Drag forces -- loftmótstaða -- vökvamótstaða

Empirically or by low order approximations to fluid dynamics it is known that for large objects of high speed in not very dense fluids one has

Drag Force

Drag force F_D is proportional to the square of the speed of the object. Mathematically,

$$F_D = \frac{1}{2} C \rho A v^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

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F_D is proportional to v^2

only an approximation -- empirical fact -- reynslulögmál....

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Object	C
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram Pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

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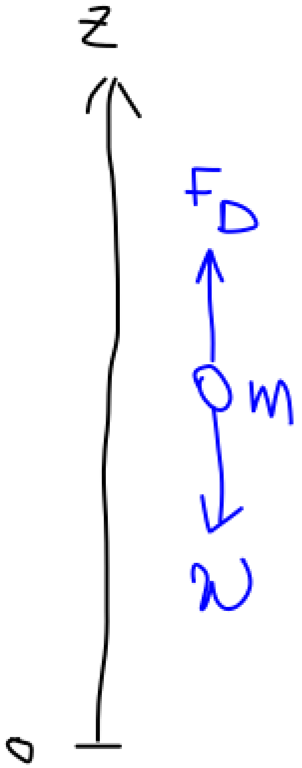
Terminal velocity - markhraði



we look at free fall in gravitational field

Newtons second law:

$$ma = m\frac{d^2z}{dt^2} = -mg + F_D$$

If the object reaches a constant velocity (terminal velocity) then

$$a = \left(\frac{d^2z}{dt^2}\right) = 0 = -mg + F_D$$

$$\rightarrow \quad F_D = mg \quad \rightarrow \quad \frac{1}{2} C \rho A v^2 = mg$$

$$\rightarrow v^2 = \frac{2mg}{\rho C A} \rightarrow \boxed{v = \sqrt{\frac{2mg}{\rho C A}} \equiv v_T}$$

markhraĝi -- terminal velocity

Skydiver

$$\left.\begin{aligned} M &= 75\ kg \\ \rho &= 1{,}21\ kg/m^3 \\ A &= 0{,}18\ m^2 \\ C &= 0{,}7 \end{aligned}\right\} \rightarrow \underline{v_T \simeq 98\ m/s}$$

$$\underline{\simeq 350\ km/hr}$$

For small spherical object at low speed in dense fluid

Stokes' Law

For a spherical object falling in a medium, the drag force is

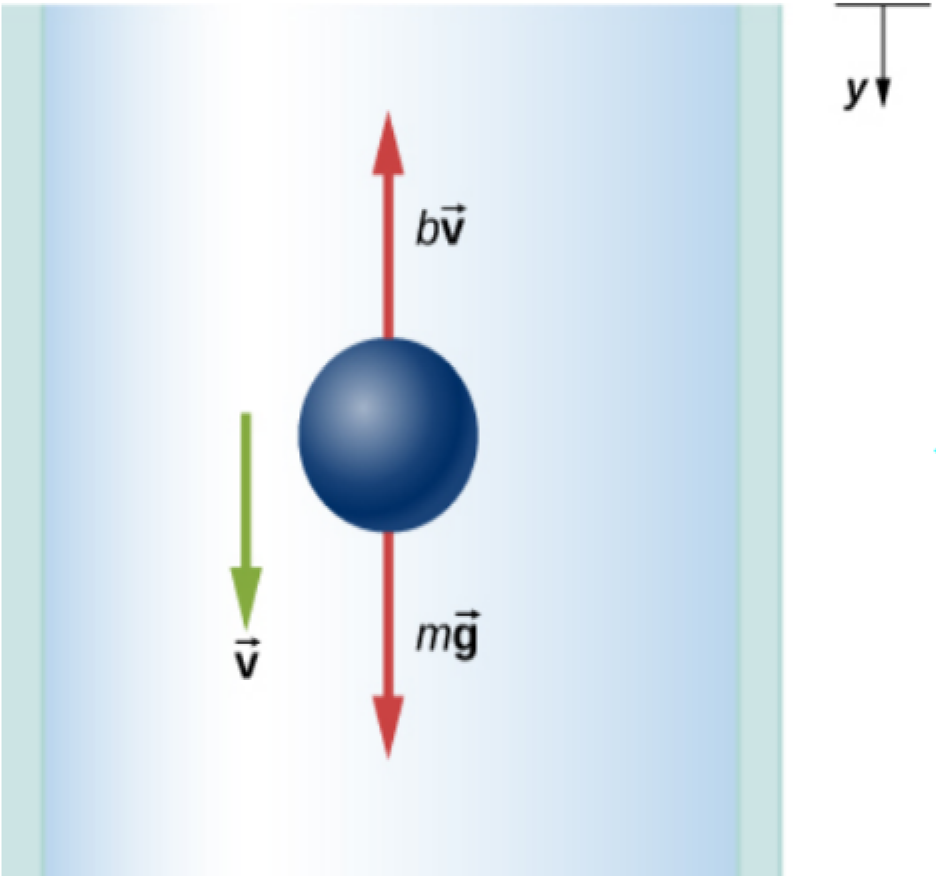
$$F_s = 6\pi r\eta v, \qquad 6.6$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

