stika)

$$\bar{r} = (x, y) = (x,\ 0{,}5x^2)$$

$$\rightarrow\ d\bar{r} = (dx,\ x\,dx)$$

$$dy = \left(\frac{dy}{dx}\right) dx$$

$$dW = \bar{F} \cdot d\bar{r} = 5y\,dx + 10x\,x\,dx = \frac{5}{2}x^2 dx + 10x^2 dx$$

$$\rightarrow\ W = \int_0^2 \left[\frac{5}{2}x^2 dx + 10x^2 dx\right] = \int_0^2 \left[\frac{25}{2}x^2\right] dx$$

$$W = \int_0^2 \frac{25}{2} x^2 dx = \frac{25}{2 \cdot 3} x^3 \Big|_0^2 = \frac{25}{6} 8 \text{ Nm} \approx 33{,}3 \text{ J} \quad 8$$

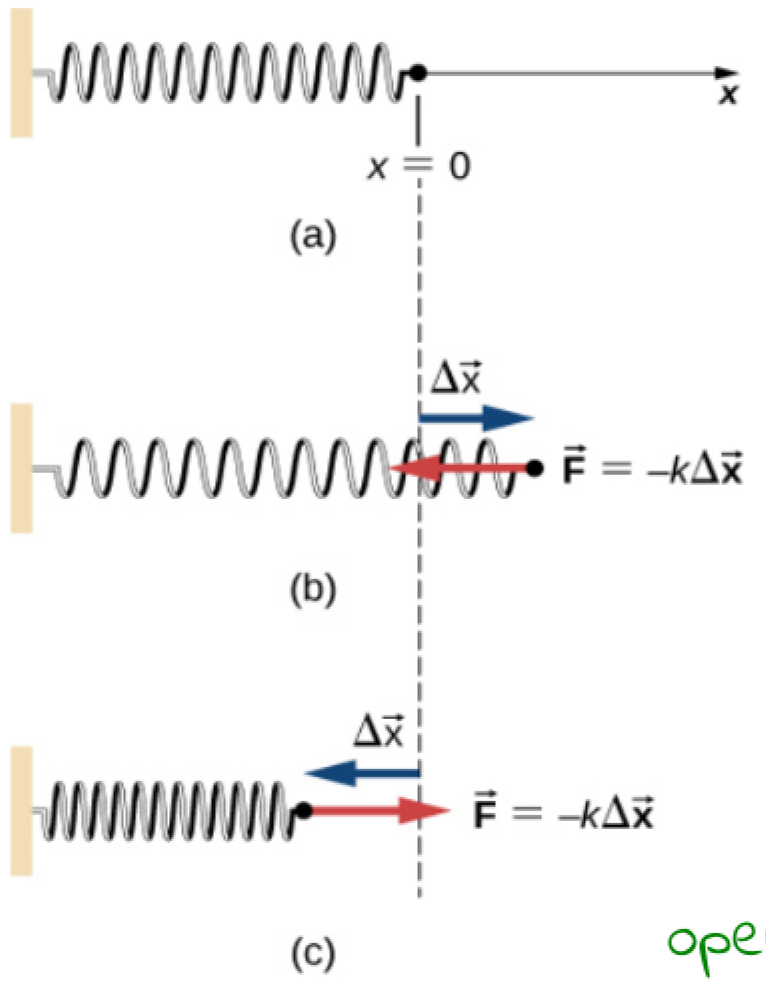
work done by a spring

$$W_{\text{spring},AB} = \int_A^B F_x dx = -k \int_A^B x dx = -k \frac{x^2}{2}\Big|_A^B = -\frac{1}{2} k \left(x_B^2 - x_A^2\right).$$

If $x_A = 0$

$\rightarrow$ $W_{\text{spring},AB} < 0$

as stretching or compressing the spring from the equilibrium requires external work



