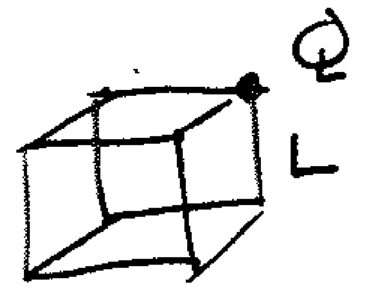
Haustpróf 1999 E-II

(1)



Allt flæðið fer í um $\underline{8}$ teninga í hverjum teningi er flæðið um $\underline{3}$ hliðar.

flæðið er „eins" um hliðarnar

flæði um eina „fjarlæga" hlið, 0 um hinar

$$\Phi = \frac{1}{8}\cdot\frac{1}{3}\frac{Q}{\epsilon_0}$$

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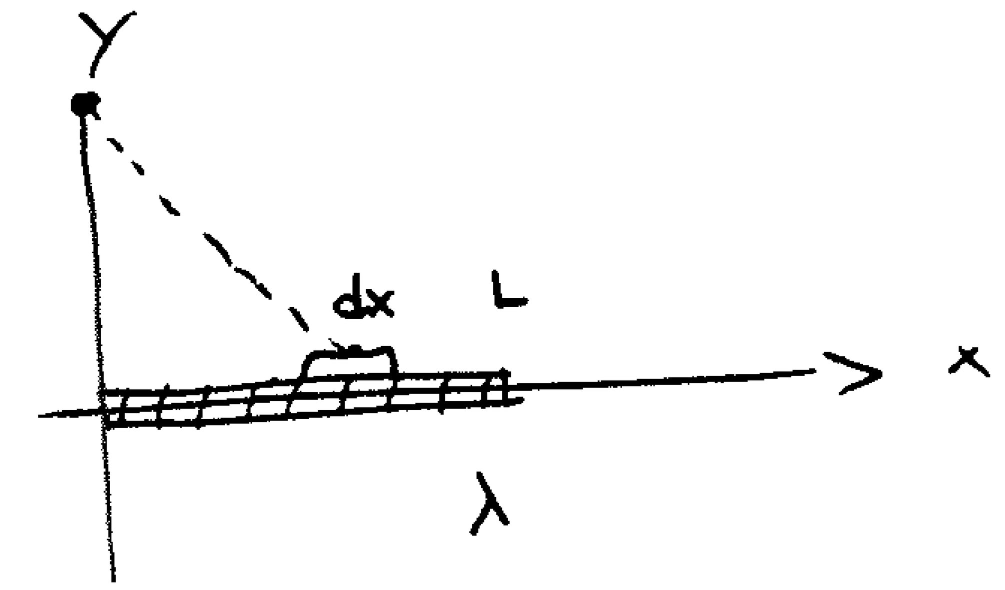
(2)



$$dV = k\frac{\lambda dx}{\sqrt{y^2+x^2}}$$

$$V(y) = k\lambda\int_0^L \frac{dx}{\sqrt{y^2+x^2}} = k\lambda\left\{\ln\left(x+\sqrt{y^2+x^2}\right)\right\}\Big|_0^L$$

$$= k\lambda\left\{\ln\left(L+\sqrt{y^2+L^2}\right)-\ln y\right\}$$

$$V(y) = k\lambda\ln\left\{\frac{L}{y}+\sqrt{1+\frac{L^2}{y}}\right\}$$

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(3)