F_s is proportional to v

Again, an approximation to fluid dynamics -- empirical law

Equation of motion -- integration -- solution



Using the coordinat system in the Figure:

$$\text{Assume: } f_R = -bv$$

$$ma = m\frac{d^2y}{dt^2} = m\frac{dv}{dt}$$

$$= mg - bv$$

use the equation of motion

$$m\frac{dv}{dt} = mg - bv$$

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with variables t and v

we can find the terminal velocity when dv/dt = 0

$$\rightarrow \quad 0 = mg - bv \quad \rightarrow \quad \boxed{v_T = \frac{mg}{b}}$$

note how it increases with m and g increasing, but decreases as b increases. very plausible behavior.

we can use separation of variables

$$m\frac{dv}{dt} = mg - bv \quad \rightarrow \quad \underbrace{\frac{m\,dv}{mg-bv}}_{\text{only } v \text{ and constants}} = \underbrace{dt}_{\text{only } t}$$

Initial values: $v(t=0) = 0,\ y(t=0) = 0$

we integrate
$$\int_0^v \frac{m\,dv'}{mg - bv'} = \int_0^t dt'$$

prime variables are dummy integration variables
Limits have to correspond

$$\rightarrow \quad -\frac{m}{b} \ln\{mg - bv'\}\Big|_0^v = t - 0$$

$$\rightarrow \quad -\frac{m}{b}\Big[\ln\{mg - bv\} - \ln\{mg\}\Big] = t$$

use $$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

to obtain

$$-\frac{m}{b}\ln\left[1-\frac{bv}{mg}\right] = t$$

$$\rightarrow \quad -\frac{m}{b}\ln\left\{1-\frac{v}{v_T}\right] = t$$

$$\rightarrow \quad \underbrace{\ln\left\{1-\frac{v}{v_T}\right] = -\frac{bt}{m}}_{\text{dimensionless...}}$$

take exponential of both sides

$$\rightarrow \quad 1 - \frac{v}{v_T} = \exp\left[-\frac{bt}{m}\right]$$

$$\rightarrow \quad \frac{v}{v_T} = -\exp\left[-\frac{bt}{m}\right] + 1$$

$$\rightarrow \quad \boxed{v(t) = v_T\left[1 - \exp\left[-\frac{bt}{m}\right]\right]}$$