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Hreyfiorka - kinetic energy

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Kinetic Energy

The kinetic energy of a particle is one-half the product of the particle's mass m and the square of its speed v:

$$K = \frac{1}{2}mv^2. \tag{7.6}$$

$$\bar{p} = m\bar{v}$$

$$\boxed{K = \frac{(mv)^2}{2m} = \frac{p^2}{m}}$$

Notice specially that the kinetic energy is always positive and grows as the square of the velocity

work - energy

The net work done on a particle

$$dW_{net} = \bar{F}_{net} \cdot d\bar{r} \quad , \quad \bar{F}_{net} = m\frac{d\bar{v}}{dt}$$

$$= m\left(\frac{d\bar{v}}{dt}\right) \cdot d\bar{r} = m\left(\frac{d\bar{v}}{dt}\right)dt \cdot \underbrace{\frac{d\bar{r}}{dt}}_{=\bar{v}}$$

$$= m\,d\bar{v} \cdot \bar{v} = m\bar{v} \cdot d\bar{v}$$

$$\rightarrow W_{net,AB} = \int_A^B m\bar{v} \cdot d\bar{v} = \int_A^B \left[m v_x dv_x + m v_y dv_y + m v_z dv_z \right]$$

$$= \frac{m}{2}\left| v_x^2 + v_y^2 + v_z^2 \right|_A^B = \boxed{\frac{1}{2}m\left|v^2\right|_A^B}$$

(11)

Work-Energy Theorem

The net work done on a particle equals the change in the particle's kinetic energy:

$$W_{\text{net}} = K_B - K_A. \tag{7.9}$$

Power - def

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Power

Power is defined as the rate of doing work, or the limit of the average power for time intervals approaching zero,

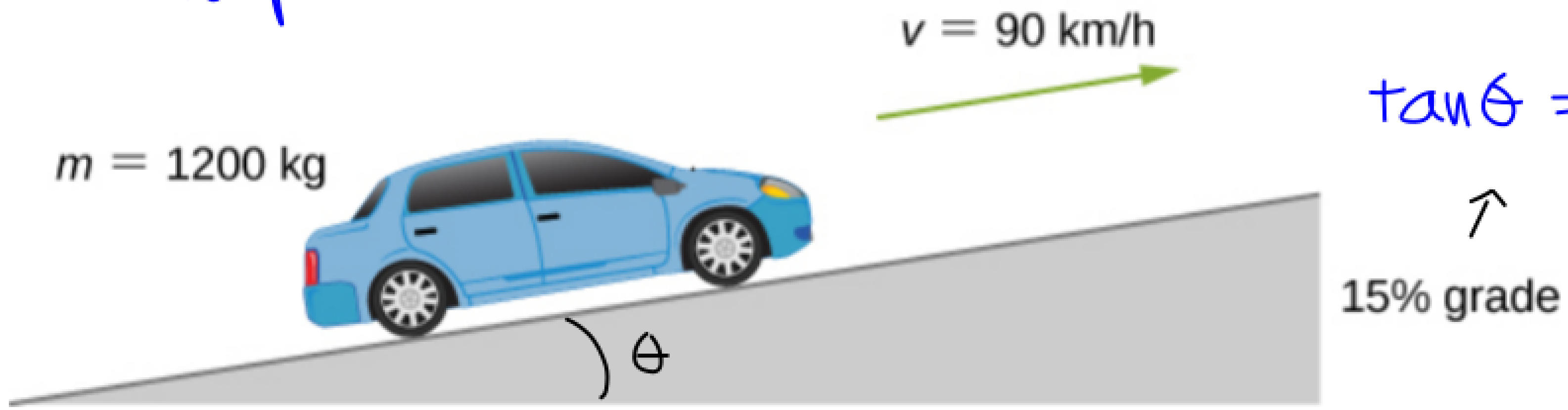
$$P = \frac{dW}{dt}. \tag{7.11}$$

$$P = \frac{dW}{dt} = \frac{\vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \left(\frac{d\vec{\mathbf{r}}}{dt}\right) = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

general equation even though
it may look as a particular result

Ex. 7.12

25 % power to friction



$\tan\theta = 0{,}15$

Figure 7.15 We want to calculate the power needed to move a car up a hill at constant speed.

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Constant v --> $\Delta K = 0$, power against gravity and friction

--> 75% of P against gravity

$$P = m\bar{g}\cdot\bar{v} = mgv\sin\theta$$

$$0{,}75\cdot P = mgv\sin\left[\arctan(0{,}15)\right]$$

$$P = \frac{1200\cdot 9{,}81\left(\frac{90}{3{,}6}\,\mathrm{m/s}\right)\sin(8{,}53^\circ)}{0{,}75} = \underline{58\,\mathrm{kW}}$$