$$R = \rho\frac{L}{A} = \frac{\rho L}{\pi(b^2-a^2)}$$

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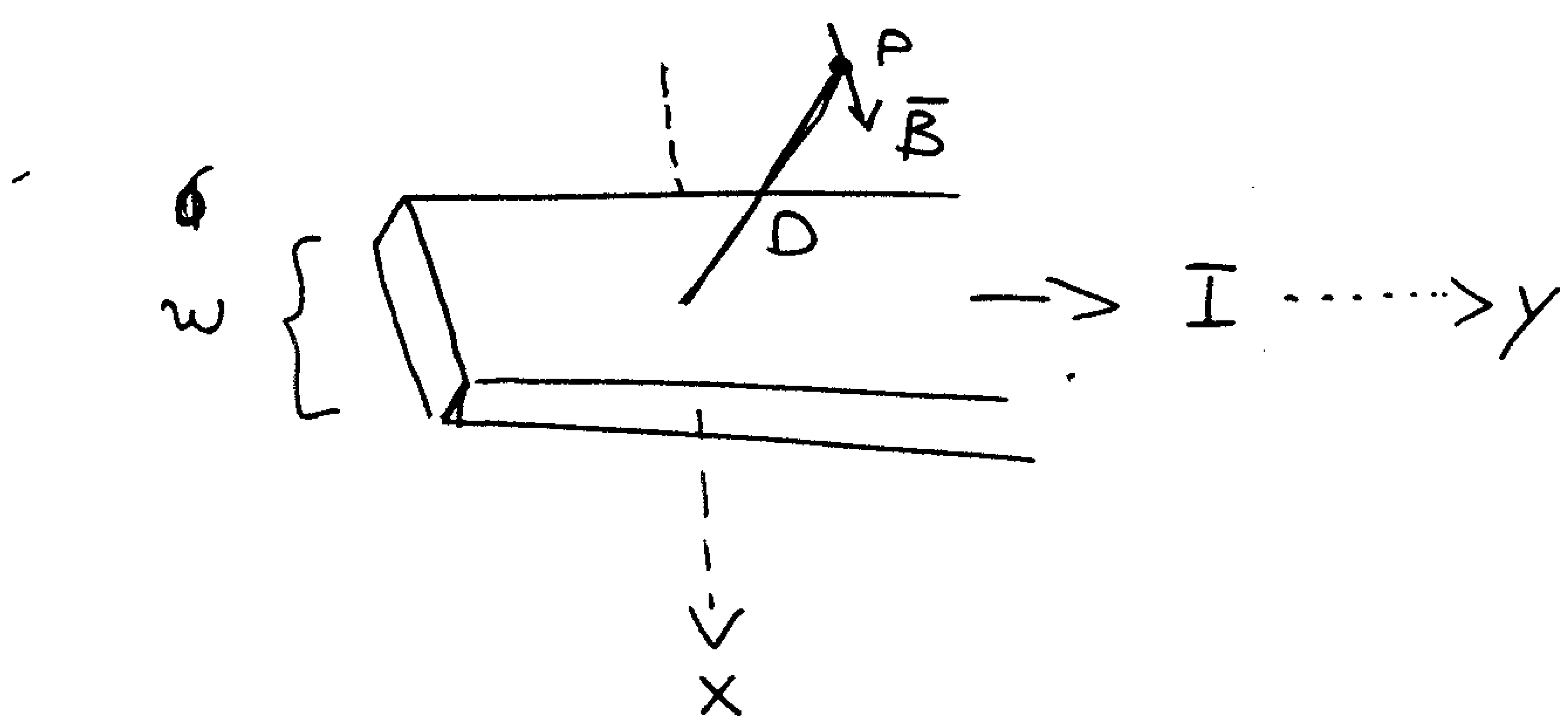
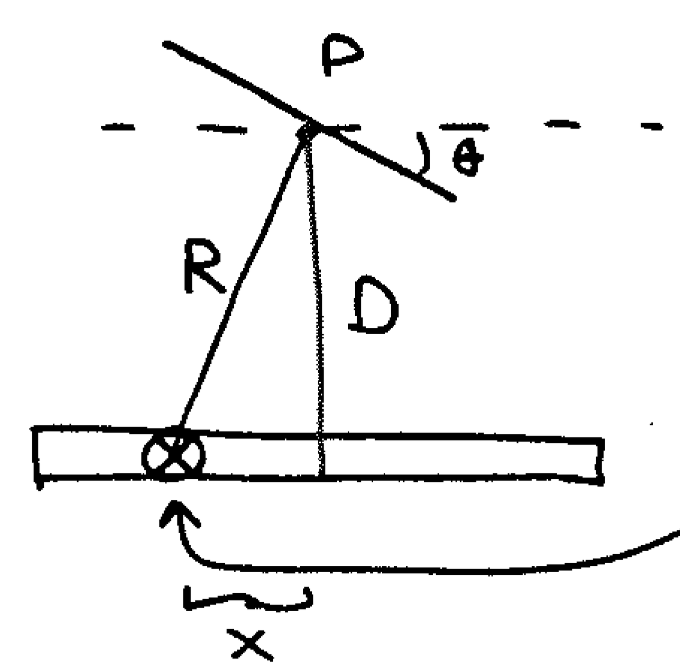
(4)