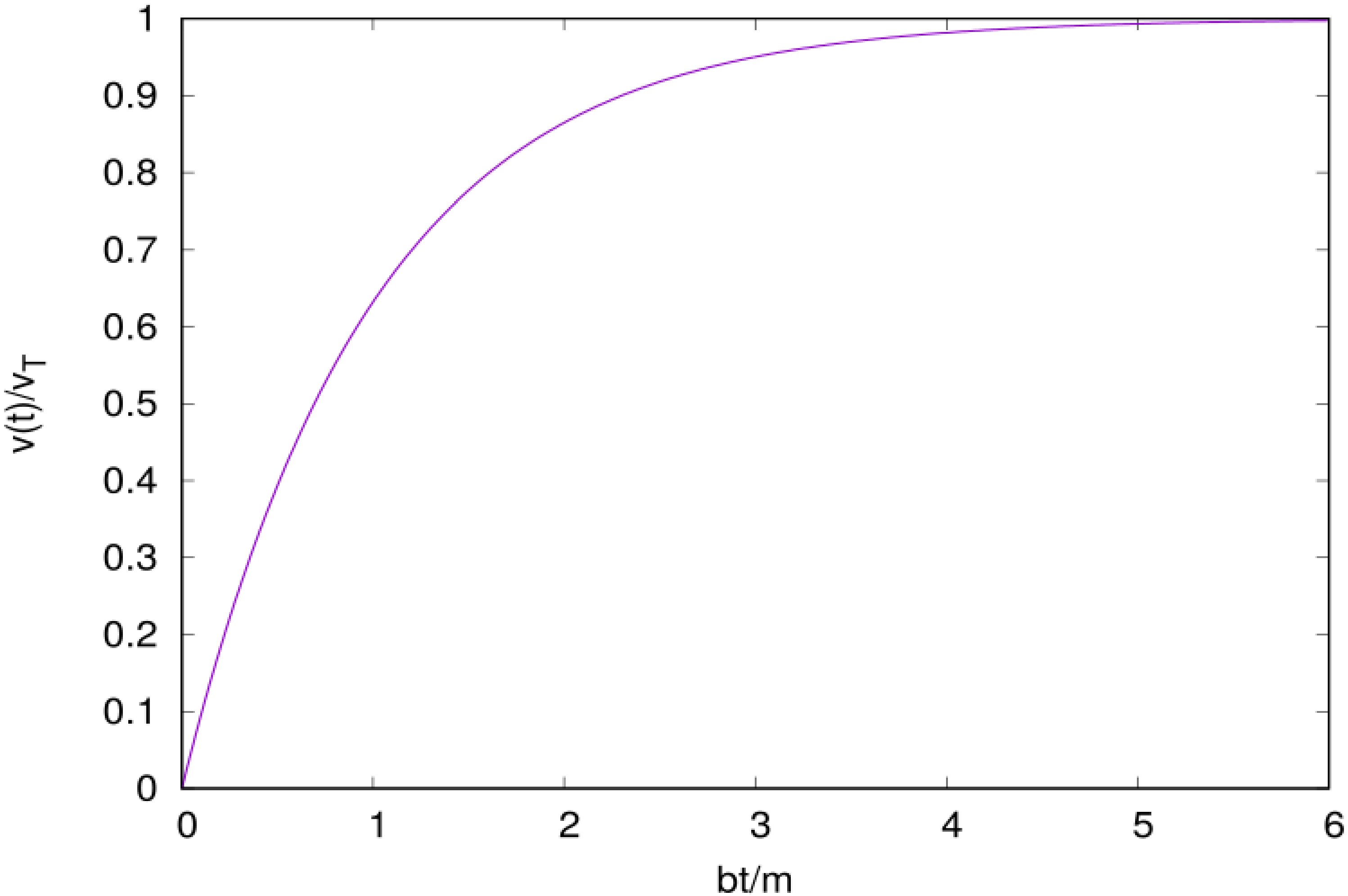
we see two characteristic scales:

$$v_T, \quad [v_T] = \frac{L}{T}, \qquad \frac{b}{m}, \quad \left[\frac{b}{m}\right] = \frac{1}{T}$$

and indeed:

$$v(t) \longrightarrow v_T \quad \text{when} \quad \frac{tb}{m} \gg 1$$

$$\text{or} \quad \text{“}t \to \infty\text{”}$$



markhraăi -
terminal velocity

Dimensionless
variables on the axes

Position

$$v(t) = \frac{dy}{dt} = v_T\left[1 - e^{-\frac{bt}{m}}\right]$$

$$\rightarrow \quad dy = v_T\left[1 - e^{-bt/m}\right]dt$$

integrate

$$\int_0^y dy' = \int_0^t v_T\left[1 - e^{-bt'/m}\right]dt'$$

$$\rightarrow \quad y - 0 = v_T \cdot t + \frac{m v_T}{b}\left[e^{-\frac{bt}{m}} - 1\right]$$

so we get

$$\boxed{y = v_T \left[t + \frac{m}{b} \left(e^{-\frac{bt}{m}} - 1 \right) \right]}$$

and we note that the dimensions add up

When $t \gg\!> \frac{m}{b}$ we get

$$y(t) \longrightarrow \underline{v_T t}$$