Fyrir óendanlegan vír gildir

$$B = \frac{\mu_0 I}{2\pi R}$$

því er hér



$$R^2 = x^2 + D^2$$

$$dI = I\frac{dx}{w}$$

$$dB_x = \frac{\mu_0 dI}{2\pi R}\cos\theta = \frac{\mu_0 dI}{2\pi R}\left(\frac{D}{R}\right)$$

$$= \frac{\mu_0 I \frac{dx}{w} D}{2\pi (x^2 + D^2)}$$

$$\rightarrow B_x = \frac{\mu_0 I D}{2\pi w}\int_{-\frac{1}{2}w}^{\frac{1}{2}w} \frac{dx}{x^2 + D^2}$$

$$= \frac{\mu_0 I}{2\pi w}\int_{-\frac{w}{2D}}^{\frac{w}{2D}} \frac{dz}{1 + z^2}$$

$$= \frac{\mu_0 I}{2\pi w}\left\{\arctan\left(\frac{w}{2D}\right) - \arctan\left(-\frac{w}{2D}\right)\right\}$$

$$= \frac{\mu_0 I}{\pi w}\arctan\left(\frac{w}{2D}\right)$$

5

Hér þurfa þau að fista upp af jöfnuninni eða reikna út hvað gerist þegar rofkladan er tekin úr rásinni og straumur flytur um R

$$I = I_0 e^{-t/\tau} \qquad \tau = \frac{L}{R}$$

$$I_0 = \frac{\mathcal{E}}{R}$$

í viðnáminu eyðist afl

$$P = I^2 R = I_0^2 R e^{-2t/\tau}$$

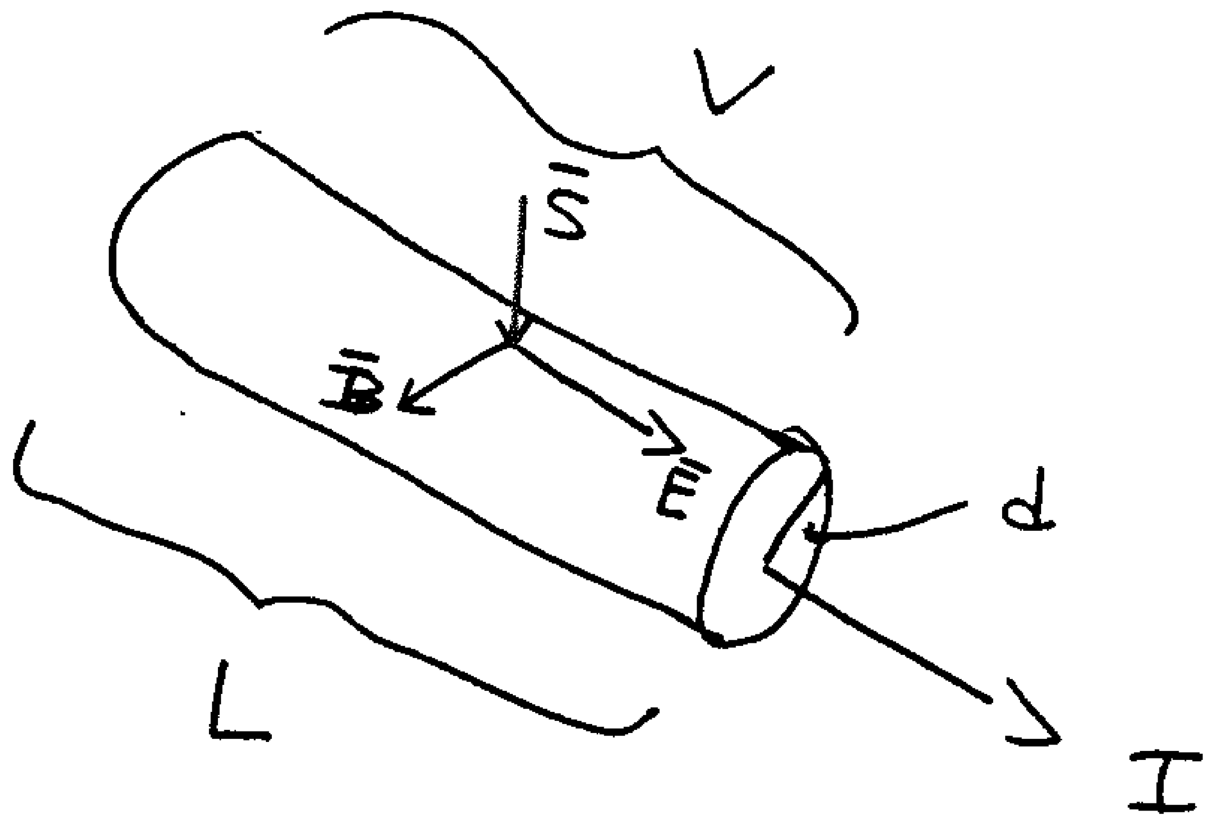
orkan sem eyðist er þá

$$E = \int_0^\infty dt\, P(t) = \int_0^\infty I_0^2 R\, dt\, e^{-2t/\tau}$$

$$= L\frac{I_0^2}{2}$$

Sem er sama og orkan í spólunni klukkan $t = 0$

6



$$V = IR \rightarrow I = \frac{V}{R}$$

a) $P = \frac{V^2}{R}$

b) sjá stefnu á teikningu

$$|S| = \frac{EB}{\mu_0} = \frac{1}{\mu_0}\left(\frac{V}{L}\right)\left(\frac{\mu_0 I}{\pi d}\right)$$

$$= \frac{1}{\mu_0}\frac{V}{L}\frac{\mu_0 V}{R\pi d} = \frac{V^2}{LR\pi d}$$

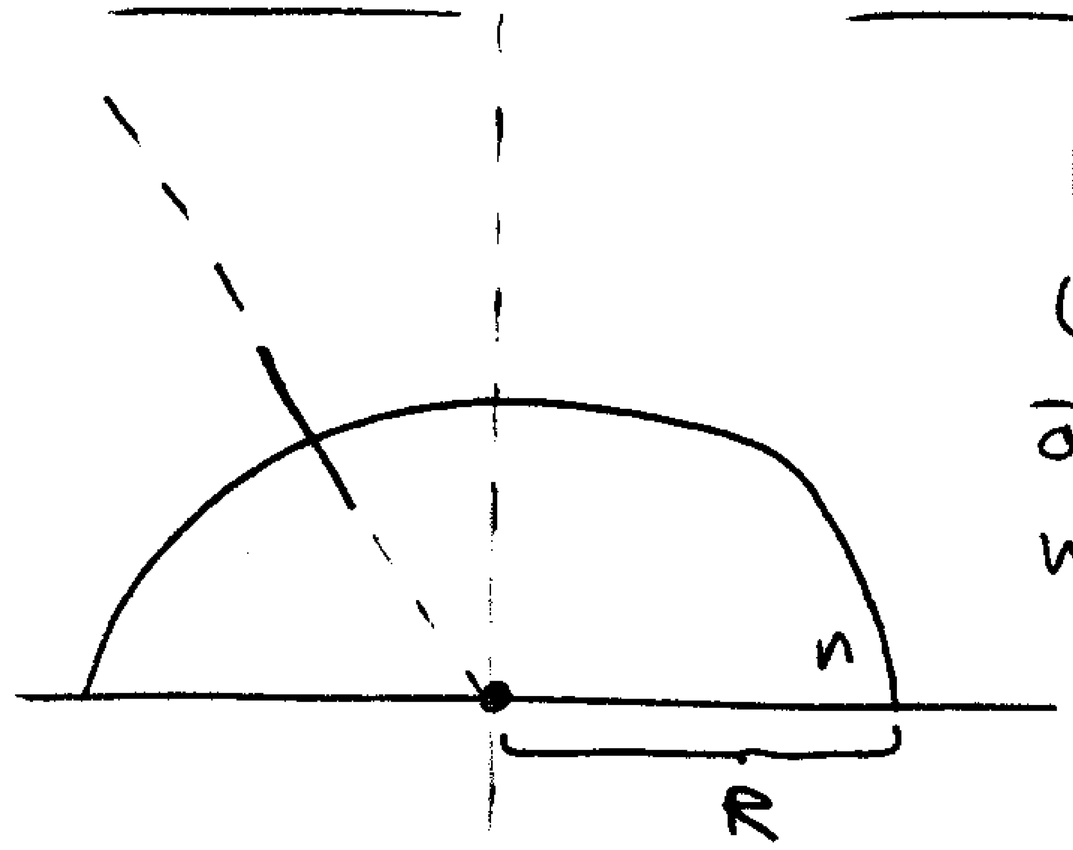
c) P vegna flæðis S

$$P = S \cdot A = S(\pi d L) = \frac{V^2 \pi d L}{L R \pi d}$$

$$= \frac{V^2}{R}$$

sama og í a)

7



þetta er eina lausnin sem ég á eftir að kynna mér betur

$$\frac{n}{q'} + \frac{1}{q} = \frac{1-n}{-R}$$

R

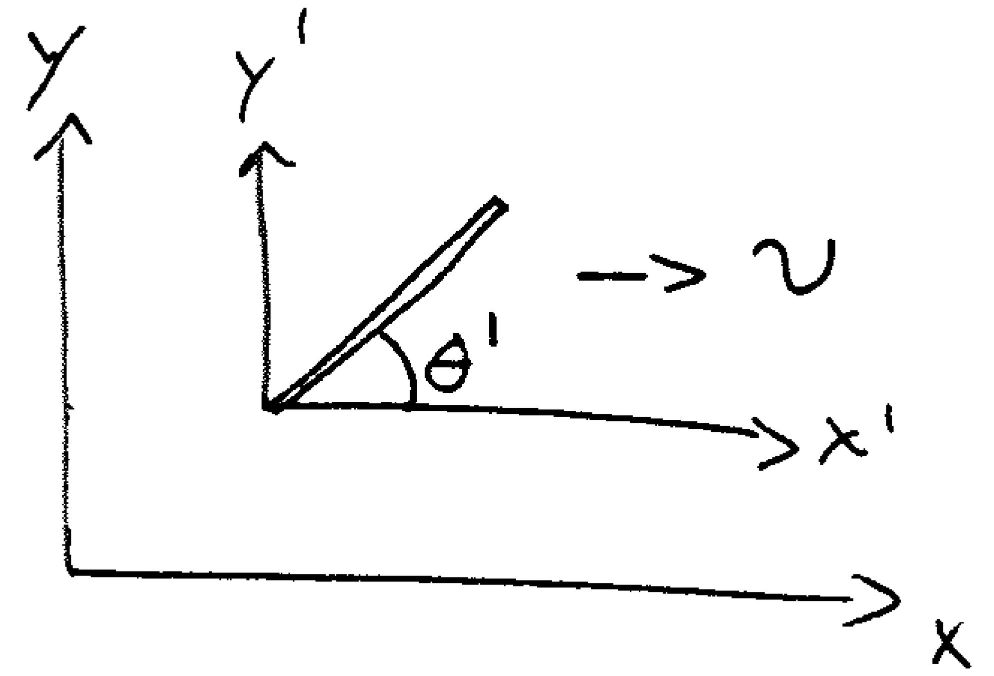
$$\frac{n}{R} + \frac{1}{q} = \frac{n-1}{R}$$

$$\rightarrow \frac{1}{q} = \frac{n-1}{R} - \frac{n}{R} = -\frac{1}{R}$$

$$\rightarrow q = -R$$

8

7



a) $\tan\theta' = \frac{dy'}{dx'}$ $\qquad \tan\theta = \frac{dy}{dx}$

en $dy = dy'$

$\rightarrow \tan\theta = \frac{dy'}{dx} = \frac{dx'}{dx}\tan\theta'$

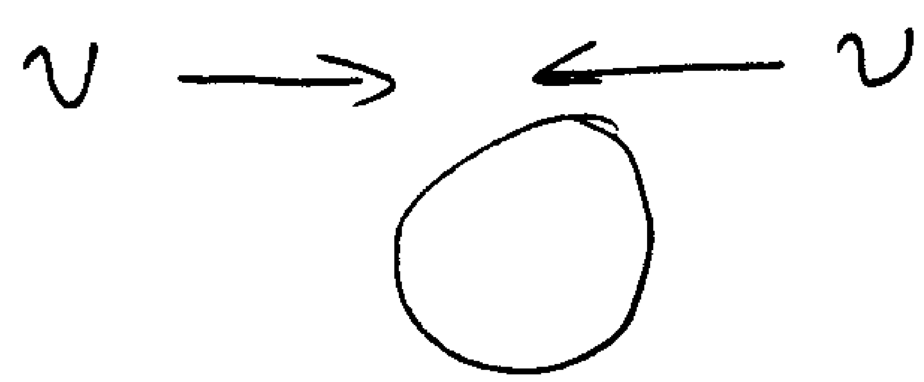
$= \gamma \tan\theta'$

b) $L^2 = L_x^2 + L_y^2 = \left(L_0 \frac{\cos\theta'}{\gamma}\right)^2 + (L_0 \sin\theta')^2$

$= L_0^2 \left\{1 - \left(\frac{v}{c}\right)^2 \cos^2\theta'\right\}$

9

8



finna v et v_r
e pelt

$$v_r = \frac{2v}{1 + \frac{v^2}{c^2}}$$

$\rightarrow \left(1 + \frac{v^2}{c^2}\right) v_r = 2v$

$\rightarrow v_r + \frac{v^2}{c^2} v_r = 2v.$

$\rightarrow v^2 - v \cdot \frac{c^2}{v_r} + c^2 = 0$

$\rightarrow v = \dots$

(10)

E_{kin} , m_0

a) $E_{total} = E_{kin} + m_0 c^2$

b) $E_t^2 = p^2c^2 + m_0^2 c^4$

$\rightarrow p^2c^2 = E_t^2 - m_0^2c^4$

$= (E_{kin} + m_0c^2)^2 - m_0^2c^4$

$= E_{kin}^2 + 2m_0c^2 E_{kin}$

$\rightarrow p = \frac{1}{c}\sqrt{E_{kin}^2 + 2m_0c^2 E_{kin}}$

$= \frac{E_{kin}}{c}\sqrt{1 + \frac{2m_0c^2}{E_{kin}}}$

c) Nu gælder $E_t = \gamma m_0 c^2 = E_{kin} + m_0 c^2$

$\rightarrow \gamma = \frac{E_{kin} + m_0c^2}{m_0c^2} = \frac{1}{\sqrt{1 - v^2/c^2}}$

(9)

$\rightarrow \left(\frac{E_{kin} + m_0c^2}{m_0c^2}\right)^2 = \frac{1}{1 - v^2/c^2}$

$\rightarrow \left(\frac{m_0c^2}{E_{kin} + m_0^2c^2}\right)^2 = 1 - \frac{v^2}{c^2}$

$\rightarrow \frac{v^2}{c^2} = 1 - \left(\frac{m_0c^2}{E_{kin} + m_0^2c^2}\right)^2$

(10)